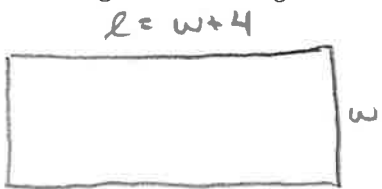


Section 4.5 – Practice Problems

1. The length of a rectangle is 4m more than the width. The area is  $320m^2$ . Find the length and the width.



$$A = lw$$

$$A = (w+4)w$$

$$320 = w^2 + 4w$$

$$w^2 + 4w - 320 = 0$$

$$(w+20)(w-16)$$

$w = -20$  ← reject negative length  
 $w = 16$

$w = 16$   
 $l = 16 + 4 = 20$

2. Find two consecutive odd whole numbers such that the sum of their squares is 130.

Let  $x+1$  be an odd #  
 $x+3$  is then the next consecutive one

$$(x+1)^2 + (x+3)^2 = 130$$

$$x^2 + 2x + 1 + x^2 + 6x + 9 - 130 = 0$$

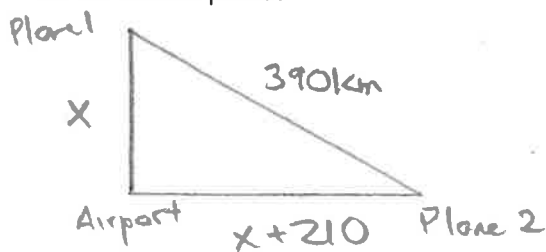
$$2x^2 + 8x - 120 = 0$$

$$x^2 + 4x - 60 = 0$$

$$(x+10)(x-6) = 0$$

$x = -10$  numbers are  $-9$  and  $-7$  ← reject not whole numbers  
 $x = 6$  numbers are  $7$  and  $9$

3. Two planes travel at right angles to each other after leaving an airport at the same time; 1 hour later, they are 390km apart. If one plane travels 210km/h faster than the other, what is the speed of the slower plane?



Let  $x$  be the speed of plane 1  
 $x + 210$  " " " plane 2

Use Pythagorean Theorem

$$x^2 + (x+210)^2 = 390^2$$

$$x^2 + x^2 + 420x + 44100 - 152100 = 0$$

$$2x^2 + 420x - 108000 = 0$$

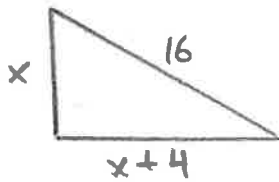
$$x^2 + 210x - 54000 = 0$$

$$(x+360)(x-150) = 0$$

$x = -360$  ← reject negative  
 $x = 150$

Slower plane is 150km/hr

4. The hypotenuse of a right triangle is 16cm long. One leg is 4cm longer than the other. Find the length of the legs.



$$x^2 + (x+4)^2 = 16^2$$

$$x^2 + x^2 + 8x + 16 - 256 = 0$$

$$2x^2 + 8x - 240 = 0$$

$$x^2 + 4x - 120 = 0$$

$$\frac{-4 \pm \sqrt{16 - 4(1)(-120)}}{2} = \frac{-4 \pm \sqrt{496}}{2}$$

$$\frac{-4 \pm 4\sqrt{31}}{2}$$

$$-2 \pm 2\sqrt{31}$$

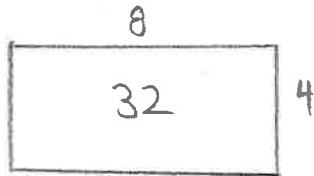
reject negative

$$-2 + 11.13$$

$$x = 9.13$$

$$x + 4 = 13.13$$

5. The length and width of a rectangular sheet of plywood is 4ft by 8ft. How much must be added equally to the length and width to double the area?



so  $(8+x)(4+x) = 64$

$$32 + 12x + x^2 = 64$$

$$x^2 + 12x - 32 = 0$$

$$\frac{-12 \pm \sqrt{144 - 4(1)(-32)}}{2}$$

$$\rightarrow \frac{-12 \pm \sqrt{272}}{2} = \frac{-12 \pm 4\sqrt{17}}{2} = -6 \pm 2\sqrt{17}$$

$$2.25 \text{ ft}$$

reject negative

6. A boat takes 1 hour longer to go 36km up a river than to go down the river. If the boat travels 15km/hr in still water, what is the speed of the current?

Let  $x$  be speed of current: Speed with current  $15+x$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\textcircled{1} \quad 15+x = \frac{36}{t_1} \rightarrow t_1 = \frac{36}{15+x}$$

$$\textcircled{2} \quad 15-x = \frac{36}{t_2} \rightarrow t_2 = \frac{36}{15-x}$$

$$t_2 - t_1 = 1$$

$$\frac{36}{15-x} - \frac{36}{15+x} = (15+x)(15-x)$$

$$36(15+x) - 36(15-x) = 225 - x^2$$

$$540 + 36x - 540 + 36x = 225 - x^2$$

$$x^2 + 72x - 225 = 0$$

$$(x+75)(x-3) = 0$$

$$x = -75 \leftarrow \text{reject}$$

$$x = 3 \checkmark$$

$$\text{current is } 3 \text{ km/hr}$$

7. The school play charges \$10 for admission, and on average 80 people attend. For each \$1 increase, attendance drops by 5 people. What price should the school charge to maximize revenue?

$(10+x)(80-5x)$  when  $x$  increases price goes up by 1 and # of people go down by 5

When finding maximum we want the vertex.

$$800 - 50x + 80x - 5x^2 = 0$$

$$-5x^2 + 30x + 800 = 0 \rightarrow -(x^2 - 6x - 160) = 0$$

$$-(5x^2 - 30x - 800) = 0 \rightarrow -(x^2 - 6x + 9 - 9 - 160) = 0$$

$$-(x^2 - 6x + 9 - 169) = 0$$

$-(x-3)^2 - 169 = 0$   
 $x = 3$   
 so  

ticket price
\$13

8. The sum of two integers is 10, and the sum of their squares is a minimum. Find the two integers.

$$x + y = 10 \rightarrow y = 10 - x$$

$$x^2 + y^2 = \min$$

$$x^2 + (10-x)^2 = \min$$

$$x^2 + 100 - 20x + x^2 = \min$$

$$2x^2 - 20x + 100 = \min$$

vertex

$$2(x^2 - 10x) + 100$$

$$2(x^2 - 10x + 25 - 25) + 100$$

$$2(x-5)^2 + 50$$

$x = 5$
$y = 5$

9. Djuna takes 4 hours to weed the garden alone, and Soo takes 6 hours to do the same job. How long does it take them together?

Full job takes Djuna 4 hours so  $\frac{1}{4}$  job in 1 hr.

" " " Soo 6 hours so  $\frac{1}{6}$  job in 1 hr

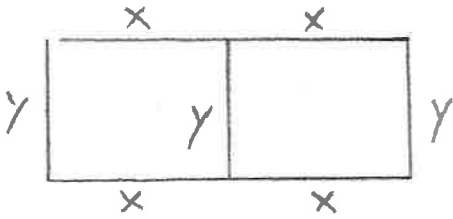
Together in one hour

they can do:  $\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$  of the job

$$\frac{5}{12} \cdot x = 1 \text{ job}$$

$$x = \frac{12}{5} \text{ hours} = 2 \frac{4}{5} = \boxed{2 \text{ hours } 24 \text{ mins}}$$

10. A ranch uses 200m of fencing to enclose two adjacent rectangular corrals. Find the dimensions that enclose a total area of 1400m<sup>2</sup>. (Drawings help)



$$4x + 3y = 200m \rightarrow y = -\frac{4}{3}x + \frac{200}{3}$$

$$2xy = 1400 \rightarrow y = \frac{700}{x}$$

$$2x \left( -\frac{4}{3}x + \frac{200}{3} \right) = 1400$$

$$-\frac{8}{3}x^2 + \frac{400x}{3} = 1400$$

$$8x^2 - 400x = -4200$$

$$8x^2 - 400x + 4200 = 0$$

$$x^2 - 50x + 525 = 0$$

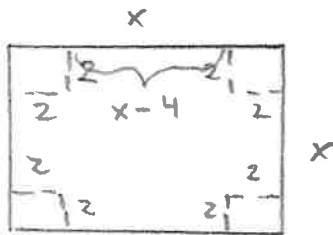
$$(x - 35)(x - 15) = 0$$

$$x = 35 \rightarrow \boxed{70 \text{ by } 20}$$

$$x = 15 \rightarrow \boxed{30 \text{ by } 46\frac{2}{3}}$$

$$525 \begin{matrix} \wedge \\ 5 \end{matrix} \begin{matrix} \wedge \\ 105 \end{matrix} \begin{matrix} \wedge \\ 5 \end{matrix} \begin{matrix} \wedge \\ 21 \end{matrix} \begin{matrix} \wedge \\ 3 \end{matrix} \begin{matrix} \wedge \\ 7 \end{matrix}$$

11. From each corner of a square piece of cardboard, a square with sides of 2cm is removed. The edges are then turned up to form an open box. If the box is to hold 200cm<sup>3</sup>, what are the dimensions of the original piece of cardboard? (Drawings help)



$$(x-4)(x-4)(2) = 200$$

$$(x^2 - 8x + 16)(2) = 200$$

$$x^2 - 8x + 16 = 100$$

$$x^2 - 8x - 84 = 0$$

$$(x - 14)(x + 6) = 0$$

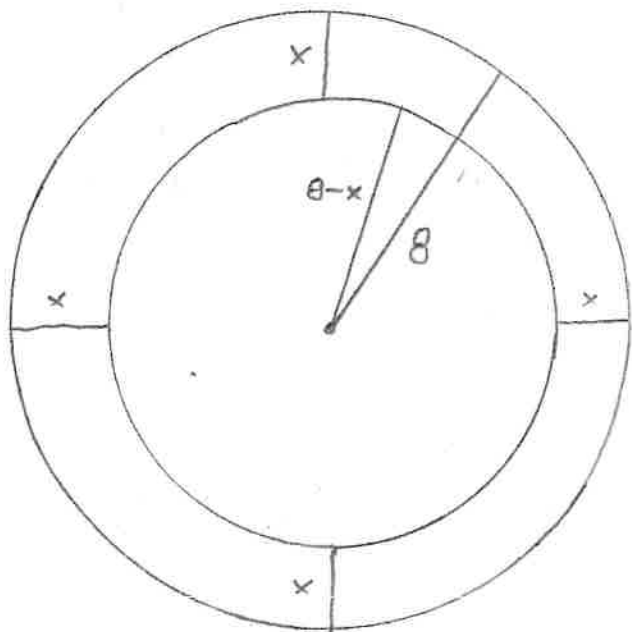
$$x = 14$$

$$x = -6 \leftarrow \text{reject negative length}$$

$$84 \begin{matrix} \wedge \\ 2 \end{matrix} \begin{matrix} \wedge \\ 42 \end{matrix} \begin{matrix} \wedge \\ 6 \end{matrix} \begin{matrix} \wedge \\ 7 \end{matrix} \begin{matrix} \wedge \\ 2 \end{matrix} \begin{matrix} \wedge \\ 3 \end{matrix}$$

Original dimensions  
14cm x 14cm

12. A circular lawn is surrounded by a flower bed of uniform width. If the flower bed has an area of  $36m^2$  and the radius of the entire garden is  $8m$ , find the width of the flower bed. (Drawings help)



$$A \text{ of Big} = \pi 8^2$$

$$A \text{ of Small} = \pi(8-x)^2$$

0.7514m

$$\pi 8^2 - \pi(8-x)^2 = 36$$

$$64\pi - \pi(64 - 16x + x^2) - 36 = 0$$

$$64\pi - 64\pi + 16\pi x - \pi x^2 - 36 = 0$$

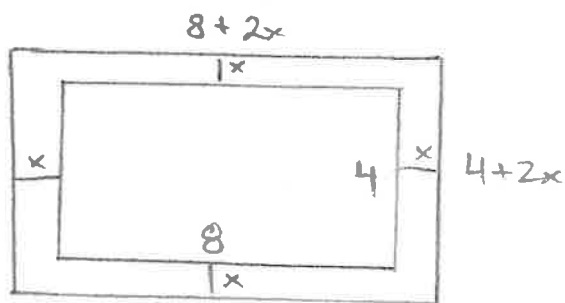
$$-x^2\pi + 16\pi x - 36 = 0$$

$$x^2 - 16x + \frac{36}{\pi} = 0$$

Reject

$$\frac{16 \pm \sqrt{(-16)^2 - 4(1)(\frac{36}{\pi})}}{2} = \frac{16 \pm \sqrt{256 - 46.8}}{2} \rightarrow \frac{16 \pm 14.5}{2} = 0.7514 \text{ and } 15.25$$

13. A gardener surrounds a  $4m \times 8m$  rectangular flower bed with a border of mulch of uniform width. If there is enough mulch to cover  $28m^2$ , how wide is the border? (Drawings help)



$$(8+2x)(4+2x) - 32 = 28 \quad (8 \cdot 4)$$

$$32 + 16x + 8x + 4x^2 - 32 - 28 = 0$$

$$4x^2 + 24x - 28 = 0$$

$$x^2 + 6x - 7 = 0$$

$$(x+7)(x-1) = 0$$

$$x = -7 \leftarrow \text{reject negative length}$$

$$x = 1$$

Border = 1m wide

14. Mahaila paddles 5km/h in still water. It takes her 1 hour longer to paddle 12km upstream than to make the same trip downstream. Find the speed of the current.

$D = 12\text{km}$

Let  $x$  be speed of current

upstream  $5 - x$

downstream  $5 + x$

$t = \frac{D}{S}$

$t_1 - t_2 = -1$   
 ↑            ↑  
 up         down

$\frac{12}{5-x} - \frac{12}{5+x} = -1$

$12(5+x) - 12(5-x) = (5+x)(5-x)$

$60 + 12x - 60 + 12x = 25 - x^2$

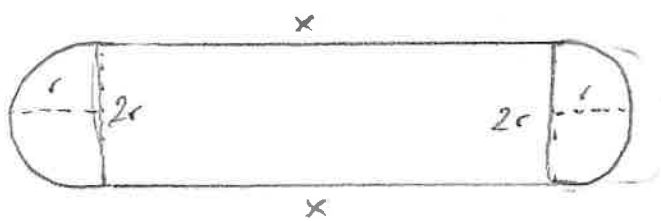
$24x - 25 + x^2 = 0$

$x^2 + 24x - 25 = 0$

$(x + 25)(x - 1)$   
 $x = -25$   
 $x = 1$

Speed of current  
 1 km/h

15. The standard running track size for track and field events is 400m. The track consists of two semi-circles connected by parallel straight lanes. If the infield of the track encloses an area of 9430m<sup>2</sup>, find the length of the straight lanes and the diameter of the track. (Drawings help)



Circumference of circle:  $2\pi r$

Area of Circle:  $\pi r^2$

Total Area:  $2rx + \pi r^2$

$2r(200 - \pi r) + \pi r^2 = 9430$

$400r - 2\pi r^2 + \pi r^2 - 9430 = 0$

$-\pi r^2 + 400r - 9430 = 0$

$\pi r^2 - 400r + 9430 = 0$

$400 \pm \sqrt{(-400)^2 - 4(\pi)(9430)} = r$

$400 \pm \sqrt{160000 - 118500} = r$

Distance:  $2x + 2\pi r = 400$

$x + \pi r = 200$

$x = 200 - \pi r$

$r = \frac{400 \pm 203.7}{2\pi}$

$r = 31.24$

$r = 96.13 \leftarrow \text{reject}$

width:  $2(31.24)$

$= 62.48\text{m}$

straight:  $200 - \pi(31.24)$

$= 101.86\text{m}$