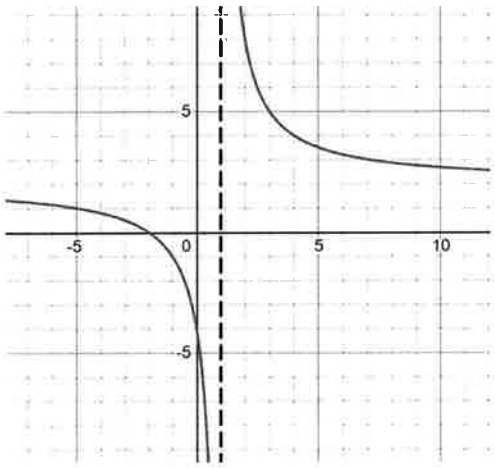
Section 4.4 – Practice Problems

1. For the following functions, find the Domain, the Vertical and Horizontal Asymptotes (if any), and approximate any $x - intercept(s)$ and $y - intercept(s)$

a) $f(x) = \frac{2x+4}{x-1}$



D: $x \neq 1$

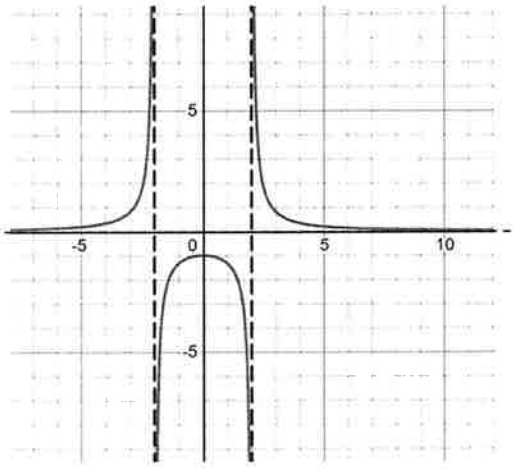
HA: $y = 2$

VA: $x = 1$

y-int: $(0, -4)$

x-int: $(-2, 0)$

b) $f(x) = \frac{4}{x^2-4}$



D: $x \neq \pm 2$

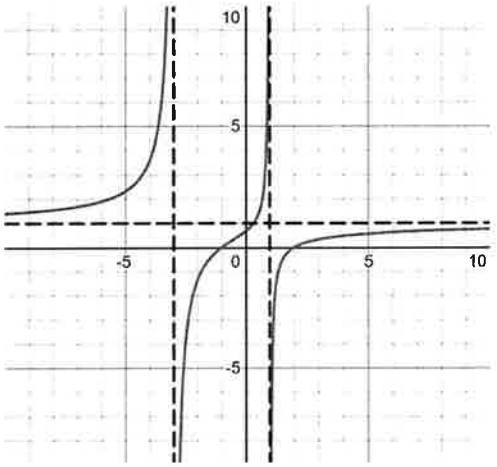
HA: $y = 0$

VA: $x = \pm 2$

x-int: None

y-int: $(0, -1)$

c) $f(x) = \dfrac{x^2 - x - 2}{x^2 + 2x - 3}$



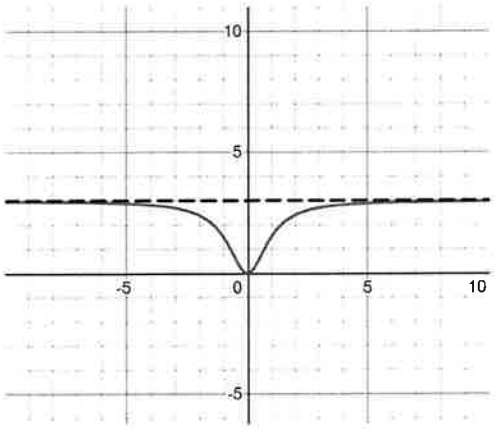
D: $x \neq 1, -3$ x-int: $(-1, 0)$ $(2, 0)$

VA: $x = 1, 3$

HA: $y = 1$

y-int: $(0, \frac{2}{3})$

d) $f(x) = \dfrac{3x^2}{x^2 + 1}$



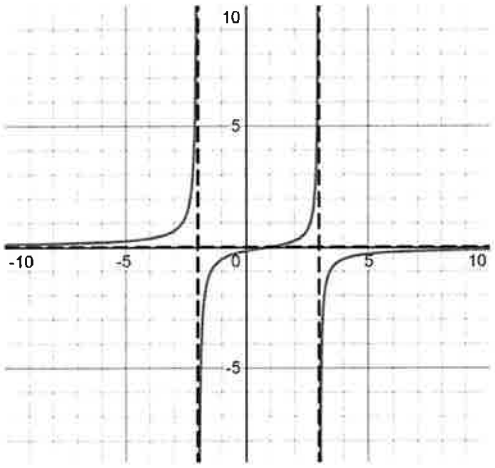
D: All Reals x-int: $(0, 0)$

VA: None

HA: $y = 3$

y-int: $(0, 0)$

e) $f(x) = \dfrac{1 - x}{x^2 - x - 6}$



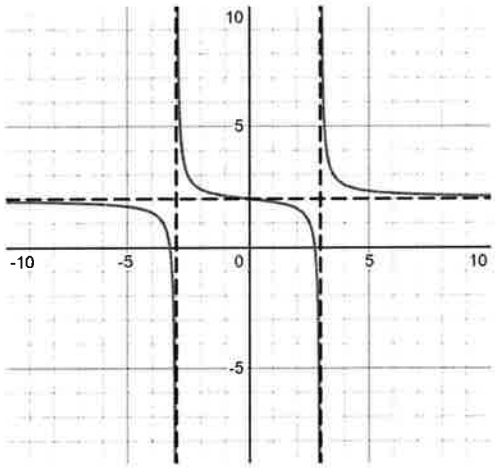
D: $x \neq -2, 3$ x-int: $(1, 0)$

VA: $x = -2, 3$

HA: $y = 0$

y-int: $(0, -\frac{1}{6})$

f) $f(x) = \dfrac{x}{x^2 - 9} + 2$



D: $x \neq \pm 3$ x-int: Approximation

$(2.9, 0)$

$(-3.1, 0)$

↑ Quadratic Equation needed for exact values

VA: $x = \pm 3$

HA: $y = 2$

y-int: $(0, 2)$

2. Find the Hole in the following Functions, sketch the graph and show where the Hole appears.

a) $f(x) = \dfrac{x^2 - 4}{x - 2}$

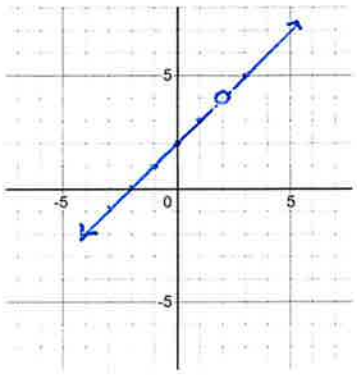
$y = \dfrac{(x+2)(x-2)}{(x-2)}$

when $x = 2$

$y = 4$

Hole at $(2, 4)$

$y = x + 2$



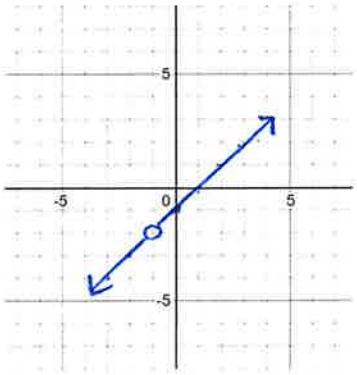
b) $f(x) = \dfrac{x^2 - 1}{x + 1}$ → $\dfrac{(x+1)(x-1)}{(x+1)}$

$y = x - 1$

when $x = -1$

$y = -2$

Hole at: $(-1, -2)$



c) $y = \dfrac{x^2 - 9}{3 - x}$

$y = \dfrac{x^2 - 9}{-x + 3}$

↓

$\dfrac{(x+3)(x-3)}{-1(x-3)}$

$y = -1(x+3)$

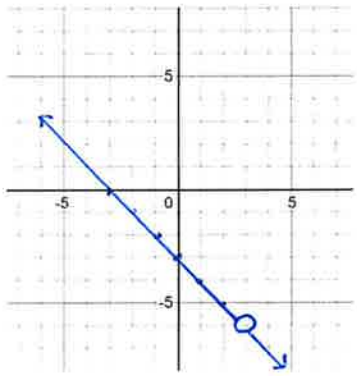
$y = -x - 3$

when $x = 3$

$y = -6$

Holes at

$(+3, -6)$



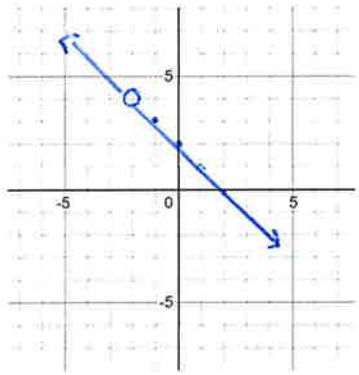
d) $y = \dfrac{4 - x^2}{x + 2}$ → $\dfrac{-x^2 + 4}{x+2}$ → $\dfrac{-1(x^2 - 4)}{(x+2)}$

$\dfrac{-1(x+2)(x-2)}{(x+2)}$

$y = -x + 2$

if $x = -2$

$y = 4$



$x \to 1^-$ means approaches 1 from left

$x \to 1^+$ means approaches 1 from right

$f(x) \to 3^-$ means from below

$f(x) \to 3^+$ means from above

3. Fill in the table of values for the following functions. Get comfortable with calculators and observe the behaviour of functions as they approach asymptotes.

a) $f(x) = \dfrac{3}{x-1}$

x	0.5	1.5	0.9	1.1	0.99	1.01
$f(x)$	-6	6	-30	30	-300	300

x	10	100	1000	−10	−100	−1000
$f(x)$	$0.\overline{3}$	$0.\overline{03}$	$0.\overline{003}$	-0.31	-0.03	-0.003

VA at $x = 1$

As $x \to 1^+$ $f(x) \to \infty$

As $x \to 1^-$ $f(x) \to -\infty$

As $x \to \infty$ $f(x) \to 0^+$

As $x \to -\infty$ $f(x) \to 0^-$

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b) $f(x) = \dfrac{3x^2-1}{x^2}$

x	−0.5	0.5	−0.1	0.1	−0.01	0.01
$f(x)$	-1	-1	-97	-97	-9997	-9997

x	10	100	1000	−10	−100	−1000
$f(x)$	2.9	2.999	2.9999	2.99	2.9999	2.99999

VA at $x = 0$

As $x \to 0^+$ $f(x) \to -\infty$

As $x \to 0^-$ $f(x) \to \infty$

As $x \to \infty$ $f(x) \to 3^-$

As $x \to -\infty$ $f(x) \to 3^-$

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c) $f(x) = \dfrac{x}{x-2}$

x	1.5	2.5	1.9	2.1	1.99	2.01
$f(x)$	-3	5	-19	21	-199	201

x	10	100	1000	−10	−100	−1000
$f(x)$	1.25	1.02	1.002	0.83	0.98	0.998

VA: $x = 2$

As $x \to 2^+$ $f(x) \to \infty$

$x \to 2^-$ $f(x) \to -\infty$

As $x \to \infty$ $f(x) \to 1^+$

$x \to -\infty$ $f(x) \to 1^-$

only interested in the numerator! Watch for Holes

4. Find the roots (zero's, solution, $x-intercepts$), if they exist, of the Rational Functions

a) $f(x)=\dfrac{x^2-4}{x+2}$ → $\dfrac{(x+2)(x-2)}{(x+2)}$

$\text{fact}: x-2$

$0=x-2$

$x=2, \quad (2,0)$

b) $g(x)=1-\dfrac{3}{x^2+2}$

$\dfrac{x^2+2}{x^2+2}-\dfrac{3}{x^2+2} \rightarrow \dfrac{x^2-1}{x^2+2} \rightarrow \dfrac{(x+1)(x-1)}{x^2+2}$

$x=\pm 1$ $\qquad 0=(x+1)(x-1)$

$(1,0) \quad (-1,0)$

c) $h(x)=1-\dfrac{3}{x-3}$

$\dfrac{x-3}{x-3}-\dfrac{3}{x-3} \rightarrow \dfrac{x-6}{x-3}$

$0=x-6$

$x=6 \quad (6,0)$

d) $f(x)=-1+\dfrac{4}{x^2+1}$

$-\dfrac{(x^2+1)}{(x^2+1)}+\dfrac{4}{(x^2+1)} \rightarrow \dfrac{-x^2-1+4}{x^2+1}$

$\dfrac{-x^2+3}{x^2+1}=\dfrac{-1(x^2-3)}{x^2+1}$

$\dfrac{-1(x+\sqrt{3})(x+\sqrt{3})}{x^2+1}$

$x=\pm\sqrt{3}$

$(\sqrt{3},0)(-\sqrt{3},0)$

e) $g(x)=1+\dfrac{4}{x^2+1}$

$\dfrac{x^2+1}{x^2+1}+\dfrac{4}{x^2+1} \rightarrow \dfrac{x^2+5}{x^2+1}$

$0=x^2+5$ has no solution in the real # system

NO x-ints

f) $h(x)=\dfrac{x^3+8}{x^2+4}$

$x^3+8=0$

$x^3=-8$

$\sqrt[3]{x^3}=\sqrt[3]{-8}$

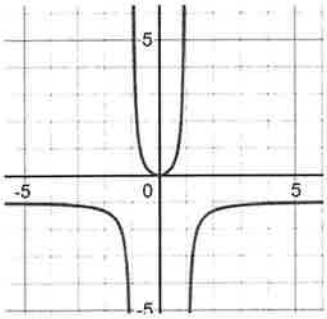
$x=-2$

$(-2,0)$

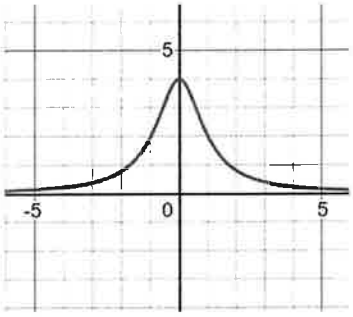
5. Match the equation with the graph.

a) $h(x) = \frac{1}{x-1}$	E	b) $h(x) = \frac{x}{x-1}$	H	c) $h(x) = \frac{-2}{x-1}$	J
d) $h(x) = \frac{1}{x^2}$	I	e) $h(x) = \frac{-x^2}{x^2-1}$	A	f) $h(x) = \frac{x^2}{x^2-1}$	C
g) $h(x) = \frac{x-3}{x-1}$	D	h) $h(x) = \frac{4}{x^2+1}$	B	i) $h(x) = \frac{x}{x^2-4}$	G
j) $h(x) = \frac{x^2-2x}{x^2+2x+1}$	F				

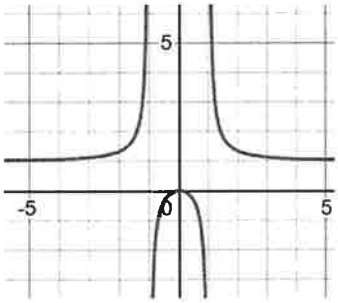
A



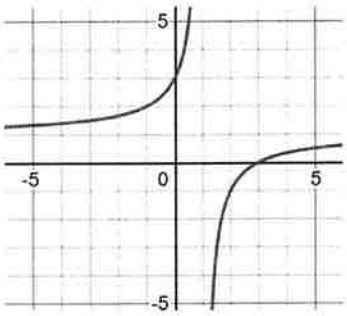
B



C



D



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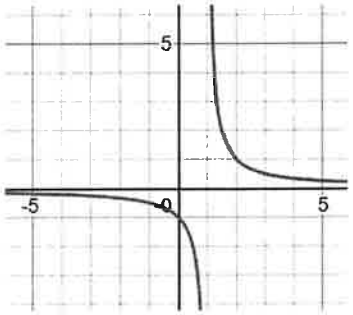
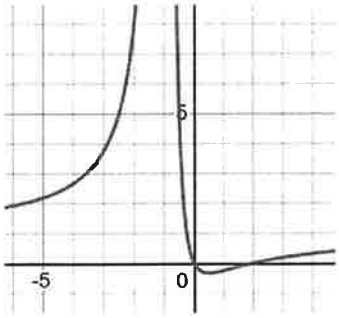
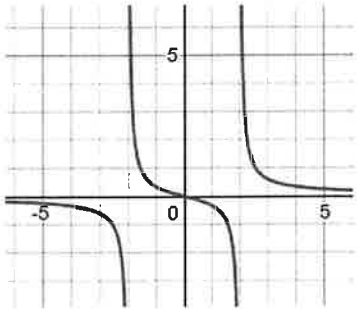


F



G



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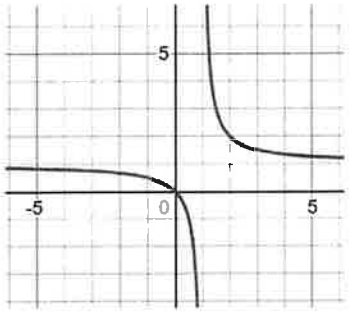
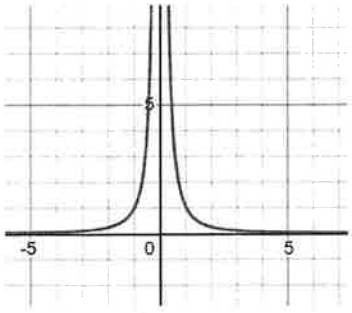
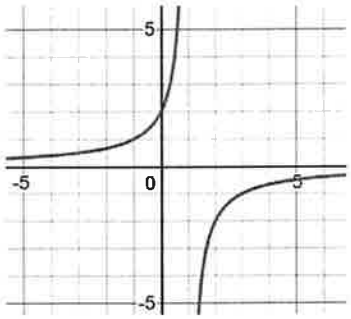


I



J



6. Sketch the following Rational Functions. State the Domain, the $x-intercepts$ and $y-intercepts$, identify the vertical asymptotes, horizontal asymptotes, and holes. Plot additional points to help generate the graph.

a) $h(x) = \frac{-x}{x+2}$

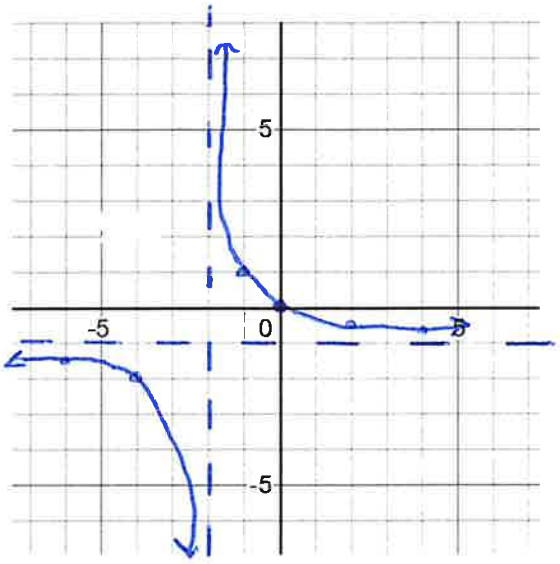
D: $x \neq -2$

VA: $x = -2$

HA: $y = -1$

x-int: $(0,0)$

y-int: $(0,0)$



x	0	-2	2	4	-4	-6	-1
$f(x)$	0	∅	$-\frac{1}{2}$	$-\frac{2}{3}$	-2	$-\frac{3}{2}$	1

b) $h(x) = \frac{x+2}{x-1}$

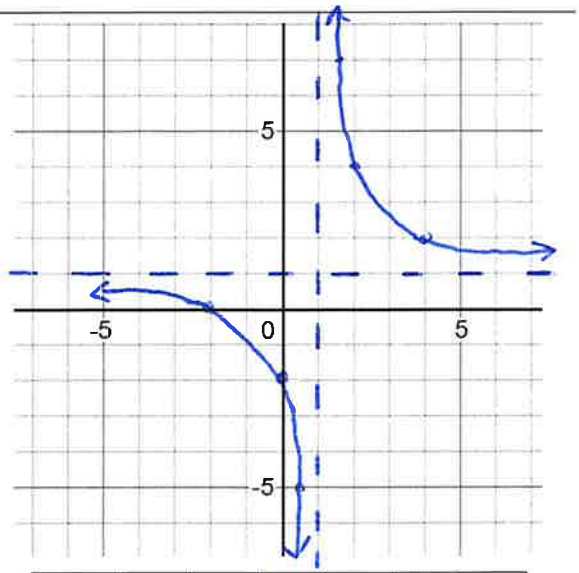
D: $x \neq 1$

VA: $x = 1$

HA: $y = 1$

x-int: $(-2,0)$

y-int: $(0,-2)$



x	0	2	-2	4	$\frac{1}{2}$	1.5
$f(x)$	-2	4	0	2	-5	7

c) $h(x) = \dfrac{x^2+3x+2}{x^2-4} \rightarrow \dfrac{(x+2)(x+1)}{(x+2)(x-2)}$

$y = \dfrac{(x+1)}{(x-2)}$

if $x = -2$, $y = \frac{1}{4}$

x-int: $(-1, 0)$

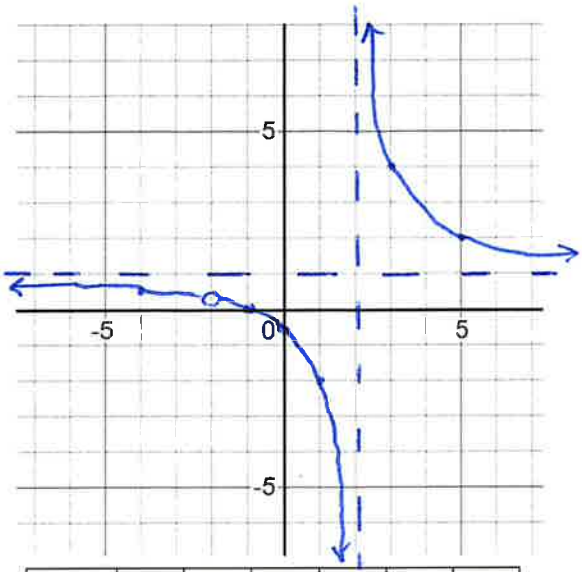
y-int: $(0, -\frac{1}{2})$

D: $x \neq \pm 2$

Hole: $(-2, \frac{1}{4})$

VA: $x = 2$

HA: $y = 1$



x	1	0	3	5	-2	-4
$f(x)$	-2	$-\frac{1}{2}$	4	2	$\frac{1}{4}$	$\frac{1}{2}$

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d) $h(x) = \dfrac{x^2}{x^3-9x} \rightarrow \dfrac{x^2}{x(x^2-9)} \downarrow \dfrac{x}{(x+3)(x-3)}$

VA: $x = \pm 3$

HA: $y = 0$

Hole: $(0,0)$

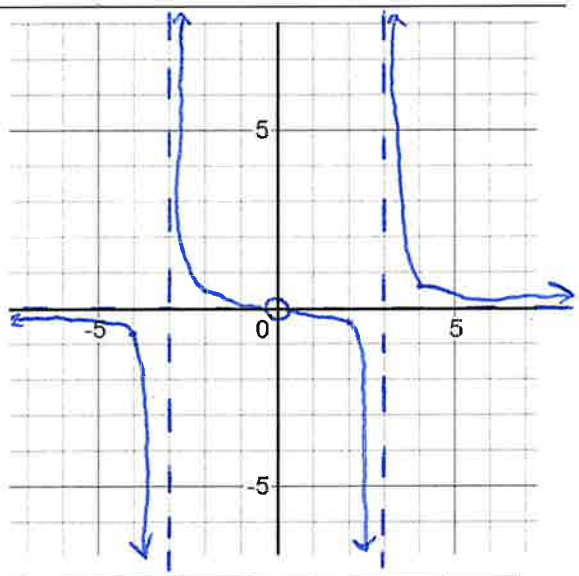
D: $x \neq 0, \pm 3$

x-int: None

y-int: None

if $x = 0$, $y = 0$

Hole at $(0,0)$



x	2	-2	4	-4	6	-6
$f(x)$	-0.4	0.4	$\frac{4}{7}$	$-\frac{4}{7}$	$0.\bar{2}$	$-0.\bar{2}$

* Notice you can cross a Horizontal Asymptote as long as x is not approaching $\pm\infty$.

See Website for Detailed Answer Key of the Remainder of the Questions

Extra Work Space