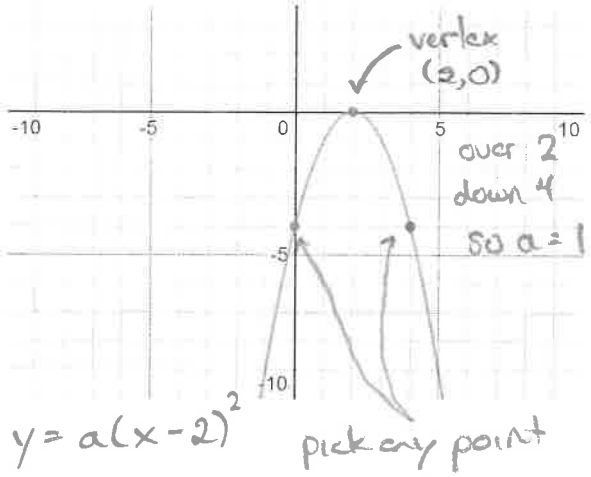
Section 4.2 – Practice Problems

Determine the equation of the following parabolas

1.



$y = a(x-2)^2$

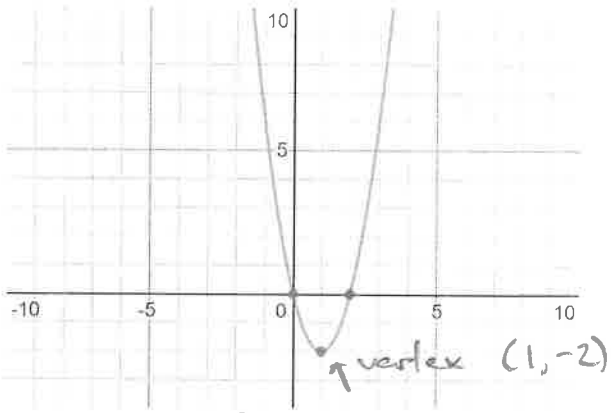
$-4 = a(0-2)^2$

$-4 = 4a$

$a = -1$

$y = -(x-2)^2$

2.



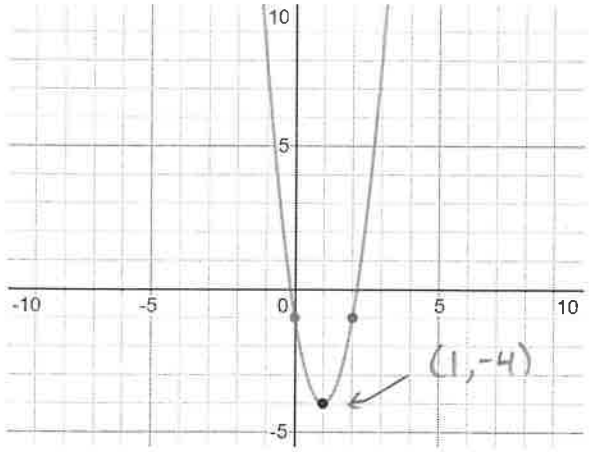
$y = a(x-1)^2 - 2$

$0 = a(2-1)^2 - 2$

$2 = a$

$y = 2(x-1)^2 - 2$

3.



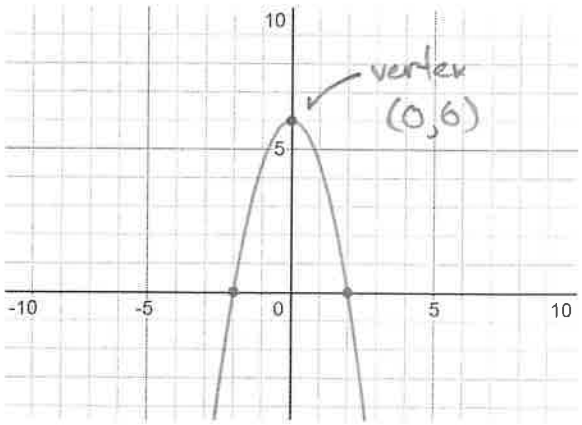
$y = a(x-1)^2 - 4$

$-1 = a(2-1)^2 - 4$

$3 = a$

$y = 3(x-1)^2 - 4$

4.



$y = ax^2 + 6$

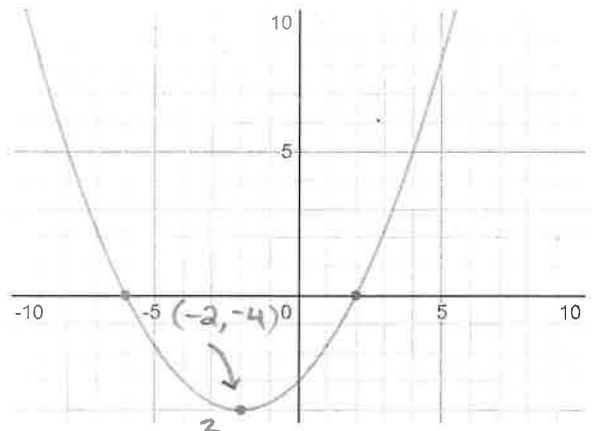
$0 = a(2)^2 + 6$

$-6 = 4a$

$a = -\frac{3}{2}$

$y = -\frac{3}{2}x^2 + 6$

5.



$y = a(x+2)^2 - 4$

$0 = a(2+2)^2 - 4$

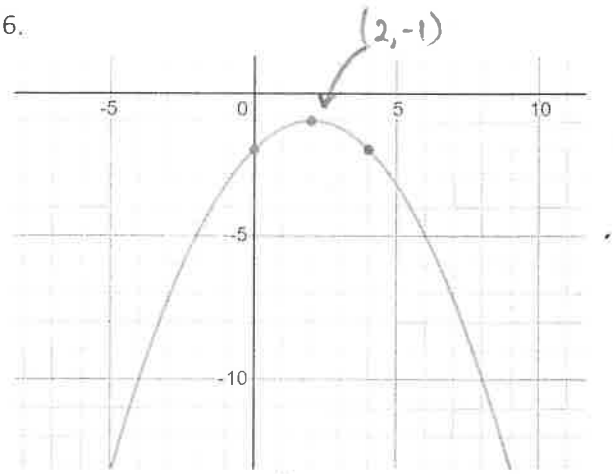
$4 = 16a$

$a = 4/16$

$a = 1/4$

$y = \frac{1}{4}(x+2)^2 - 4$

6.



$y = a(x-2)^2 - 1$

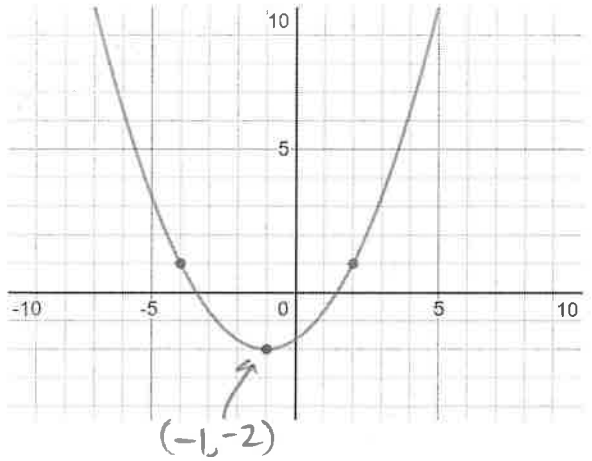
$-2 = a(0-2)^2 - 1$

$-1 = a(4)$

$-\frac{1}{4} = a$

$y = -\frac{1}{4}(x-2)^2 - 1$

7.



$y = a(x+1)^2 - 2$

$1 = a(2+1)^2 - 2$

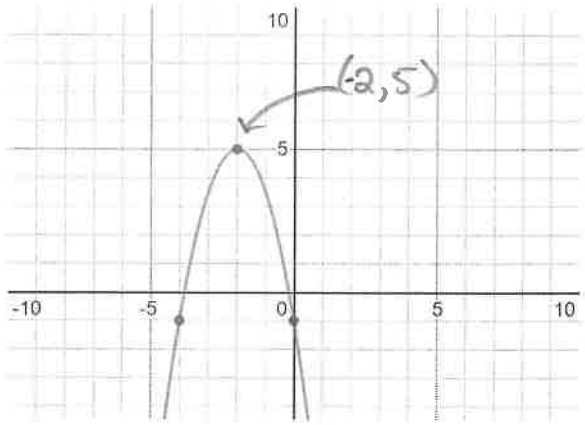
$1 = 9a - 2$

$3 = 9a$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x+1)^2 - 2$

8.



$y = a(x+2)^2 + 5$

$-1 = a(0+2)^2 + 5$

$-6 = a(4)$

$a = -6/4$

$a = -3/2$

$y = -\frac{3}{2}(x+2)^2 + 5$

Find the equation of a quadratic function whose graph satisfies the given coordinates.

9. *vertex*: $(2, 9)$ $x-intercept$: 5

$y = a(x-2)^2 + 9$ $(5,0)$

$0 = a(5-2)^2 + 9$

$-9 = a(9)$

$a = -1$

$y = -(x-2)^2 + 9$

10. *vertex*: $(-2, 12)$ $x-intercept$: -4

$y = a(x+2)^2 + 12$ $(-4,0)$

$0 = a(-4+2)^2 + 12$

$0 = 4a + 12$

$-\frac{12}{4} = a$

$a = -3$

$y = -3(x+2)^2 + 12$

11. *vertex*: $(1, -4)$ $x-intercept$: -2

$y = a(x-1)^2 - 4$ $(-2,0)$

$0 = a(-2-1)^2 - 4$

$4 = 9a$

$a = \frac{4}{9}$

$y = \frac{4}{9}(x-1)^2 - 4$

12. *vertex*: $(-4, 12)$ $x-intercept$: 4

$y = a(x+4)^2 + 12$ $(4,0)$

$0 = a(4+4)^2 + 12$

$-12 = 64a$

$-\frac{12}{64} = a$

$-\frac{3}{16} = a$

$y = -\frac{3}{16}(x+4)^2 + 12$

13. *vertex*: $(-3, -5)$ $y-intercept$: 1

$y = a(x+3)^2 - 5$ $(0,1)$

$1 = a(0+3)^2 - 5$

$6 = 9a$

$\frac{6}{9} = a$

$\frac{2}{3} = a$

$y = \frac{2}{3}(x+3)^2 - 5$

14. *vertex*: $(2, 4)$ $y-intercept$: -3

$y = a(x-2)^2 + 4$ $(0,-3)$

$-3 = a(0-2)^2 + 4$

$-7 = 4a$

$a = -\frac{7}{4}$

$y = -\frac{7}{4}(x-2)^2 + 4$

15. *vertex*: $(1, 4)$ *point*: $(2, 3)$

$y = a(x-1)^2 + 4$

$3 = a(2-1)^2 + 4$

$-1 = a$

$\boxed{y = -(x-1)^2 + 4}$

16. *vertex*: $(-2, -4)$ *point*: $(-3, -1)$

$y = a(x+2)^2 - 4$

$-1 = a(-3+2)^2 - 4$

$-1 = a - 4$

$a = 3$

$\boxed{y = 3(x+2)^2 - 4}$

Find the Vertex by completing the square and using the vertex formula

17. $f(x) = x^2 + 4x + 3$

$y = x^2 + 4x + 3$

$y - 3 = x^2 + 4x$

$y - 3 = x^2 + 4x + 4 - 4$

$y + 1 = x^2 + 4x + 4$

$y + 1 = (x+2)^2$

$\boxed{y = (x+2)^2 - 1}$

vertex: $(-2, -1)$

VERTEX FORMULA

$\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

$\frac{-4}{2(1)}, 3 - \frac{16}{4}$

$\boxed{(-2, -1)}$

---

18. $f(x) = x^2 - 8x + 15$

Another style:

$y = x^2 - 8x + 15$

$= x^2 - 8x + 16 - 16 + 15$

$y = (x-4)^2 - 1$

vertex: $(4, -1)$

$\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$\left(\frac{8}{2}, 15 - \frac{64}{4}\right)$

$(4, 15 - 16)$

$\boxed{(4, -1)}$

$\left(\frac{-b}{2a}, c-\frac{b^2}{4a}\right)$

19. $f(x) = x^2 + 3x - 8$

$y = x^2 + 3x + \frac{9}{4} - \frac{9}{4} - 8$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - \frac{32}{4}$

$y = \left(x + \frac{3}{2}\right)^2 - \frac{41}{4}$

vertex: $\left(-\frac{3}{2}, -\frac{41}{4}\right)$

$\left(\frac{-3}{2}, -8 - \frac{9}{4}\right)$

$\left(-\frac{3}{2}, -\frac{32}{4} - \frac{9}{4}\right)$

$\left(-3/2, -\frac{41}{4}\right)$

20: $f(x) = 3x^2 - 18x + 25$

$y = 3x^2 - 18x + 25$

$y - 25 = 3x^2 - 18x$

$y - 25 = 3(x^2 - 6x)$

$y - 25 = 3(x^2 - 6x + 9 - 9)$ ← multiply this by the a term to take it out of the brackets

$y - 25 = 3(x^2 - 6x + 9) - 27$

$y = 3(x-3)^2 - 2$

vertex: $(3, -2)$

$\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$\left(\frac{18}{6}, 25 - \frac{324}{12}\right)$

$(3, 25 - 27)$

$(3, -2)$

---

21. $f(x) = \frac{1}{2}x^2 - 3x + 4$

$y - 4 = \frac{1}{2}x^2 - 3x$

$y - 4 = \frac{1}{2}(x^2 - 6x)$

$y - 4 = \frac{1}{2}(x^2 - 6x + 9 - 9)$

$y - 4 = \frac{1}{2}(x^2 - 6x + 9) - \frac{9}{2}$

$y - 4 = \frac{1}{2}(x-3)^2 - \frac{9}{2}$

$y = \frac{1}{2}(x-3)^2 - \frac{9}{2} + 4$

$y = \frac{1}{2}(x-3)^2 - \frac{9}{2} + \frac{8}{2}$

$y = \frac{1}{2}(x-3)^2 - \frac{1}{2}$

$\left(3, -\frac{1}{2}\right)$

$\left(-\frac{b}{2a}, c - \frac{b^3}{4a}\right)$

$\left(\frac{3}{1}, 4 - \frac{9}{2}\right)$

$\left(3, -\frac{1}{2}\right)$

22. $f(x) = 0.6x^2 + 2x - 3$

$0.6 = \frac{6}{10} = \frac{3}{5}$

$\frac{2}{\frac{3}{5}} = 2 \cdot \frac{5}{3} = \frac{10}{3}$

$y + 3 = 0.6\left(x^2 + \frac{10}{3}x\right)$

$y + 3 = \frac{3}{5}\left(x^2 + \frac{10}{3}x + \frac{100}{36} - \frac{100}{36}\right)$

$y + 3 = 0.6\left(x^2 + \frac{10}{3}x + \frac{100}{36}\right) - \frac{5}{3}$

$y = 0.6\left(x + \frac{5}{3}\right)^2 - \frac{5}{3} - 3$

$y = 0.6\left(x + \frac{5}{3}\right)^2 - \frac{5}{3} - \frac{9}{3}$

$\rightarrow y = 0.6\left(x + \frac{5}{3}\right)^2 - \frac{14}{3}$

vertex: $\left(-\frac{5}{3}, -\frac{14}{3}\right)$

$\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$\left(\frac{-2}{2(0.6)}, -3 - \frac{4}{4(0.6)}\right)$

$\rightarrow \left(\frac{-2}{\frac{6}{5}}, -3 - \frac{1}{\frac{3}{5}}\right)$

$\left(-\frac{10}{6}, -3 - \frac{5}{3}\right)$

$\boxed{\left(-\frac{5}{3}, -\frac{14}{3}\right)}$

Sketch the Graph. Label the Vertex and at least four other points

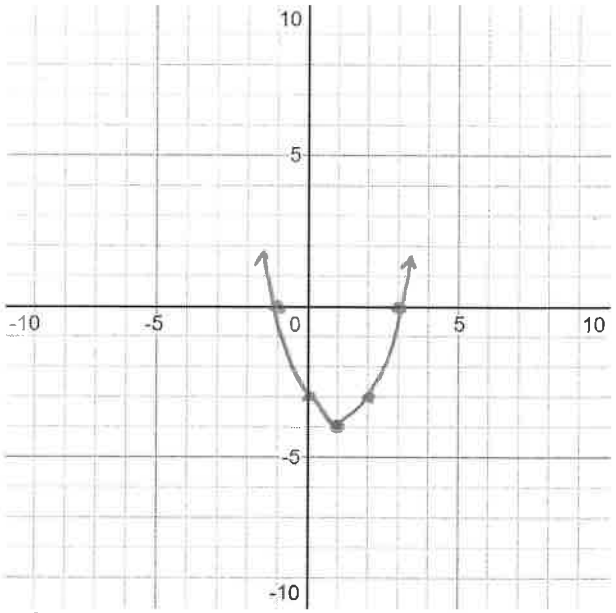
23. $f(x) = x^2 - 2x - 3$

↓

factor

$(x - 3)(x + 1)$

these are x-ints



vertex:

$x^2 - 2x + 1 - 1 - 3$

$(x^2 - 1)^2 - 4$

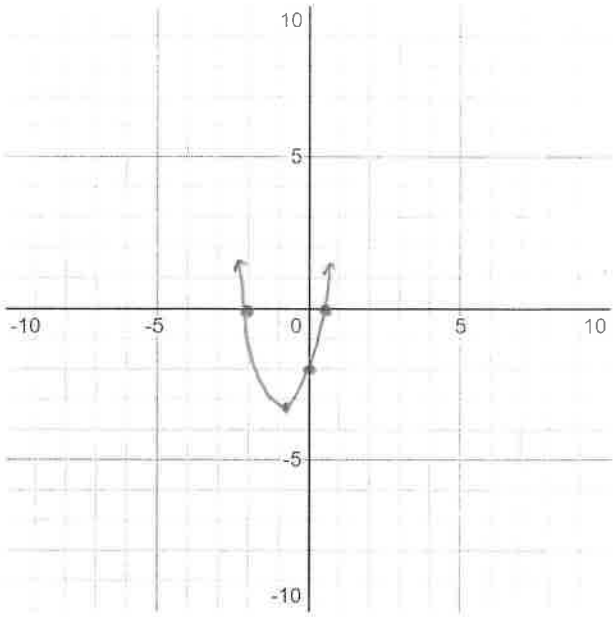
$(1, -4)$

$a = 1$

so

regular shape.

24. $f(x) = 2x^2 + 3x - 2$



factor: AC Method

$x^2 + 3x - 4$

$(x + \frac{4}{2})(x - \frac{1}{2})$

$(x+2)(2x-1)$

roots: $x = -2$

$x = \frac{1}{2}$

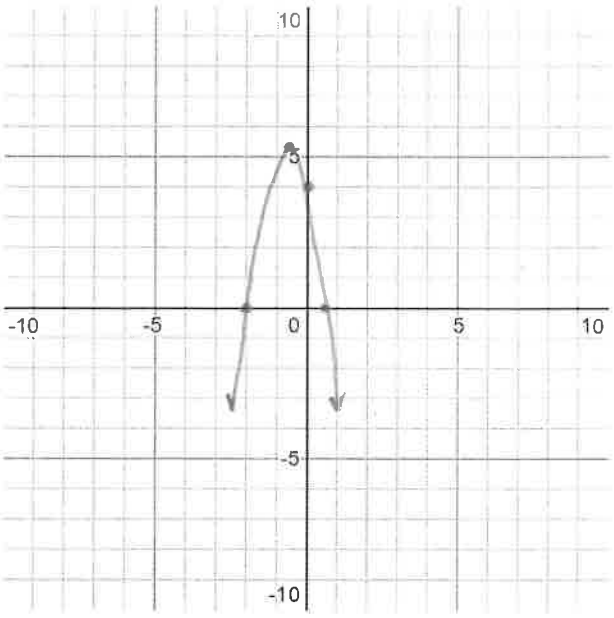
y-int: -2

vertex: $\frac{-b}{2a}, c - \frac{b^2}{4a}$

$(-\frac{3}{4}, -2 - \frac{9}{8}) \rightarrow (-\frac{3}{4}, -3\frac{1}{8})$

---

25. $f(x) = -3x^2 - 4x + 4$



$y = -3(x^2 + \frac{4}{3}x) + 4$

$y = -3(x^2 + \frac{4}{3}x + \frac{4}{9} - \frac{4}{9}) + 4$

$y = -3(x^2 + \frac{4}{3}x + \frac{4}{9}) + \frac{4}{3} + 4$

$y = -3(x + \frac{2}{3})^2 + 5\frac{1}{3}$

$a = -3$

vertex: $(-\frac{2}{3}, 5\frac{1}{3})$

AC Method:

$-(x^2 + 4x - 12)$

$-(x + \frac{6}{3})(x - \frac{2}{3}) \rightarrow -(x+2)(3x-2)$

$x = -2, x = \frac{2}{3}$

y-int: 4

26. $f(x) = -4x^2 + 12x - 5$

Factor: $-(x^2 - 12x + 20)$

$-(x - \frac{2}{4})(x - \frac{10}{4})$

$-(x - \frac{1}{2})(x - \frac{5}{2})$ $x = \frac{1}{2}$

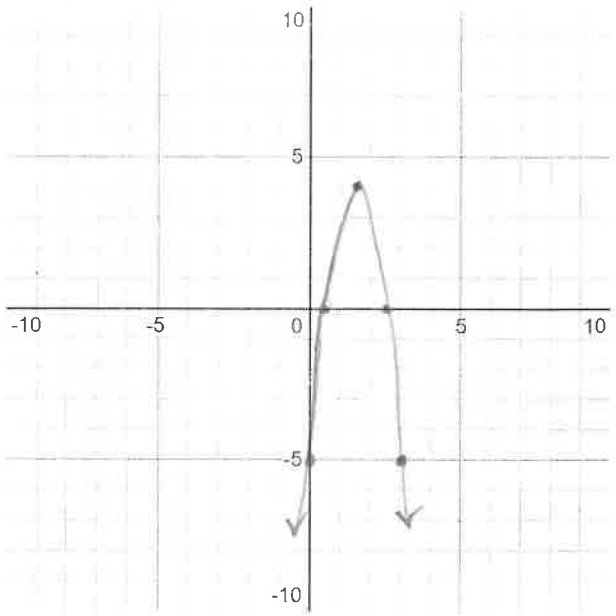
$(2x - 1)(2x - 5)$ $x = \frac{5}{2}$

y-int: $(0, -5)$

vertex: $\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$

$\left(\frac{-12}{-8}, -5 - \frac{144}{-16}\right)$

$\left(\frac{3}{2}, 4\right)$



---

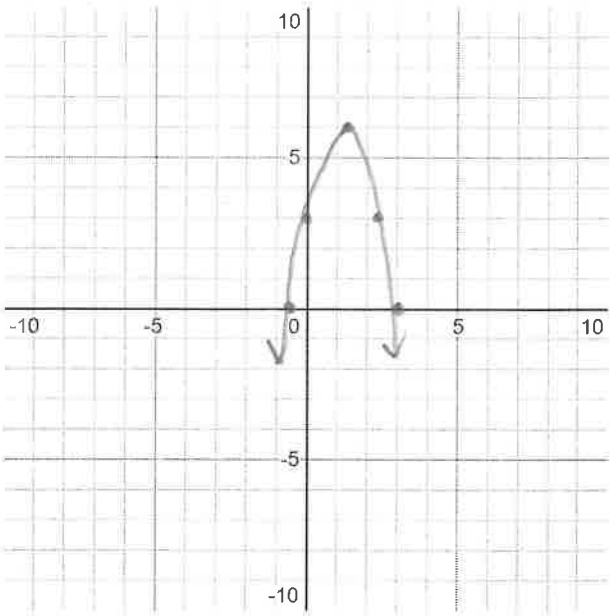
27. $f(x) = 3 + 5x - 2x^2$

$-2x^2 + 5x + 3$

Factor: $-(x^2 - 5x - 6)$

$-(x + \frac{1}{2})(x - \frac{6}{2})$

$-(2x + 1)(x - 3)$



vertex:

$y - 3 = -2(x^2 - \frac{5}{2}x)$

$y - 3 = -2(x^2 - \frac{5}{2}x + \frac{25}{16} - \frac{25}{16})$

$y - 3 = -2(x^2 - \frac{5}{2} + \frac{25}{16}) + \frac{25}{8}$

$y = -2(x - \frac{5}{4})^2 + \frac{25}{8} + 3 \rightarrow y = -2(x - \frac{5}{4})^2 + \frac{49}{8}$

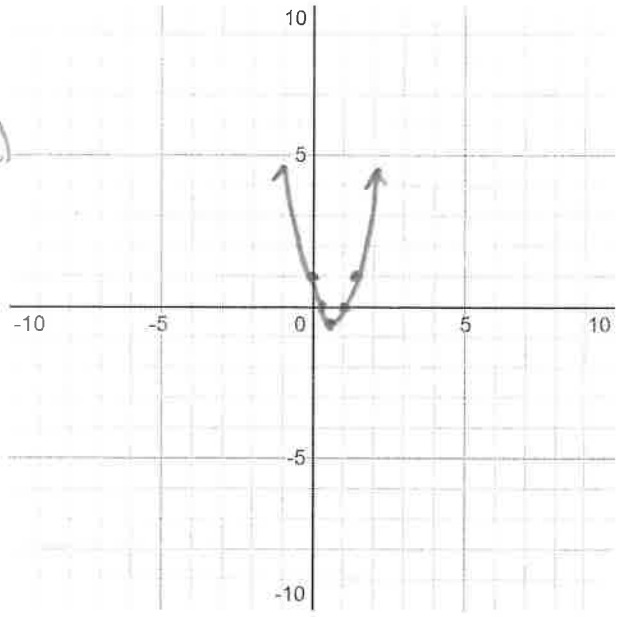
y-int: $(0, 3)$

x-int: $(-\frac{1}{2}, 0)$ $(3, 0)$

vertex: $(\frac{5}{4}, 6.125)$

$\frac{49}{8} = 6.125$

28. $f(x) = 3x^2 - 4x + 1$



y-int: $(0,1)$

x-int: $x^2 - 4x + 3$ (AC Method)

$(x - \frac{3}{3})(x - \frac{1}{3})$

$(x-1)(3x-1)$

x-int: $(1,0)(\frac{1}{3},0)$

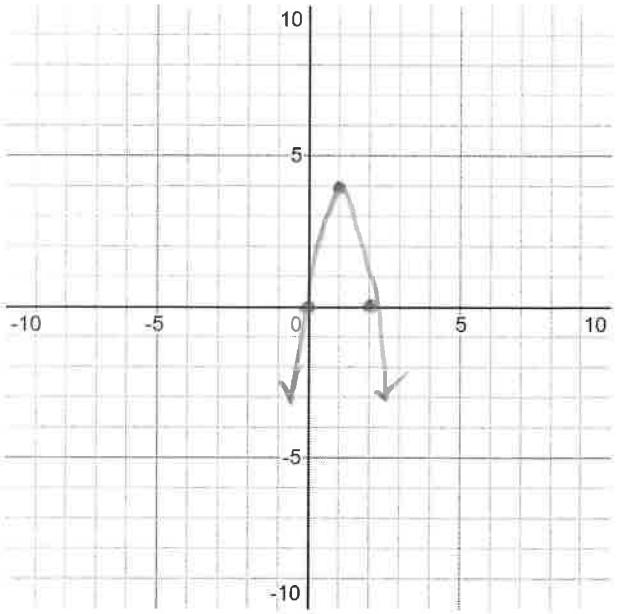
vertex: $\frac{-b}{2a}, c - \frac{b^2}{4a}$

$\frac{4}{6}, 1 - \frac{16}{12}$

$(\frac{2}{3}, 1 - \frac{4}{3}) \rightarrow (\frac{2}{3}, -\frac{1}{3})$

---

29. $f(x) = -4x^2 + 8x$



$-4x(x-2)$

x-int: $(0,0)$ $(2,0)$

y-int: $(0,0)$

vertex: $-4(x^2 - 2x)$

$-4(x^2 - 2x + 1 - 1)$

$-4(x^2 - 2x + 1) + 4$

$-4(x-1)^2 + 4$

30. $f(x) = 2x^2 + 4x + 5$

$2(x^2+2x)+5$

$2(x^2+2x+1-1)+5$

$2(x^2+2x+1)-2+5$

$2(x+1)^2+3$

vertex: $(-1,3)$

opens up so no x-ints

y-int: $(0,5)$