# Section 4.4 – The Quadratics Equation and the Discriminant

**This Booklet Belongs to: ____________ Block: ______**

## Solving Quadratic Equations with the Quadratic Equation

- When quadratics **do not factor easily** we can use the **Quadratic Equation**
- It works every time and will give us **NO SOLUTION** if it cannot be factored

**Quadratic Equation**

The solution to the quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

## Use The Quadratic Equation to Solve the Following Quadratics

**Example 1:** Solve $2x^2 + x - 6 = 0$

**Solution 1:**

$$a = 2, b = 1, c = -6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-6)}}{2(2)}$$

$$x = \frac{-1 \pm \sqrt{49}}{4} \quad \rightarrow \quad x = \frac{-1 \pm 7}{4} \quad \rightarrow \quad x = -\frac{8}{4} and \frac{6}{4}$$

**Solutions are:** $\boldsymbol{-2 \quad and \quad \frac{\sqrt{3}}{2}}$

**Example 2:** Solve $3x^2 + 2x - 4 = 0$

**Solution 2:**

$$a = 3, b = 2, c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{52}}{6} \quad \rightarrow \quad x = \frac{-2 \pm \sqrt{4 \cdot 13}}{6} \quad \rightarrow \quad x = \frac{-2 \pm 2\sqrt{13}}{6}$$

$$x = \frac{2(-1 \pm \sqrt{13})}{6} \quad \rightarrow \quad x = \frac{-1 \pm \sqrt{13}}{3}$$

**Solutions are:** $\boldsymbol{\frac{-1 - \sqrt{13}}{3} \quad and \quad \frac{-1 + \sqrt{13}}{3}}$

**Example 3:** Solve $2x^2 - 3x = -4$

**Solution 3:**

$$Rewrite\ as: 2x^2 - 3x + 4 = 0 \qquad a = 2, b = -3, c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{9 - 32}}{3} \quad \rightarrow \quad x = \frac{3 \pm \sqrt{\mathbf{-21}}}{3}$$

$$x = \emptyset \quad \rightarrow \quad We\ can't\ take\ the\ Square\ Root\ of - 21$$

**Solutions are:** $\boldsymbol{None}$

**Example 4:** Solve $9x^2 - 12x + 4 = 0$

**Solution 4:**

$$9x^2 - 12x + 4 = 0 \qquad a = 9, b = -12, c = 4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \rightarrow \quad x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(9)(4)}}{2(9)}$$

$$x = \frac{12 \pm \sqrt{144 - 144}}{18} \quad \rightarrow \quad x = \frac{12 \pm \sqrt{\mathbf{0}}}{18}$$

$$x = \frac{12}{18} = \frac{2}{3} \quad \rightarrow \quad \boldsymbol{Solution\ is}: \boldsymbol{x} = \frac{\mathbf{2}}{\mathbf{3}}$$

**The Discriminant**

- Solving a quadratic can be predicted by looking at the part under to radical --- **The Discriminant**
- We know consider the answer in the previous examples and compare them to the info below

If $\boldsymbol{b^2 - 4ac > 0}$ **we have 2 real solutions** $\rightarrow$ $rational\ if\ b^2 - 4ac > 0\ is\ a\ perfect\ square$

$\rightarrow$ $irrational\ if\ b^2 - 4ac > 0\ is\ not\ a\ perfect\ square$

If $\boldsymbol{b^2 - 4ac = 0}$ **we have 1 real solutions** $\rightarrow$ $rational\ if\ \dfrac{b}{a}\ is\ rational$

$\rightarrow$ $irrational\ if\ \dfrac{b}{a}\ is\ irrational$

If $\boldsymbol{b^2 - 4ac < 0}$ **we have 0 real solutions**

# Section 4.4 – Practice Problems

Find the Discriminant, use it to determine the number of real roots of each equation.

1. $x^2 - 8x + 16 = 0$

2. $2x^2 - x - 3 = 0$

3. $3x^2 - 4x + 5 = 0$

4. $3x^2 - 5x + 1 = 0$

5. $2x^2 - 6x = 0$

6. $\dfrac{3x-2}{2x-1} = \dfrac{1}{2-x}$

7. $(2x+3)(x-1) = x + 5$

8. $x^2 - 2\sqrt{2}x + 2 = 0$

Determine the value of $k$ so that the equation had the indicated number of solutions

9. $kx^2 + x + k = 0; two\ real\ solutions$

10. $kx^2 + x + k = 0; one\ real\ solution$

11. $x^2 - kx + 4 = 0; two\ real\ solutions$

12. $x^2 - kx + 4 = 0; no\ real\ solutions$

Solve the following questions using the Quadratic Equation. Leave answers in radical form.

13. $x^2 = -4x - 1$

14. $x^2 = 4 - 4x$

15. $x^2 = -4x + 1$

16. $2x^2 = 3x + 1$

17. $\frac{x^2}{4} + \frac{1}{8} = \frac{x}{2}$

18. $8x^2 - 20x - 3 = 0$

19. $\frac{x(2x+1)}{x-2} = \frac{10}{x-2}$

20. $(x-2)(x+4) = 2x(x-3)$

21. $\frac{x^2}{12} + \frac{x}{4} = -\frac{1}{3}$

22. $(x+3)^2 = 6x(x+1)$

Solve using any method.

23. $\frac{x}{3} - 2 = -\frac{3}{x}$

24. $\frac{2x}{3x-1} = \frac{2x-3}{x+1}$

25. $\dfrac{x+2}{x} + \dfrac{x}{x-2} = 5$

26. $\dfrac{2}{x+4} - \dfrac{3}{x+1} = 4$

27. $\sqrt{2x-1} = x-2$

28. $\sqrt{x^2+1} = \sqrt{3x+2}$

29. $\left(\frac{x^2+2}{x}\right)^2 - 6\left(\frac{x^2+2}{x}\right) + 5 = 0$

30. $\left(\frac{x^2+1}{x}\right)^2 + 4\left(\frac{x^2+1}{x}\right) - 12 = 0$

# Answer Key – Section 4.4

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|---|---|
| 1. $D = 0; 1\ real\ rational\ root$ | 23. 3 |
| 2. $D = 25; 2\ real\ rational\ roots$ | 24. $3, \frac{1}{4}$ |
| 3. $D = -44; 0\ real\ roots$ | 25. $\frac{5 \pm \sqrt{13}}{3}$ |
| 4. $D = 13; 2\ real\ irrational\ roots$ | 26. $-2, -\frac{13}{4}$ |
| 5. $D = 36;\ 2\ real\ rational\ roots$ | 27. 5 |
| 6. $D = 0; 1\ real\ rational\ root$ | 28. $\frac{3 \pm \sqrt{13}}{2}$ |
| 7. $D = 64; 2\ real\ rational\ roots$ | 29. $\frac{5 \pm \sqrt{17}}{2}$ |
| 8. $D = 0; 1\ real\ irrational\ roots$ | 30. $1, -3 \pm 2\sqrt{2}$ |
| 9. $-\frac{1}{2} < k < \frac{1}{2}$ | |
| 10. $k = \pm\frac{1}{2}$ | |
| 11. $k < -4, k > 4$ | |
| 12. $-4 < k < 4$ | |
| 13. $-2 \pm \sqrt{3}$ | |
| 14. $-2 \pm 2\sqrt{2}$ | |
| 15. $-2 \pm \sqrt{5}$ | |
| 16. $\frac{3 \pm \sqrt{17}}{4}$ | |
| 17. $\frac{2 \pm \sqrt{2}}{2}$ | |
| 18. $\frac{5 \pm \sqrt{31}}{4}$ | |
| 19. $-\frac{5}{2}$ | |
| 20. $4 \pm 2\sqrt{2}$ | |
| 21. $\emptyset$ | |
| 22. $\pm\frac{3\sqrt{5}}{5}$ | |