Section 4.3– Practice Problems

1. Answer the following questions to lockdown your vocabulary.

a) A rational expression is defined as the ratio of two polynomials with the denominator not equal to zero.

b) A function defined by $f(x) = \frac{g(x)}{h(x)}$ with $g(x)$ and $h(x)$ Polynomials functions and $h(x) \neq 0$, is called a Rational function

c) To determine the excluded values of the domain of a rational function, we find the values for where the Denominator is equal to 0

d) A vertical line that a graph approaches but never touches is called a vertical asymptote

e) A horizontal line that a graph approached as $x \to \pm\infty$ is called a Horizontal Asymptote

f) The graph of a Rational Function $f(x) = \frac{3}{x-2}$ will have a vertical asymptote of $x = 2$ and a horizontal asymptote of $y = 0$

numerator only → when $y = 0$

when $x = 0$

2. Find the Domain, the $x - intercept(s), y - intercepts$, and any *holes* for the following

a) $y = \frac{3x-9}{4x+12}$

D: $x \neq -3$

Holes: None

$\frac{3(x-3)}{4(x+3)}$

x-int: (3,0)

y-int: $(0, -3/4)$

$\frac{3(0)-9}{4(0)+12} = -\frac{9}{12}$

b) $y = \frac{(x+6)(x+3)}{(x-2)^2}$ → $\frac{(x+6)(x+3)}{(x-2)(x-2)}$

D: $x \neq 2$

Holes: None

x-int: (-6,0) and (-3,0)

y-int: $(0, 9/2)$

$\frac{(6)(3)}{(-2)(-2)} = \frac{18}{4} \downarrow \frac{9}{2}$

c) $y = \dfrac{x^2 - 8x - 9}{x^2 - x - 6}$ → $\dfrac{(x-9)(x+1)}{(x-3)(x+2)}$

D: $x \neq 3$, $x \neq -2$ Holes: None

x-int: (9,0) and (-1,0)

y-int: $(0, 3/2)$

$\dfrac{(-9)(1)}{(-3)(2)} = \dfrac{-9}{-6} = \dfrac{3}{2}$

d) $y = \dfrac{(x^2-1)(x+1)}{x^3}$ → $\dfrac{(x+1)(x-1)(x+1)}{x^3}$

D: $x \neq 0$

Holes: None

x-int: (1,0) and (-1,0)

y-int: None

$\dfrac{(1)(-1)(1)}{0}$ ← not allowed

e) $y = \dfrac{x+2}{x^2+4}$ ← no restriction

D: All Real #'s

Holes: None

x-int: (-2,0)

y-int: $(0, 1/2)$

f) $y = \dfrac{-3x^2 + 12}{x^2 - 9}$ → $\dfrac{-3(x^2-4)}{x^2-9}$

$\dfrac{-3(x+2)(x-2)}{(x-3)(x+3)}$

D: $x \neq 3$, $x \neq -3$

x-int: (2,0) and (-2,0) Holes: Nones

y-int: $(0, -4/3)$

$\dfrac{-3(2)(-2)}{(-3)(3)} = \dfrac{12}{-9} = -\dfrac{4}{3}$

g) $y = \frac{4}{(x+4)^2}$

D: $x \neq -4$

Holes: None

x-int: None

y-int: $(0, \frac{1}{4})$

$\frac{4}{4^2} = \frac{1}{4}$

h) $y = \frac{-x^2+9}{-2x^2+8} \rightarrow \frac{-1(x^2-9)}{-2(x^2-4)}$

$\frac{(x+3)(x-3)}{2(x+2)(x-2)}$

D: $x \neq 2$, $x \neq -2$

Holes: None

x-int: $(3,0)$ and $(-3,0)$

y-int: $(0, \frac{3}{8})$

$\frac{(3)(-3)}{2(2)(-2)} = \frac{-9}{-8} = \frac{9}{8}$

i) $y = \frac{2+x}{x^2+4}$

D: All Real #'s

Hols: None

x-int: $(-2,0)$

y-int: $(0, \frac{1}{2})$

$\frac{2}{4} = \frac{1}{2}$

j) $y = \frac{x^2-3x-4}{4+3x-x^2} \rightarrow \frac{(x-4)(x+1)}{-x^2+3x+4}$

$\frac{(x-4)(x+1)}{-1(x^2-3x-4} \rightarrow \frac{(x-4)(x+1)}{-1(x-4)(x+1)} = \frac{1}{-1}$

Domain: $x \neq 4$, $x \neq -1$

Holes: $x = 4$, $x = -1$

x-int: None

y-int: $(0,-1)$

3. Find the $y-intercept$ and $x-intercept(s)$ vertical and horizontal asymptotes of the following

a) $y=-\frac{4}{x}$

VA: $x=0$

y-int: Does not Exist (DNE)

x-int: DNE

HA: $\frac{-\frac{4}{x}}{\frac{x}{x}} \rightarrow \frac{\frac{4}{\infty}}{1} \rightarrow \frac{0}{1}=0$

HA: $y=0$

b) $y=-\frac{3}{x^2}$

VA: $x=0$

y-int: DNE

x-int: DNE

HA: $y=0$

c) $y=1+\frac{1}{x} \rightarrow \frac{x+1}{x}$

VA: $x=0$

HA: $\frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}} \rightarrow \frac{1+\frac{1}{\infty}}{1}=\frac{1}{1}=1$

HA: $y=1$

y-int: DNE

x-int: $(-1,0)$

d) $y=2-\frac{1}{x} \rightarrow \frac{2x-1}{x}$

VA: $x=0$

HA: $y=2$

y-int: DNE

x-int: $x=\frac{1}{2}$ $(\frac{1}{2},0)$

e) $y = -4 + \frac{1}{x^2}$ → $\frac{-4x^2+1}{x^2}$

VA: $x = 0$

HA: $y = -4$

y-int: DNE

x-int: $(\frac{1}{2}, 0)$ $(-\frac{1}{2}, 0)$

$-4x^2+1=0$

$4x^2-1=0$

$(2x+1)(2x-1)=0$

f) $y = -1 - \frac{1}{x^2}$ → $\frac{-x^2-1}{x^2}$

VA: $x = 0$

HA: $y = -1$

y-int: DNE

x-int: DNE

$-x^2-1=0$

$x^2+1=0$

no real roots

g) $y = -\frac{1}{x+1}$

VA: $x = -1$

HA: $y = 0$

y-int: $(0, -1)$

x-int: DNE

$-\frac{\frac{1}{x}}{\frac{x}{x}+\frac{1}{x}} \rightarrow \frac{0}{1+0}$

h) $y = -\frac{2}{(x+1)^2}$

VA: $x = -1$

HA: $y = 0$

x-int: DNE

y-int: $(0, -2)$

i) $y = 3 + \frac{2}{x-1}$

$\frac{3(x-1)}{x-1} + \frac{2}{x-1}$

$\frac{3x-3+2}{x-1} \rightarrow \frac{3x-1}{x-1}$

VA: $x = 1$

HA: $y = 3$

y-int: $(0, 1)$

x-int: $(\frac{1}{3}, 0)$

j) $y = 1 - \frac{2}{(x+1)^2}$

x-int: $\frac{-2 \pm \sqrt{4-4(1)(-1)}}{2}$

$\frac{-2 \pm \sqrt{8}}{2}$, $-1 \pm \sqrt{2}$

$\frac{(x+1)^2 - 2}{(x+1)^2} \rightarrow \frac{x^2+2x+1-2}{(x+1)^2}$

$\frac{x^2+2x-1}{(x+1)^2} \rightarrow \frac{x^2+2x-1}{(x+1)^2}$

VA: $x = -1$

HA: $y = 1$

y-int: $(0, -1)$

x-int: $(-1+\sqrt{2}, 0)$

$(-1-\sqrt{2}, 0)$

y-int

$1 - \frac{2}{1}$

$1 - 2 = -1$

4. What are the Vertical Asymptote, Horizontal Asymptote, and any holes of the following.

a) $y = \frac{1}{x}$

VA: $x = 0$

HA: $y = 0$

Holes: None

$\frac{\frac{1}{x}}{\frac{x}{x}} \rightarrow \frac{\frac{1}{\infty}}{1} = \frac{0}{1}$

b) $y = \frac{2}{x+3}$

VA: $x = -3$

HA: $y = 0$

Holes: None

$\frac{\frac{2}{x}}{\frac{x}{x} + \frac{3}{x}} \rightarrow \frac{0}{1+0} = 0$

c) $y = \frac{1}{x^2 - 7x + 12}$

$\frac{1}{(x-3)(x-4)}$

VA: $x = 3$
$x = 4$

HA: $y = 0$

Holes: None

d) $y = \frac{x^2}{x^2 - 9}$ → $\frac{x^2}{(x-3)(x+3)}$

VA: $x = 3$
$x = -3$

HA: $y = 1$

Holes: None

$\frac{\frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{9}{x^2}} \rightarrow \frac{1}{1-0} = 1$

e) $y = \frac{x}{x^2 + 1}$ ← no real roots

VA: None

HA: $y = 0$

Holes: None

$\frac{\frac{x}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}}$

↓

$\frac{\frac{1}{x}}{1 + \frac{1}{x^2}} = \frac{0}{1}$

f) $y = \frac{x^3}{x^2 - x - 20}$ → $\frac{x^3}{(x-5)(x+4)}$

VA: $x = 5$
$x = -4$

HA: None

Holes: None

$\frac{\frac{x^3}{x^2}}{\frac{x^2}{x^2} - \frac{x}{x^2} - \frac{20}{x^2}}$

$\frac{x}{1 - \frac{1}{x} - \frac{20}{x^2}}$

$\frac{\infty}{1 - 0 - 0} = \infty$

g) $y = \dfrac{x^2 + 3x - 1}{4 - x^2}$ → not easily factorable in numerator

$-\dfrac{(x^2+3x-1)}{x^2-4}$ → $-\dfrac{(x^2+3x-1)}{(x-2)(x+2)}$

VA: $x = 2$, $x = -2$

HA: $y = -1$

Holes: None

$\dfrac{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2}}{\frac{4}{x^2} - \frac{x^2}{x^2}}$ ↓ $\dfrac{1 - 0 - 0}{0 - 1}$ $\dfrac{1}{-1} = -1$

h) $y = \dfrac{2x^3 - 18x}{x^3 - 3x^2 - 4x}$ → $\dfrac{2x(x^2-9)}{x(x^2-3x-4)}$

$\dfrac{2x(x+3)(x-3)}{x(x-4)(x+1)}$

$\dfrac{2(x+3)(x-3)}{(x-4)(x+1)}$

VA: $x = 4$, $x = -1$

Holes: $x = 0$

$(0, 9/2)$

HA: $y = 2$

↓ for Hole set $x = 0$

$\dfrac{2(3)(-3)}{(-4)(1)} = \dfrac{-18}{-4} = \dfrac{9}{2}$

i) $y = \dfrac{x^2 - 4}{2x^3 + 7x^2 - 4x}$

$\dfrac{(x+2)(x-2)}{x(2x^2+7x-4)}$

$\dfrac{(x+2)(x-2)}{x(2x-1)(x+4)}$

VA: $x = 0$, $x = \frac{1}{2}$, $x = -4$

HA: $y = 0$

Holes: None

j) $y = \dfrac{9 - 6x}{4x^2 - 9}$ → $\dfrac{-6x+9}{4x^2-9}$ → $\dfrac{-3(2x-3)}{(2x+3)(2x-3)}$

$\dfrac{-3}{2x+3}$

VA: $x = -3/2$

HA: $y = 0$

Holes: $(3/2, -1/2)$

↓ for Holes set $x = 3/2$

$\dfrac{-3}{2(\frac{3}{2})+3}$

$\dfrac{-3}{3+3} = -\dfrac{3}{6}$

$-\dfrac{1}{2}$

k) $y = \dfrac{16x - x^3}{2x^3 + 7x^2 - 4x}$

$\dfrac{-x^3 + 16x}{2x^3 + 7x^2 - 4x} \rightarrow \dfrac{-x(x^2 - 16)}{x(2x^2 + 7x - 4)}$

$\dfrac{-x(x+4)(x-4)}{x(2x-1)(x+4)} \rightarrow -\dfrac{(x-4)}{(2x-1)}$

VA: $x = \frac{1}{2}$

HA: $y = -1$

Holes: $(0, -4)$

$(-4, -8/9)$

Holes

set $x = 0$

$x = -4$

if $x = 0$

$-\frac{-4}{-1} = -4$

if $x = -4$

$\frac{-(-8)}{-9} = -8/9$

l) $y = 1 - \dfrac{3}{x^2 - 1} \rightarrow \dfrac{x^2 - 1}{x^2 - 1} - \dfrac{3}{x^2 - 1}$

$\dfrac{x^2 - 1 - 3}{x^2 - 1} \rightarrow \dfrac{x^2 - 4}{x^2 - 1} \rightarrow \dfrac{(x+2)(x-2)}{(x+1)(x-1)}$

HA: $y = 1$

VA: $x = 1$

$x = -1$

Holes: None

See Website for Detailed Answer Key of the Remainder of the Questions

Extra Work Space