Section 4.2 – Proofs

- Proofs are an integral part of mathematics, particularly geometry
- Using **postulates** (assumptions accepted without proof), **theorems** (statements that have been proved), and **definitions**, we can justify steps in our proof
- Let's first look at some required vocabulary

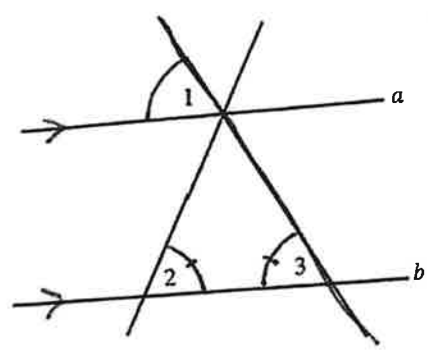
Terminology

1. $AB \| \| CD$	- Line AB is parallel to line CD
2. $\overline{AB}$ *bisects* $\angle BAC$	- Line segment AB divides angle BAC into two equal part
3. $l_1 \perp l_2$	- This symbol means perpendicular, the two lines form a right angle
4. Complementary	- Means $\angle 1 + \angle 2 = 90°$
5. Supplementary	- Means $\angle 1 + \angle 2 = 180°$
6. Perpendicular Bisector	- Means a line segment that is $\perp$ to the segment at its midpoint

Example: Given: $a \| b$ and $\angle 2 = \angle 3$

Prove: $\angle 1 = \angle 2$

a = b
b = c
a = c !



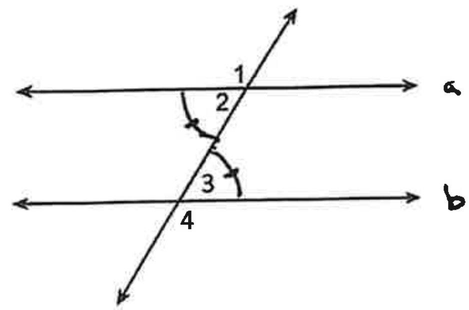
Solution:

Proof

Statement	Reason
a ∥ b	Given
∠1 = ∠3	Corresponding angles equal
∠2 = ∠3	Given
∠1 = ∠2	substitution

Example: Given: $\angle 2 = \angle 3$ $a || b$

Prove: $\angle 1 = \angle 4$



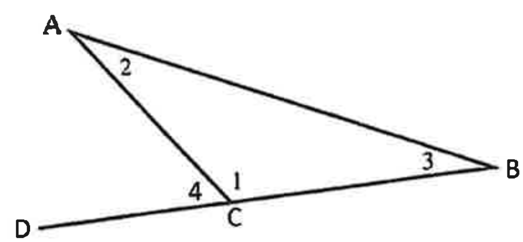
Solution:

Proof

Statement	Reason
$\angle 2 = \angle 3$	Given
$a \parallel b$	Given
$\angle 1 + \angle 2 = 180°$	Supplementary Angles
$\angle 3 + \angle 4 = 180°$	" "
$\angle 1 + \angle 2 = \angle 3 + \angle 4$	Both equal 180°
$\angle 1 + \angle 3 = \angle 3 + \angle 4$	Substitution
$\angle 1 = \angle 4$	Subtraction (algebra)

Example: Given: $\Delta ABC\ with\ \overline{DCB}$

Prove: $\angle 4 = \angle 2 + \angle 3$



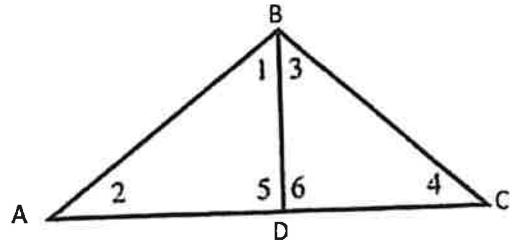
Solution:

Proof

Statement	Reason
$\angle 1 + \angle 2 + \angle 3 = 180°$	sum angles in Δ
$\angle 4 + \angle 1 = 180°$	supplementary angles
$\angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 1$	Both equal 180°
$\angle 2 + \angle 3 = \angle 4$	subtraction
$\angle 4 = \angle 2 + \angle 3$	rewrite

Example: Given: $BD \perp AC$ and $\angle 1 = \angle 3$

Prove: $\angle 2 = \angle 4$

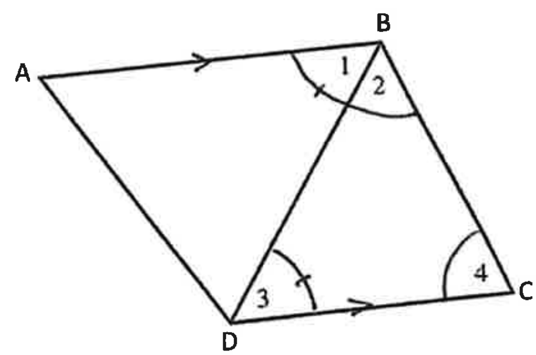


Solution:

Proof Statement	Reason
$\angle 1 + \angle 2 + \angle 5 = 180°$	sum angles in a triangle
$\angle 3 + \angle 4 + \angle 6 = 180°$	sum angles in a triangle
$\angle 1 + \angle 2 + \angle 5 = \angle 3 + \angle 4 + \angle 6$	Both sum to 180°
$\angle 1 = \angle 3$	Given
$\angle 5 = \angle 6 = 90°$	$BD \perp AC$
$\cancel{\angle 1} + \angle 2 + \cancel{\angle 5} = \cancel{\angle 1} + \angle 4 + \cancel{\angle 5}$	Substitution
$\angle 2 = \angle 4$	subtraction (algebra)

Example: Given: $AB \parallel CD$ in ABCD

Prove: $\angle ABC$ is Supplementary to $\angle 4$



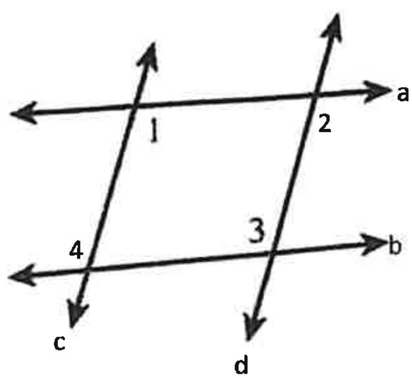
Solution:

Proof Statement	Reason
$AB \parallel CD$	Given
$\angle 1 = \angle 3$	Alternate interior angles
$\angle 2 + \angle 3 + \angle 4 = 180°$	Sum angles of Δ
$\angle 2 + \angle 1 + \angle 4 = 180°$	substitution
$\angle ABC = \angle 1 + \angle 2$	Angle sum
$\angle ABC + \angle 4 = 180°$	substitution
$\angle ABC$ supp $\angle 4$	By definition

Practice Problems #1-8

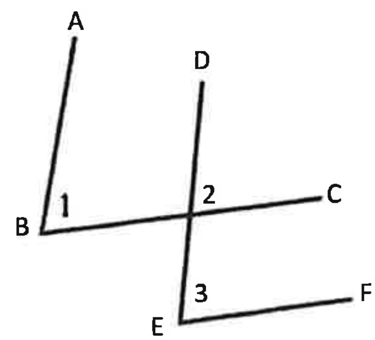
Section 4.2 – Practice Problems

1. Given: $c||d$ and $\angle 1 = \angle 3$
   Prove: $a||b$



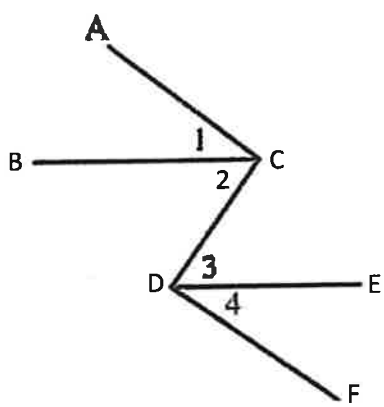
Proof	Statement	Reason
1.	$c \parallel d$	
2.		Given
3.	$\angle 3 = \angle 4$	
4.	$\angle 1 = \angle 4$	
5.	$a \parallel b$	Alternate Interior $\angle$'s

2. Given: $\angle 1 = \angle 2$ and $\angle 1 = \angle 3$
   Prove: $BC||EF$



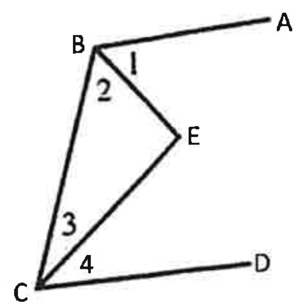
Proof	Statement	Reason
1.	$\angle 1 = \angle 2$	
2.	$\angle 1 = \angle 3$	
3.	$\angle 2 = \angle 3$	
4.		

3. Given: $\angle ACD = \angle CDF$ and $\angle 1 = \angle 4$
   Prove: $BC||DE$



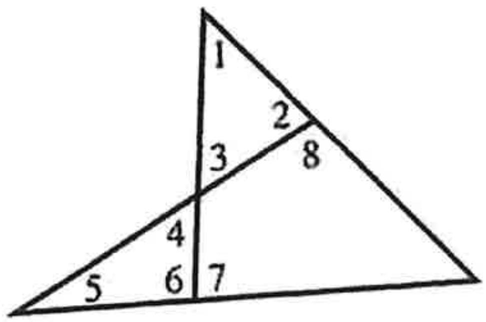
Proof	Statement	Reason
1.	$\angle ACD = \angle CDF$	
2.	$\angle 1 + \angle 2 = \angle 3 + \angle 4$	Addition of Angles
3.	$\angle 1 = \angle 4$	
4.	$\angle 1 + \angle 2 = \angle 3 + \angle 1$	
5.	$\angle 2 = \angle 3$	
6.	$BC \parallel DE$	

4. Given: BE *bisects* $\angle ABC$ and CE *bisects* $\angle BCD$
   $\angle 2 + \angle 3 = 90°$
   Prove: $AB||CD$



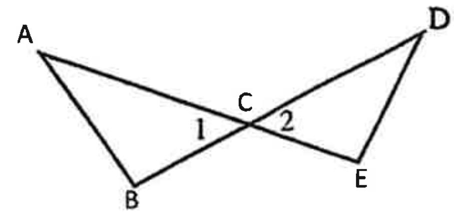
Proof	Statement	Reason
1.	BE bisects $\angle ABC$	
2.	$\angle 1 = \angle 2$	Definition of Bisect
3.	CE bisects $\angle BCD$	
4.		
5.	$\angle 2 + \angle 3 = 90°$	
6.	$\angle 2 + \angle 2 + \angle 3 + \angle 3 =$	
7.	$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$	Addition
8.	$AB \parallel CD$	

5. Given: $\angle 1 = \angle 5$
Prove: $\angle 7 = \angle 8$



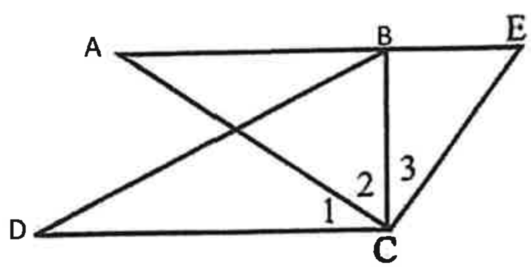
Proof	Statement	Reason
1.	$\angle 1 = \angle 5$	
2.		Vertical Angles
3.		Third Angle of a Triangle
4.		Supplementary Angles
5.		Equal Angles from Supplementary

6. Given: $AB \perp BD$ and $DE \perp AE$
Prove: $\angle A = \angle D$



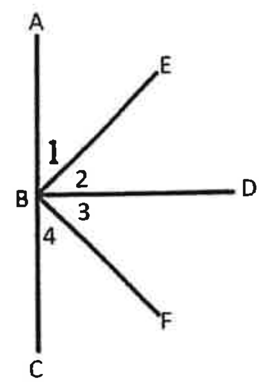
Proof	Statement	Reason
1.	$AB \perp BD$	
2.	$DE \perp AE$	
3.	$\angle B = \angle E$	
4.	$\angle 1 = \angle 2$	
5.	$\angle A = \angle D$	

7. Given: $BC \perp CD$ and $AC \perp CE$
Prove: $\angle 1 = \angle 3$



Statement	Reason

8. Given: $AC \perp BD$ and BD *bisects* $\angle EBF$
Prove: $\angle 1 = \angle 4$



Statement	Reason