Section 4.3 – Practice Problems

1. Find the local maximum and minimum values of f.

a) $f(x) = 3x^2 - 4x + 13$

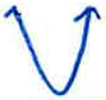
$f'(x) = 6x - 4$

$0 = 6x - 4 \rightarrow \frac{4}{6} = x$

$x = 2/3$

$f'(x) < 0$ $f'(x) > 0$

$x > 2/3$ $x < 2/3$



$\boxed{\text{local min } f(2/3) = 35/3}$

b) $f(x) = x^3 - 12x - 5$

$f'(x) = 3x^2 - 12 \rightarrow 12 = 3x^2$

$x^2 = 4$ $x = \pm 2$

Interval	$3x^2-12$	
$(-\infty, -2)$	+	inc
$(-2, 2)$	−	dec
$(2, \infty)$	+	inc

local max at $f(-2) = 11$

local min at $f(2) = -21$

c) $f(x) = 2 + 5x - x^5$

$f'(x) = -5x^4 + 5$

$= -5(x^4 - 1)$

crit points: $x = \pm 1$

local min $f(-1) = -2$

local max $f(1) = 6$

d) $f(x) = x^4 - x^3$

$f'(x) = 4x^3 - 3x^2$

$x^2(4x - 3)$

crit points: $0, 3/4$

no max

local min $f(3/4) = -\frac{27}{256}$

2. Find the critical numbers, intervals of increase and decrease, and local maximum values of the function. Then use this information to sketch the graph of f.

a) $f(x) = 2 + 6x - 6x^2$

$f'(x) = 6 - 12x$

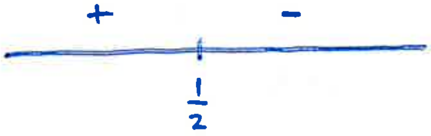
$f'(x) = 0 = 6 - 12x$

$0 = 6 - 12x$

$x = \frac{1}{2}$

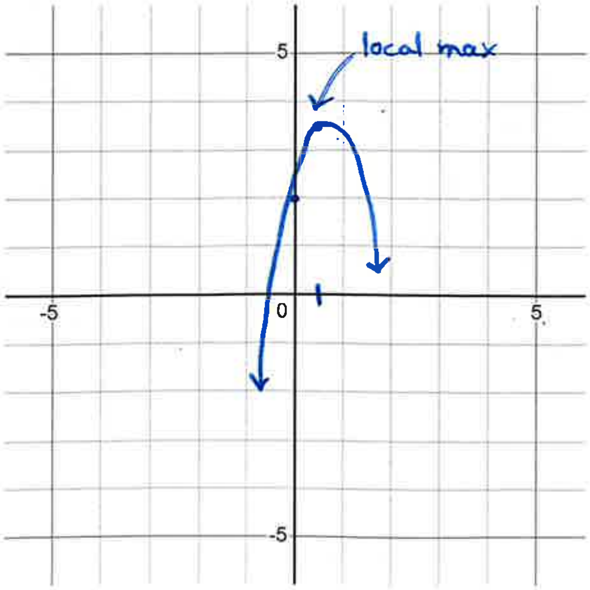
crit point $\frac{1}{2}$

$f(\frac{1}{2}) = \frac{7}{2}$



Increase: $(-\infty, \frac{1}{2})$

Decrease: $(\frac{1}{2}, \infty)$



$f(2) = 10$ $f(4) = 6$

b) $f(x) = x^3 - 9x^2 + 24x - 10$

$f'(x) = 3x^2 - 18x + 24$

$= 3(x^2 - 6x + 8) = 3(x-2)(x-4)$

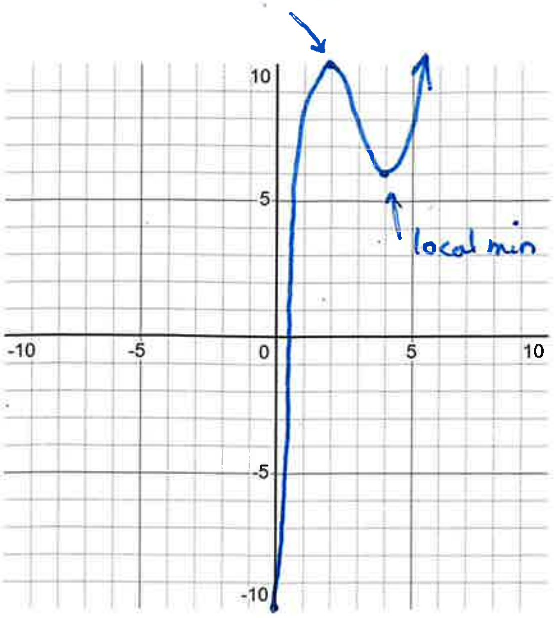
crit pts: 2, 4

+ − +

2 4

increases: $(-\infty, 2)$ $(4, \infty)$

decreases: $(2, 4)$



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c) $g(x) = 1 + 3x^2 - 2x^3$

$g'(x) = -6x^2 + 6x$

$= -6x(x-1)$

crit points: 0, 1

− + −

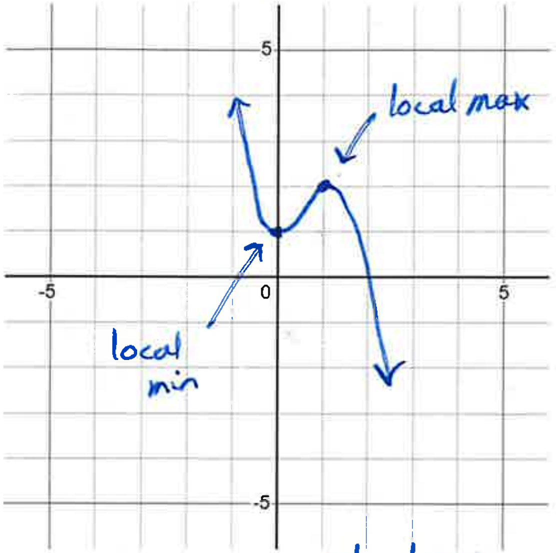
0 1

$g(0) = 1$

$g(1) = 2$

Dec: $(-\infty, 0)$ and $(1, \infty)$

Inc: $(0, 1)$



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d) $g(x) = 3x^4 - 16x^3 + 18x^2 + 1$

$g'(x) = 12x^3 - 48x^2 + 36x$

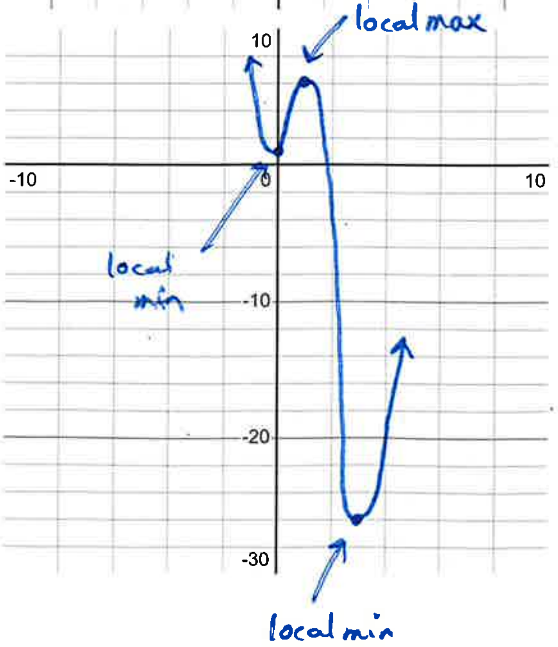
$= 12x(x^2 - 4x + 3)$

$12x(x-1)(x-3)$

crit points 0, 1, 3

− + − +

0 1 3



Increase: $(0,1)$ and $(3, \infty)$

Decrease: $(-\infty, 0)$ and $(1, 3)$

$g(0) = 1$ $g(1) = 6$ $g(3) = -26$

e) $h(x) = x^4 - 8x^2 + 6$

$h'(x) = 4x^3 - 16x$

$= 4x(x^2 - 4) \rightarrow 4x(x+2)(x-2)$

crit points

$0, -2, 2$

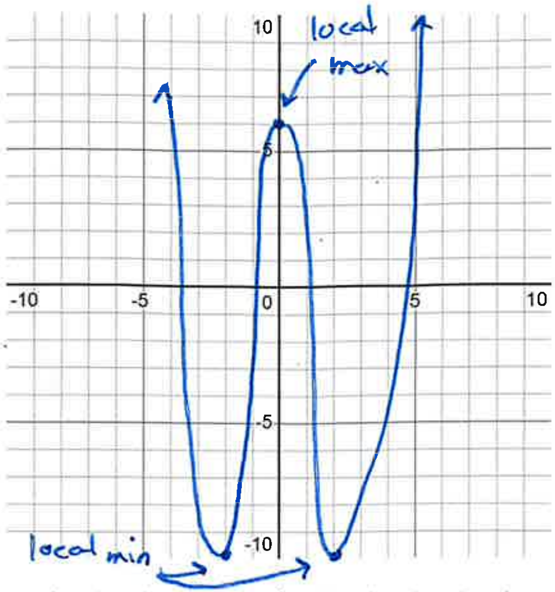
$-\quad +\quad -\quad +$

$-2 \quad 0 \quad 2$

Dec: $(-\infty, -2)$
$(0, 2)$

Inc: $(-2, 0)$
$(2, \infty)$

$h(-2) = -10$ $h(0) = 6$
$h(2) = -10$



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f) $h(x) = 3x^5 - 5x^3$

$h'(x) = 15x^4 - 15x^2$

$= 15x^2(x^2 - 1)$

crit pts:
$0, \pm 1$

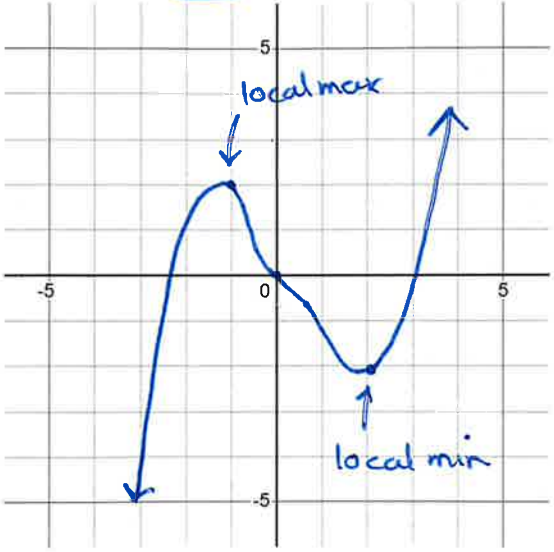
$+\quad -\quad -\quad +$

$-1 \quad 0 \quad 1$

inc: $(-\infty, -1)$
$(1, \infty)$

dec: $(-1, 1)$

$h(0) = 0$
$h(1) = -2$
$h(-1) = 2$



3. Find the local maximum and minimum values of f.

a)

$$f(x) = 2x^{\frac{2}{3}}\left(3 - 4x^{\frac{1}{3}}\right)$$

$f'(x) = 2x^{2/3}\left(-\frac{4}{3}x^{-2/3}\right) + (3 - 4x^{\frac{1}{3}})\frac{4}{3}x^{-\frac{1}{3}}$

$= -\frac{8x^{2/3}}{3x^{2/3}} + 3.\frac{4}{3x^{\frac{1}{3}}} - \frac{16x^{\frac{1}{3}}}{3x^{1/3}}$

crit points
$0, \frac{1}{8}$

$= -\frac{8}{3} - \frac{16}{3} + \frac{4}{x^{1/3}} \rightarrow \frac{4}{x^{1/3}} - 8$

$-\quad +\quad -$

$0 \quad \frac{1}{8}$

$= \frac{4 - 8x^{\frac{1}{3}}}{x^{1/3}}$

$4 - 8x^{\frac{1}{3}} = 0$

$x^{1/3} = \frac{1}{2}$ $x = \frac{1}{8}$

$f(0) = 0$
$f(\frac{1}{8}) = \frac{1}{2}$

$\boxed{\text{local max: } f(\tfrac{1}{8}) = \tfrac{1}{2}}$

$\boxed{\text{local min: } f(0) = 0}$

b)

$$f(x) = \frac{x^2}{x^2 - 1}$$

$f'(x) = \frac{(x^2-1)2x - x^2(2x)}{(x^2-1)^2}$

$= \frac{2x^3 - 2x - 2x^3}{(x^2-1)^2} \rightarrow \frac{-2x}{(x^2-1)^2}$

crit pt:
$0, 1, -1$

$+\quad +\quad -\quad -$

$-1 \quad 0 \quad 1$

$f(0) = 0$

$\boxed{\text{local max at } f(0) = 0}$

c)

D: $x \leq 4$

$$f(x) = x\sqrt{4-x}$$

$$f'(x) = x\left(\frac{1}{2\sqrt{4-x}} \cdot -1\right) + \sqrt{4-x}$$

$$= \frac{-x}{2\sqrt{4-x}} + \sqrt{4-x} \Rightarrow \frac{-x + (4-x)2}{2\sqrt{4-x}}$$

$$\rightarrow \frac{-3x+8}{2\sqrt{4-x}}$$

$\boxed{\text{local max at } f(8/3) = \frac{16\sqrt{3}}{9}}$

ct pts: 4, 8/3

d)

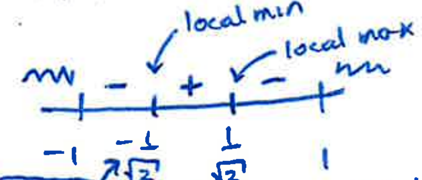
D: $\{x \mid -1 \leq x \leq 1\}$

$$f(x) = x\sqrt{1-x^2}$$

$$f'(x) = x\left(\frac{1}{2\sqrt{1-x^2}} \cdot -2x\right) + \sqrt{1-x^2}$$

$$= \frac{-2x^2}{2\sqrt{1-x^2}} + \sqrt{1-x^2} = \frac{-x^2 + 1 - x^2}{\sqrt{1-x^2}}$$

$$= \frac{-2x^2+1}{\sqrt{1-x^2}}$$



$f(-\frac{1}{\sqrt{2}}) = -\frac{1}{2}$

crunch the y-value.

$\boxed{f(\frac{1}{\sqrt{2}}) = \frac{1}{2}}$

ct pts: $\pm \frac{1}{\sqrt{2}}$

4. Find the absolute maximum or minimum value of the function.

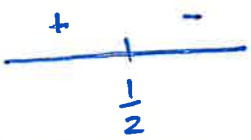
a)

$$f(x) = 27 + x - x^2$$

$$f'(x) = 1 - 2x$$

$0 = 1 - 2x$

$x = \frac{1}{2}$



ct pt.

$x = \frac{1}{2}$

$\boxed{\text{abs max at } f(\tfrac{1}{2}) = \frac{109}{4}}$

b)

$$f(x) = 3 - \frac{1}{\sqrt{x^2+1}} = 3 - (x^2+1)^{-\frac{1}{2}}$$

$$f'(x) = \frac{1 \cdot 2x}{2\sqrt{x^2+1}^{\,3}} = \frac{x}{\sqrt{x^2+1}^{\,3}}$$

ct pt: 0

$\boxed{\text{Abs min}: f(0) = 2}$

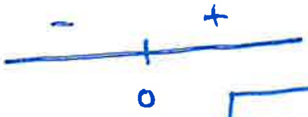
c)

$$g(x) = \frac{x^2-1}{x^2+1}$$

$$g'(x) = \frac{(x^2+1)2x - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

ct pt: 0



$\boxed{\text{Abs min}: g(0) = -1}$

d)

$$g(x) = \frac{x^2-x+1}{x^2+1}, \quad \boxed{x \geq 0}$$

$$g'(x) = \frac{(x^2+1)(2x-1) - (x^2-x+1)(2x)}{(x^2+1)^2}$$

$$= \frac{2x^3 - x^2 + 2x - 1 - [2x^3 - 2x^2 + 2x]}{(x^2+1)^2}$$

$$= \frac{x^2-1}{(x^2+1)^2} \rightarrow \frac{(x+1)(x-1)}{(x^2+1)^2}$$

ct pt:

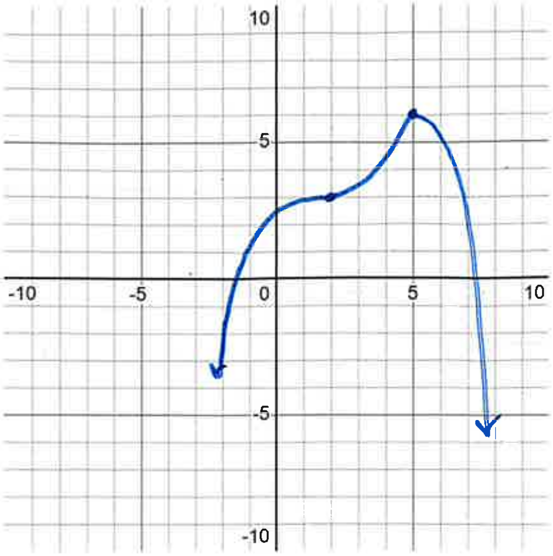
$x = -1$ ← outside domain

$x = 1$

$\boxed{\text{Abs min } g(1) = \frac{1}{2}}$

5. Sketch the graph of a function f that satisfies all of the following conditions.

a) $f(2) = 3, f(5) = 6$
b) $f'(2) = f'(5) = 0$
c) $f'(x) \geq 0$ for $x < 5$
d) $f'(x) < 0$ for $x > 5$



6. Find the local maximum and minimum values of the function f defined by:

$$f(x) = \begin{cases} -x & \text{if } x < 0 \\ 2x^3 - 15x^2 + 36x & \text{if } 0 \leq x \leq 4 \\ 216 - x & \text{if } x > 4 \end{cases}$$

$6x^2 - 30x + 36 = 0$

$6(x^2 - 5x + 6) = 0$

$6(x-2)(x-3) = 0$

$f'(x) = -1$ if $x < 0$

$6x^2 - 30x + 36$ if $0 \leq x \leq 4$

-1 if $x > 4$

ct pt: 2 3 0 4

$f(2) = 28$ ← local max

$f(3) = 27$ local mins

$f(0) = 0$

$f(4) = 32$ ← abs max