Section 4.5 – Applications of Quadratics Functions and Equations

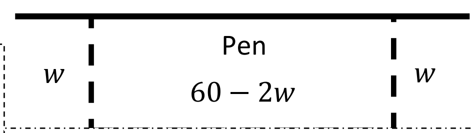
This Booklet Belongs to: ________________ Block: ______

- If we can maximize or minimize the quadratic we can solve many types of problems
- Solving for the vertex, and knowing if the vertex is a maximum or a minimum is the key to solving quadratic formula problems

Example 1:

A rectangular pen is to be built along the side of a barn. Find the maximum area that can be enclosed with $60m$ of fencing if the barn is one side of the enclosure.

Solution 1:



Let $w = width\ of\ the\ pen$

Then $60 - 2w = length\ of\ the\ pen$

$$A = w(60 - 2w) \quad \rightarrow \quad A = 60w - 2w^2$$

$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{60}{2(-2)}, 0 - \frac{(60)^2}{4(-2)}\right)$$

$$= (\mathbf{15}, \mathbf{450})$$

So the area is a maximum at the vertex.

$$w = 15\ and\ A = 450m^2$$

$$l = 60 - 2(15) = 30$$

$$A = l \cdot w = 15 \cdot 30 = 450m^2$$

Example 2:

Mary stands on the top of a building and fires a gun upwards. The bullet travels according to the equation $h = -16t^2 + 384t + 50$, where h is the height of the bullet off the ground in metres at t seconds after it was fired.

a) How far is Mary above the ground when she fires the gun?
b) What is the bullet's maximum height above the ground?
c) How long does it take for the bullet to reach its greatest height?

Solution 2:

a) When she fires the gun, $t = 0$

$h(0) = -16(0)^2 + 384(0) + 50$, so when $t = 0, \boldsymbol{h = 50m}$

b) $Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{384}{2(-16)}, 50 - \frac{(384)^2}{4(-16)}\right) \qquad = (\mathbf{12}, \mathbf{2354})$

So the height is a maximum at the vertex, $\boldsymbol{h = 2354\ meters}$

c) So the height is a maximum at the vertex. $\boldsymbol{t = 12\ seconds}$

Example 3:

Bob's Rent-a-Wreck rents 300 $cars\ at$ \$40 per day. For each \$1 increase in cost of renting, 5 fewer cars are rented. For what rate should the cars be rented to produce the maximum income, and what is that income?

Solution 3:

- Let $R(x) = income\ from\ renting\ cars$
- If there is no change in rates, then $R(x) = 40 \cdot 300 = \$12\,000$ in income
- Let $x = increase\ in\ rate\ (in\$)$
- The new cost of renting a car is $(40 + x)$
- The number of cars rented is $(300 - 5x)$

$R(x) = (40 + x)(300 - 5x) \quad = 12000 - 200x + 300x - 5x^2 \quad = \mathbf{-5x^2 + 100x + 12000}$

$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{(100)}{2(-5)}, 12000 - \frac{(100)^2}{4(-5)}\right) \qquad = (\mathbf{10}, \mathbf{12\,500})$$

So the income is a maximum at the vertex, $\boldsymbol{x} = \mathbf{10}, \$\mathbf{12500}$

The maximum Income of $\$\mathbf{12\,500}$ occurs when $x = 10$, so the cars should be rented for $(40 + 10) = \$\mathbf{50}$

Example 4:

Find two numbers whose **difference** is 100 and the **sum of whose squares** is minimum.

Solution 4:

- Let $x = larger\ number$
- Let $y = smaller\ number$
- Then $x - y = 100 \quad \rightarrow \quad x = y + 100$

$$Sum = s = x^2 + y^2$$

$$= (y + 100)^2 + y^2$$

$$= y^2 + 200y + 10000 + y^2$$

$$2y^2 + 200y + 10\,000 \quad \rightarrow \quad 2(y^2 + 100y + 5000)$$

$$y = -\frac{b}{2a} = \frac{-100}{2(1)} = -50$$

$$x - y = 100 \quad \rightarrow \quad x - (-50) = 100 \quad \rightarrow \quad x = 50$$

The two numbers are : $\mathbf{50\ and - 50}$

Example 5:

The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number.

Solution 5:

- Let $x = the\ number$
- Then $\frac{1}{x}$ is $the\ reciprocal\ of\ a\ number$

The numbers can be:

$$\frac{1}{2} \ \ or \ \ 4$$

$x + \frac{2}{x} = \frac{9}{2}$

Multiply by LCM

$2x\left(x + \frac{2}{x} = \frac{9}{2}\right)$

Re-arrange

$2x^2 + 4 = 9x$

AC-Method

$2x^2 - 9x + 4 = 0$

Factor

$x^2 - 9x + 8 \rightarrow (x-1)(x-8)$

$\left(x - \frac{1}{2}\right)\left(x - \frac{8}{2}\right) \rightarrow \left(x - \frac{1}{2}\right)(x-4)$

$\boldsymbol{x = \frac{1}{2}} \ \ \boldsymbol{or} \ \ \boldsymbol{x = 4}$

- Check Answer to Confirm they are valid

Example 6:

Ray and Ann ride a bicycle a distance of $4km$ each morning. They both finish at the same time but Ann starts $1\ minute$ before Ray, and Ray travels $1km/h$ faster than Ann. What speeds are they travelling at?

Solution 6:

- Ann's time − Ray's time is $= 1\ min$ or $(1/60hr)$
- Remember: $Speed = \frac{Distance}{Time}$
- So: $Time = \frac{Distance}{Speed}$

	Speed (km/hr)	*Distance (km)*	*Time (hrs)*
Ann	x	4	$\frac{4}{x}$
Ray	$x+1$	4	$\frac{4}{x+1}$

$\frac{4}{x} - \frac{4}{(x+1)} = \frac{1}{60}$

$60x(x+1)\left(\frac{4}{x} - \frac{4}{(x+1)} = \frac{1}{60}\right)$

$240(x+1) - 240x = x(x+1)$

$240x + 240 - 240x = x^2 + x$

$x^2 + x - 240 = 0 \rightarrow (x+16)(x-15) = 0$

$\boldsymbol{x = \frac{15km}{hr}} \ \ \boldsymbol{or - \frac{16km}{hr}}$

The speeds can be:

$$\boldsymbol{\frac{15km}{hr} \ \ or - \frac{16km}{hr}}$$

But we **reject** $-\frac{16km}{hr}$.

Ann travels at $\frac{15km}{hr}$ and Ray at $\frac{16km}{hr}$

Section 4.5 – Practice Problems

1. The length of a rectangle is $4m$ more than the width. The area is $320m^2$. Find the length and the width.

2. Find two consecutive odd whole numbers such that the sum of their squares is 130.

3. Two planes travel at right angles to each other after leaving an airport at the same time; 1 hour later, they are $390km$ apart. If one plane travels $210km/h$ faster than the other, what is the speed of the slower plane?

4. The hypotenuse of a right triangle is $16cm$ long. One leg is $4cm$ longer than the other. Find the length of the legs.

5. The length and width of a rectangular sheet of plywood is $4ft \ by \ 8ft$. How much must be added equally to the length and width to double the area?

6. A boat takes 1 hour longer to go $36km$ up a river than to go down the river. If the boat travels $15km/hr$ in still water, what is the speed of the current?

7. The school play charges $\$10$ for admission, and on average 80 people attend. For each $\$1$ increase, attendance drops by 5 people. What price should the school charge to maximize revenue?

8. The sum of two integers is 10, and the sum of their squares is a minimum. Find the two integers.

9. Djuna takes $4\ hours$ to weed the garden alone, and Soo takes $6\ hours$ to do the same job. How long does it take them together?

10. A ranch uses $200m$ of fencing to enclose two adjacent rectangular corrals. Find the dimensions that enclose a total area of $1400m^2$. (Drawings help)

11. From each corner of a square piece of cardboard, a square with sides of $2cm$ is removed. The edges are then turned up to form an open box. If the box is to hold $200cm^3$, what are the dimensions of the original piece of cardboard? (Drawings help)

12. A circular lawn is surrounded by a flower bed of uniform width. If the follower bed has an area of $36m^2$ and the radius of the entire garden is $8m$, find the width of the flower bed. (Drawings help)

13. A gardener surrounds a $4m\ x\ 8m$ rectangular flower bed with a border of mulch of uniform width. If there is enough mulch to cover $28m^2$, how wide is the border? (Drawings help)

14. Mahaila paddles $5km/h$ in still water. It takes her $1\ hour$ longer to paddle $12km$ upstream than to make the same trip downstream. Find the speed of the current.

15. The standard running track size for track and field events is $400m$. The track consists of two semi-circles connected by parallel straight lanes. If the infield of the track encloses an area of $9430m^2$, find the length of the straight lanes and the diameter of the track. (Drawings help)

Answer Key – Section 4.5

1.	$w = 16, l = 20$
2.	$7, 9$
3.	$150 km/hr$
4.	$9.14, 13.14$
5.	$2.25 ft$
6.	$3 km/hr$
7.	$\$13$
8.	$5\ and\ 5$
9.	$2.4\ hours; 2\ hours\ 24\ minutes$
10.	$20m\ x\ 70m\quad or\quad 30m\ x\ \frac{140}{3}m$
11.	$14cm\ x\ 14cm$
12.	$0.7514m$
13.	Border is $1m\ wide$
14.	Speed of the Current is $1 km/h$
15.	$Straight\ Section\ is\ 101.86m$ $Width\ is\ 62.48m$

Extra Work Space