Section 4.2 – The Equation of a Parabola

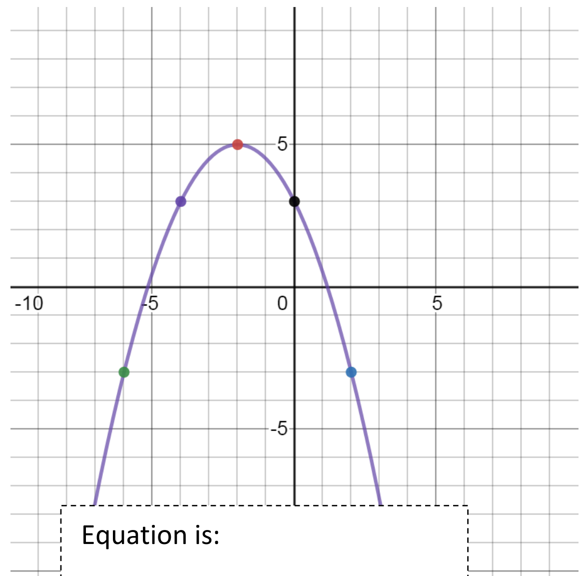
This Booklet Belongs to: ______________________ Block: ________

Finding the Equation of a Parabola from a Graph

Finding the equation of a parabola from a graph requires two things:

1. The vertex
2. The value that determines the shape and direction of the parabola

Example 1: Determine an equal for the parabola.



Solution 1:

Vertex: $(-2,5)$

So, $y = a(x+2)^2 + 5$

- Now, solve for a, by plugging in any point on the line, other than the vertex
- We can use: $(0,3), (-4,3), or\ (2,-3)$

$$y = a(x+2)^2 + 5$$

$$3 = a(0+2)^2 + 5$$

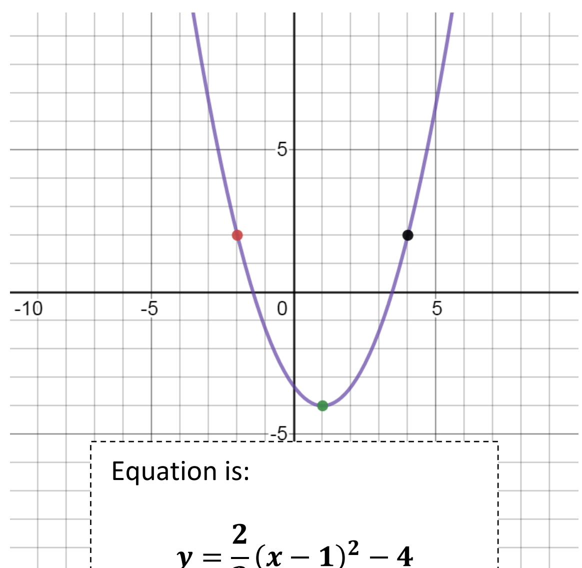
$$3 = 4a + 5$$

$$-2 = 4a \quad \rightarrow \quad a = -\frac{1}{2}$$

Equation is:

$$\boldsymbol{y = -\frac{1}{2}(x+2)^2 + 5}$$

Example 2: Determine an equal for the parabola.



Solution 2:

Vertex: $(1,-4)$

So, $y = a(x-1)^2 - 4$

- Now, solve for a, by plugging in any point on the line, other than the vertex
- We can use: $(4,2)\ or\ (-2,2)$
- I will use (-2.2)

$$y = a(x-1)^2 - 4 \quad \rightarrow \quad 2 = a(-2-1)^2 - 4$$

$$2 = 9a - 4$$

$$6 = 9a \quad \rightarrow \quad a = \frac{2}{3}$$

Equation is:

$$\boldsymbol{y = \frac{2}{3}(x-1)^2 - 4}$$

General Form to Standard Form (Completing the Square)

- Generally quadratic equations come in **General Form** $\boldsymbol{f(x) = ax^2 + bx + c}$
- The good news is that we can change them to **Standard Form** $\boldsymbol{f(x) = a(x-h)^2 + k}$
- We use a technique called **completing the square,** follow the guidelines below

How the equation changes	Steps
$f(x) = ax^2 + bx + c$	• Given Standard Form
$y = ax^2 + bx + c$	• Replace $f(x)$ with y to simplify
$y - c = a(x^2 + \frac{b}{a}x)$	• Add $-c$ to both sides • Factor a out of the right side
$y - c + a\left(\frac{b}{2a}\right)^2 = a(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2)$	• Add $\left(\frac{b}{2a}\right)^2$ to the right side • Add $a * \left(\frac{b}{2a}\right)^2$ to the left to balance
$y - c + \frac{b}{4a}^2 = a(x + \frac{b}{2a})^2$	• Simplify to a perfect square
$y = a(x + \frac{b}{2a})^2 + c - \frac{b}{4a}^2$	• Add c and subtract $\frac{b}{4a}^2$ from both sides
$f(x) = a(x + \frac{b}{2a})^2 + c - \frac{b}{4a}^2$	• Write in the form $f(x) = ax^2 + bx + c$

<u>The Vertex Formula</u>

The Graph $f(x) = ax^2 + bx + c, \ a \neq 0$ is a parabola with **vertex** $(\boldsymbol{h}, \boldsymbol{k})$ and **axis of symmetry of** $\boldsymbol{x = h}$,

where $\boldsymbol{h = -\frac{b}{2a}}$ **and** $\boldsymbol{k = c - \frac{b^2}{2a}}$

If $\boldsymbol{a > 0}$, the parabola has a **minimum value and opens upward**

If $\boldsymbol{a < 0}$, the parabola has a **maximum value and opens downward**

Example 3: Determine the vertex of the equation $f(x) = 2x^2 - 4x - 3$

Solution 3: **Completing the Square**

$$f(x) + 3 = 2x^2 - 4x \quad \rightarrow \quad f(x) + 3 = 2(x^2 - 2x)$$

$$f(x) + 3 + 2(-1)^2 = 2(x^2 - 2x + (-1)^2)$$

$$f(x) + 5 = 2(x^2 - 2x + 1)$$

$$f(x) + 5 = 2(x - 1)^2$$

$$f(x) = 2(x - 1)^2 - 5$$

$$\boldsymbol{Vertex}: (\mathbf{1}, -\mathbf{5})$$

Vertex Formula $f(x) = 2x^2 - 4x - 3$ has $a = 2, b = -4, and\ c = -3$

$$Vertex \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{-4}{2(2)}, -3 - \frac{(-4)^2}{4(2)}\right) = (\mathbf{1}, -\mathbf{5})$$

Therefore the vertex is: $(\mathbf{1}, -\mathbf{5})$

Example 4: Determine the vertex and axis of symmetry for $f(x) = 2x^2 - 4x - 1$

Solution 4: **Completing the Square**

$$f(x) + 1 = 2x^2 - 4x \quad \rightarrow \quad f(x) + 1 = 2(x^2 - 2x)$$

$$f(x) + 1 + 2(-1)^2 = 2(x^2 - 2x + (-1)^2)$$

$$f(x) + 3 = 2(x^2 - 2x + 1)$$

$$f(x) + 3 = 2(x - 1)^2$$

$$f(x) = 2(x - 1)^2 - 3$$

$$\boldsymbol{Vertex}: (\mathbf{1}, -\mathbf{3})$$

Vertex Formula $f(x) = 2x^2 - 4x - 1$ has $a = 2, b = -4, and\ c = -1$

$$Vertex \left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{-4}{2(2)}, -1 - \frac{(-4)^2}{4(2)}\right) = (\mathbf{1}, -\mathbf{3})$$

Therefore the axis of symmetry is: $\boldsymbol{x = 1}$

Example 5: Given the following quadratic $f(x) = -2x^2 + 8x - 3$, determine the vertex, axis of symmetry, max/min, domain and range

Solution 5: $f(x) = -2x^2 + 8x - 3$ has $a = -2, b = 8, and\ c = -3$

Completing the Square

$$f(x) + 3 = -2x^2 + 8x \quad \rightarrow \quad f(x) + 3 = -2(x^2 - 4x)$$

$$f(x) + 3 + (-2)(-2)^2 = -2(x^2 - 4x + (-2)^2)$$

$$f(x) + 3 - 8 = -2(x^2 - 4x + 4)$$

$$f(x) + 3 - 8 = -2(x - 2)^2$$

$$f(x) = -2(x - 2)^2 + 5$$

$$\boldsymbol{Vertex}: (\mathbf{2}, \mathbf{5})$$

$$Vertex\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right) = \left(-\frac{8}{2(-2)}, -3 - \frac{(8)^2}{4(-2)}\right) = (\mathbf{2}, \mathbf{5})$$

Therefore the vertex is: $(\mathbf{2}, \mathbf{5})$

Plotting 4 other Points: $f(x) = -2x^2 + 8x - 3$

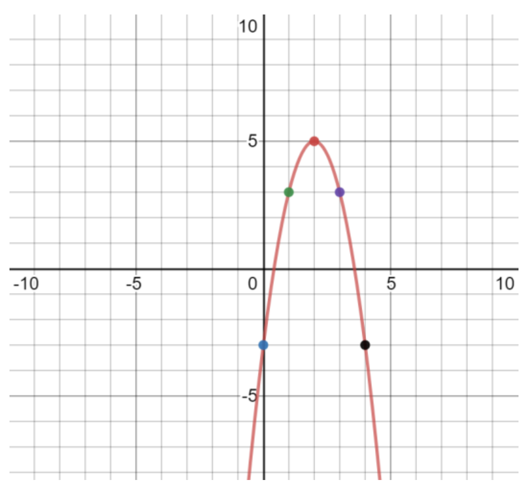
$f(0) = -2(0)^2 + 8(0) - 3 = -3$

$f(1) = -2(1)^2 + 8(1) - 3 = 3$

$f(3) = -2(3)^2 + 8(3) - 3 = 3$

$f(4) = -2(4)^2 + 8(4) - 3 = -3$

x	$f(x)$
0	-3
1	3
3	3
4	-3



Vertex: $(\mathbf{2}, \mathbf{5})$ **Axis of Symmetry:** $\boldsymbol{x = 2}$
Max/Min: $\boldsymbol{Maximum\ at\ 5}$
Domain: $\boldsymbol{All\ Real\ Numbers}$ **Range:** $\boldsymbol{y \leq 5}$

Example 6: Given that $f(x)$ is a quadratic function with minimum $f(1) = -3$, find the vertex, axis of symmetry, domain and range

Solution 6: $f(1) = -3$ means the point $(1, -3)$, so the **vertex is** $(\mathbf{1}, -\mathbf{3})$

The **Axis of Symmetry is:** $\boldsymbol{x = 1}$

Domain: All Real Numbers **Range:** $\boldsymbol{y \geq 3}$

Example 7: Determine a quadratic function with vertex $(2, 1)$ and $y - intercept: -3$

Solution 7:

The **Standard Form** of a Quadratic Function is: $y = \boldsymbol{a}(\boldsymbol{x} - \boldsymbol{h})^2 + \boldsymbol{k}$, **so** $\boldsymbol{y = a(x - 2)^2 + 1}$

The $\boldsymbol{y - int}$ means that the **graph crosses the y-axis**, at $\boldsymbol{x = 0}$, so it **crosses at:** $(\boldsymbol{0}, \boldsymbol{-3})$

So now we can **input** and **solve for $\boldsymbol{a}$**:

$-3 = a(0 - 2)^2 + 1$

$-3 = 4a + 1$

$-4 = 4a \quad \rightarrow \quad \boldsymbol{a = -1}$ **Thus:** $\boldsymbol{y = -(x - 2)^2 + 1}$

Example 8: Find the vertex and $x - intercepts$ of $f(x) = 2x^2 + 5x - 3$

Solution 8:

$$f(x) = 2x^2 + 5x - 3 \quad \rightarrow \quad f(x) + 3 = 2(x^2 + \frac{5}{2})$$

$$f(x) + 3 + 2\left(\frac{5}{4}\right)^2 = 2(x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2)$$

$$f(x) + 3 + \frac{25}{8} = 2(x^2 + \frac{5}{2}x + \frac{25}{16})$$

$$f(x) + \frac{49}{8} = 2\left(x + \frac{5}{4}\right)^2 \quad \rightarrow \quad \boldsymbol{f(x) = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}}$$

$$x - intercepts\ found\ by\ factoring\ (AC\ Method) \quad \rightarrow \quad 2x^2 + 5x - 3 = 0$$

$$x^2 + 5x - 6 = 0 \quad \rightarrow \quad (x + 6)(x - 1) = 0$$

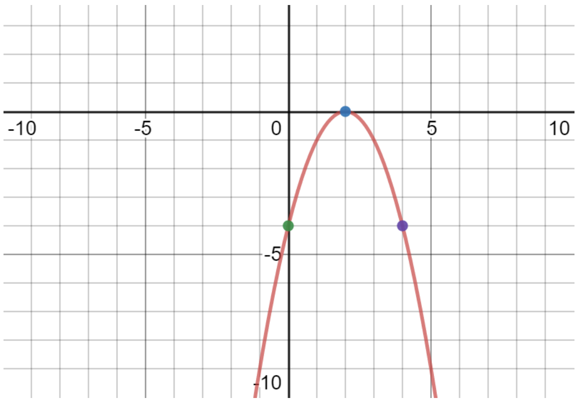
$$\left(x + \frac{6}{2}\right)\left(x - \frac{1}{2}\right) = 0 \quad \rightarrow \quad (x + 3)(2x - 1) = 0$$

So $\boldsymbol{Vertex\ is}$: $\boldsymbol{(-\frac{5}{4}, -\frac{49}{8})}$ $\boldsymbol{x - intercepts\ are}$: $\boldsymbol{x = -3,\ x = \frac{1}{2}}$

Section 4.2 – Practice Problems

Determine the equation of the following parabolas

1.



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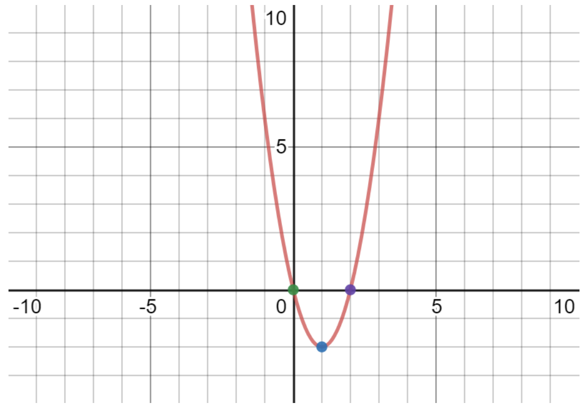

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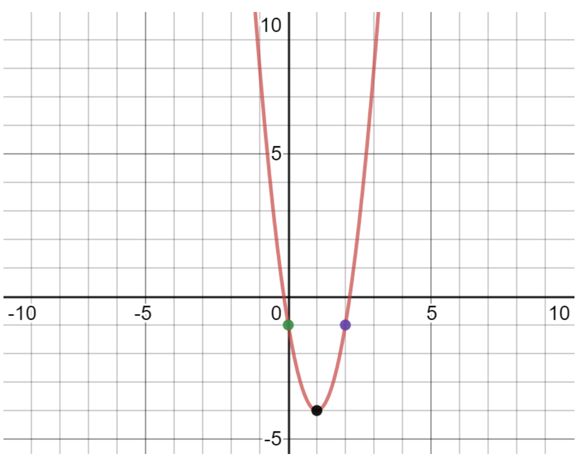

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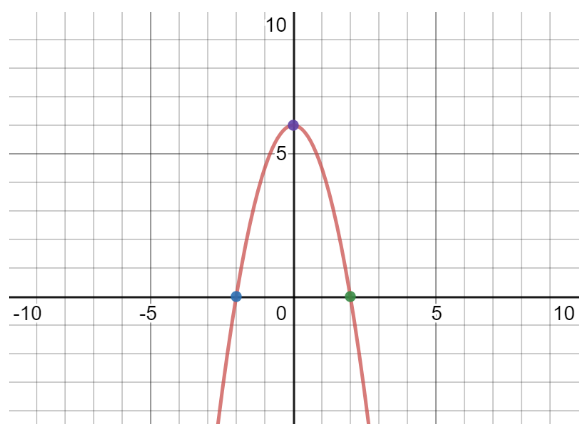

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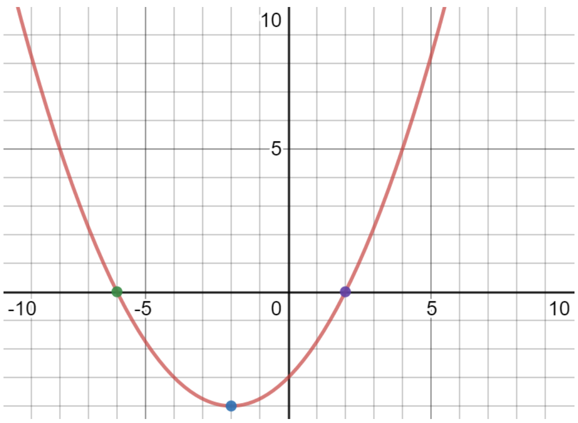

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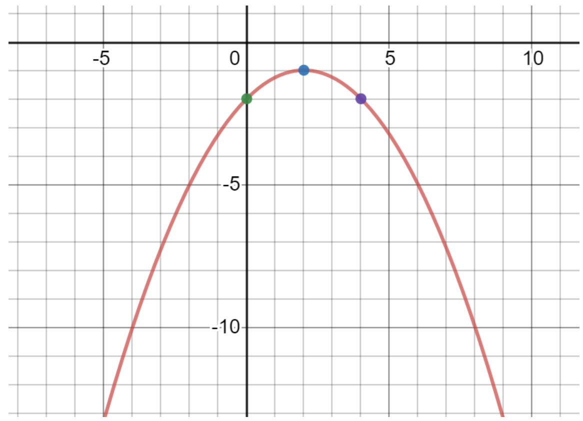

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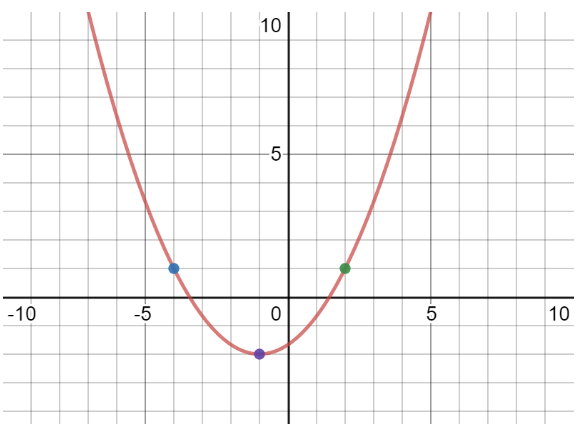

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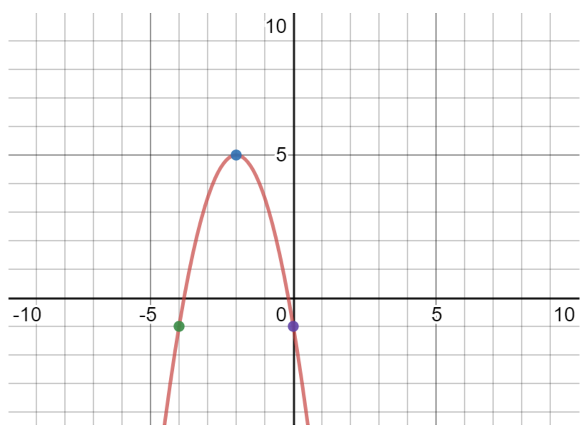


Find the equation of a quadratic function whose graph satisfies the given coordinates.

9. $vertex: (2, 9)$ $x - intercept:\ 5$

10. $vertex: (-2, 12)$ $x - intercept:\ -4$

11. $vertex: (1, -4)$ $x - intercept:\ -2$

12. $vertex: (-4, 12)$ $x - intercept: 4$

13. $vertex: (-3, -5)$ $y - intercept: 1$

14. $vertex: (2, 4)$ $y - intercept:\ -3$

15. $vertex: (1, 4)$ $point$: (2, 3)

16. $vertex: (-2, -4)$ $point: (-3, -1)$

Find the Vertex by completing the square and using the vertex formula

17. $f(x) = x^2 + 4x + 3$

18. $f(x) = x^2 - 8x + 15$

19. $f(x) = x^2 + 3x - 8$

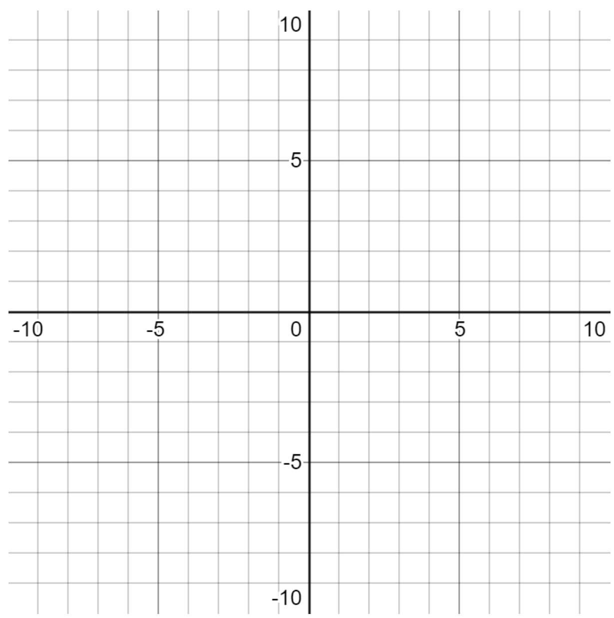
20. $f(x) = 3x^2 - 18x + 25$

21. $f(x) = \frac{1}{2}x^2 - 3x + 4$

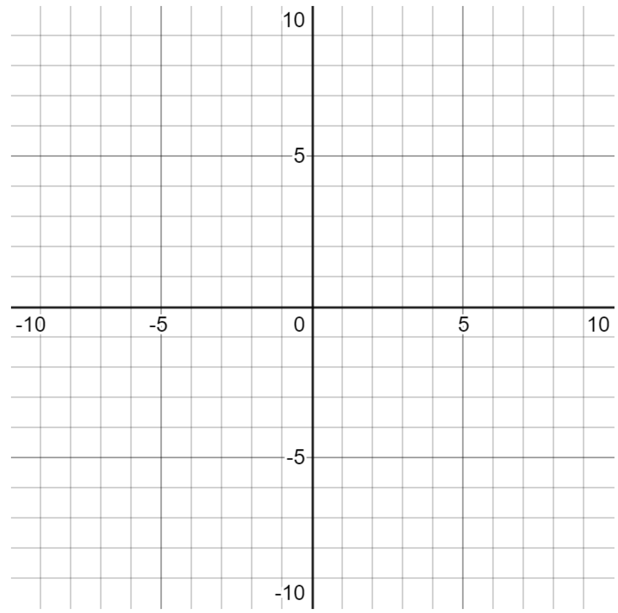
22. $f(x) = 0.6x^2 + 2x - 3$

Sketch the Graph. Label the Vertex and at least four other points

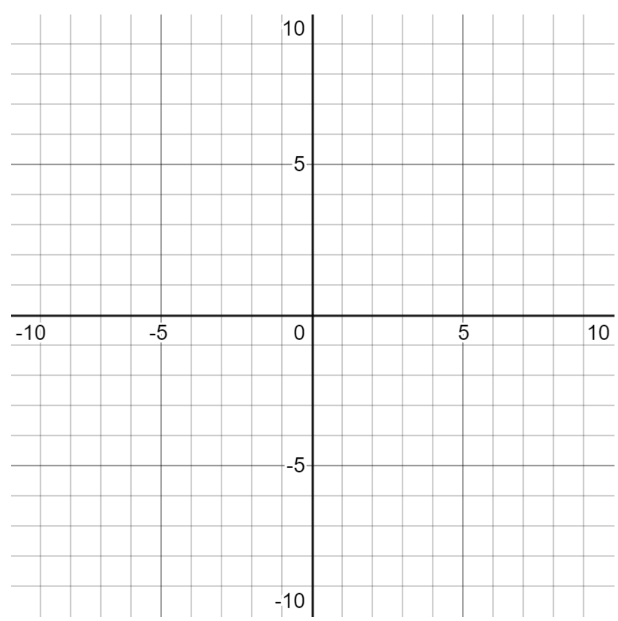
23. $f(x) = x^2 - 2x - 3$



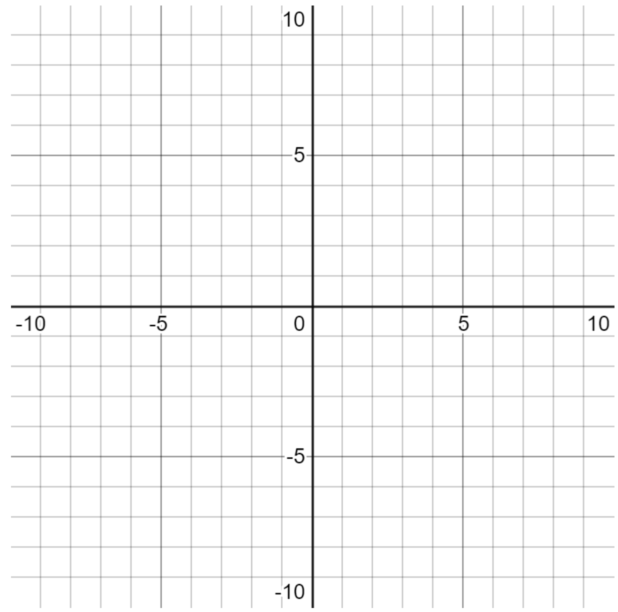
24. $f(x) = 2x^2 + 3x - 2$



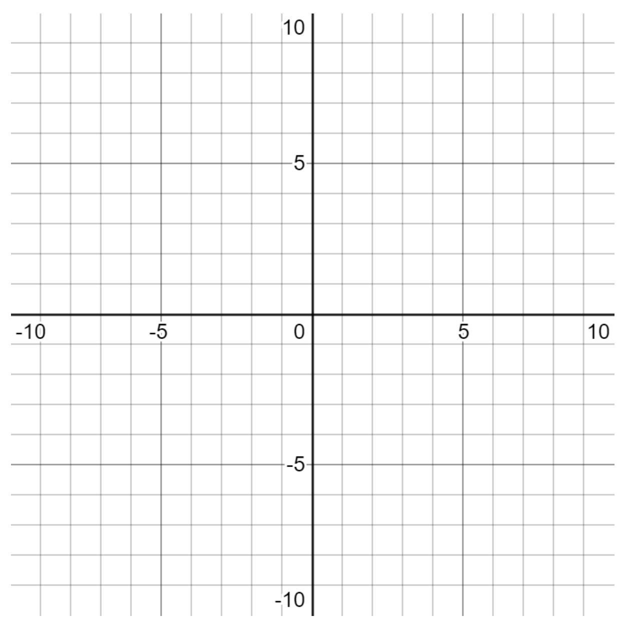
25. $f(x) = -3x^2 - 4x + 4$



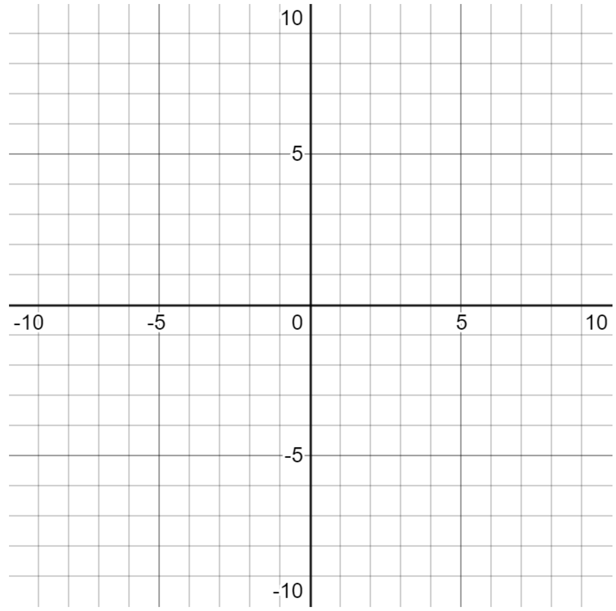
26. $f(x) = -4x^2 + 12x - 5$



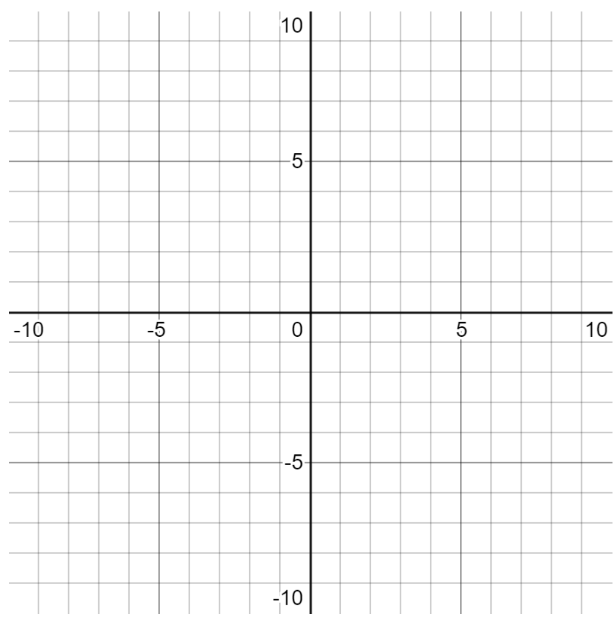
27. $f(x) = 3 + 5x - 2x^2$



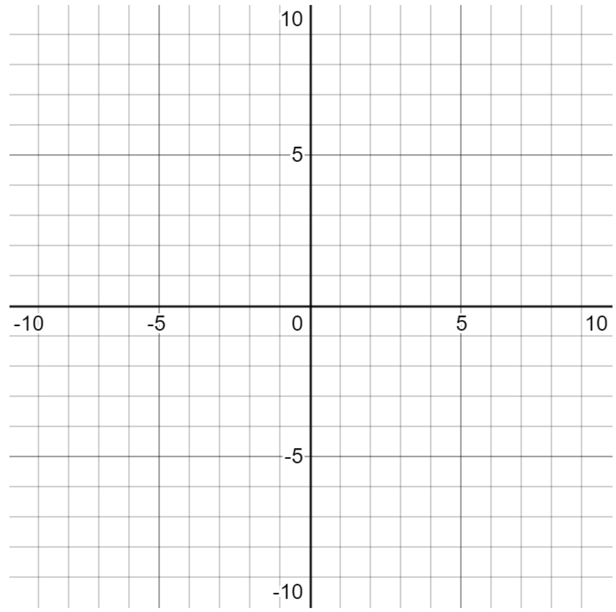
28. $f(x) = 3x^2 - 4x + 1$



29. $f(x) = -4x^2 + 8x$



30. $f(x) = 2x^2 + 4x + 5$



Section 4.2 – Answer Key

1. $f(x) = -(x-2)^2$	23. $See\ Website$
2. $f(x) = 2(x-1)^2 - 2$	24. $See\ Website$
3. $f(x) = 3(x-1)^2 - 4$	25. $See\ Website$
4. $f(x) = -\frac{3}{2}(x)^2 + 6$	26. $See\ Website$
5. $f(x) = \frac{1}{4}(x+2)^2 - 4$	27. $See\ Website$
6. $f(x) = -\frac{1}{4}(x-2)^2 - 1$	28. $See\ Website$
7. $f(x) = \frac{1}{3}(x+1)^2 - 2$	29. $See\ Website$
8. $f(x) = -\frac{3}{2}(x+2)^2 + 5$	30. $See\ Website$
9. $f(x) = -(x-2)^2 + 9$	
10. $f(x) = -3(x+2)^2 + 12$	
11. $f(x) = \frac{4}{9}(x-1)^2 - 4$	
12. $f(x) = -\frac{3}{16}(x+4)^2 + 12$	
13. $f(x) = \frac{2}{3}(x+3)^2 - 5$	
14. $f(x) = -\frac{7}{4}(x-2)^2 + 4$	
15. $f(x) = -(x-1)^2 + 4$	
16. $f(x) = 3(x+2)^2 - 4$	
17. $(-2, -1)$	
18. $(4, -1)$	
19. $(-\frac{3}{2}, -\frac{41}{4})$	
20. $(3, -2)$	
21. $(3, -\frac{1}{2})$	
22. $-\frac{5}{3}, -\frac{14}{3})$	