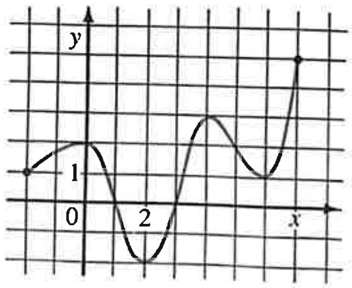
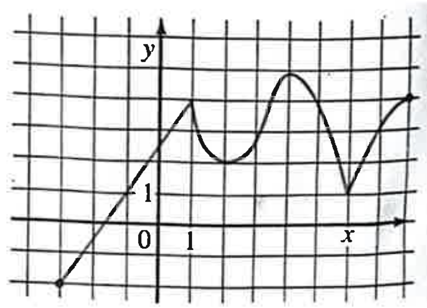
## Section 4.2 – Practice Problems

1. Given the functions below:

i.




ii.




State the…

a) Absolute Maximum Value

i) $f(7) = 5$ ii) $f(4) = 5$

b) Absolute Minimum Value

i) $f(2) = -2$ ii) $f(-3) = -2$

c) Local Maximum Values

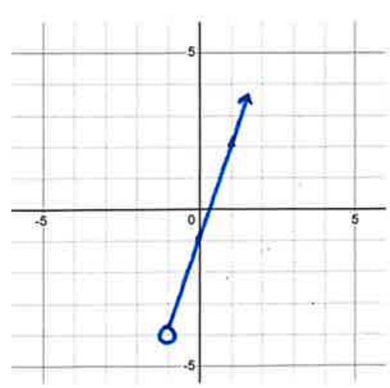
i) $f(0) = 2$, $f(4) = 3$ ii) $f(1) = 4$, $f(4) = 5$

d) Local Minimum Values

i) $f(2) = -2$, $f(6) = 1$ ii) $f(2) = 2$, $f(6) = 1$

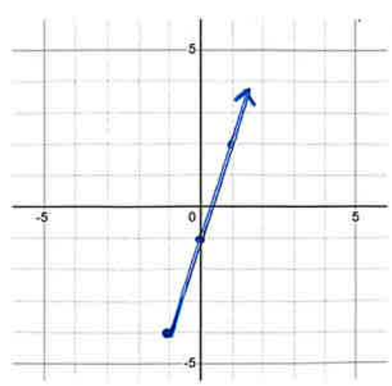
2. Sketch the graph of each function and use it to state the absolute and local maximum and minimum values of the functions.

a) $f(x) = 3x - 1, x > -1$




no min or max

b) $g(x) = 3x - 1, x \geq -1$




absolute min: $g(-1) = -4$

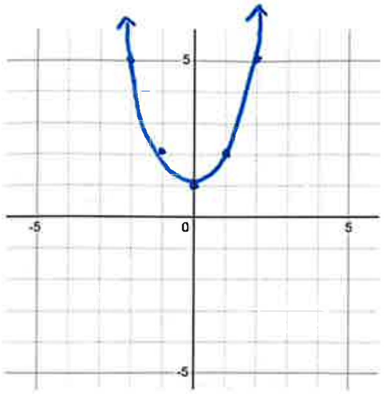
no max

c) $f(x) = x^2 + 1$

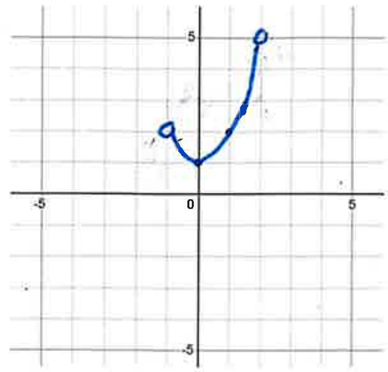
local/absolute min

$f(0) = 1$

no max



d) $y = x^2 + 1, -1 < x < 2$



local/absolute min

$f(0) = 1$

no max

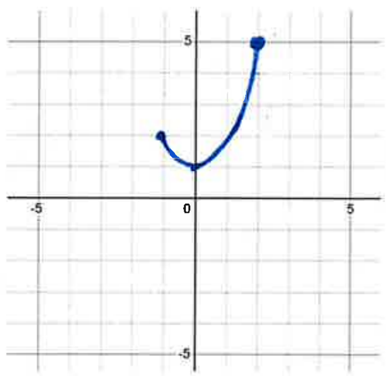
e) $y = x^2 + 1, -1 \leq x \leq 2$

absolute/local min

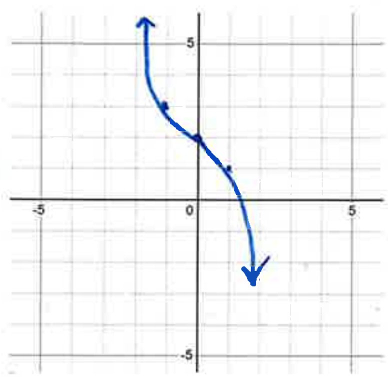
$f(0) = 1$

abs max

$f(2) = 5$



f) $y = 2 - x^3$



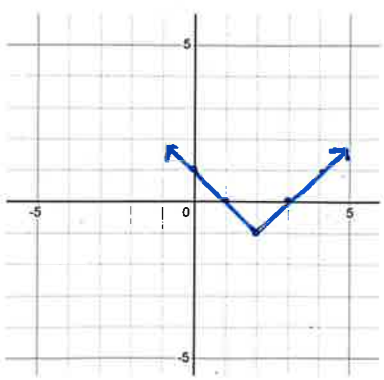
no max or min

g) $y = |x - 2| - 1$

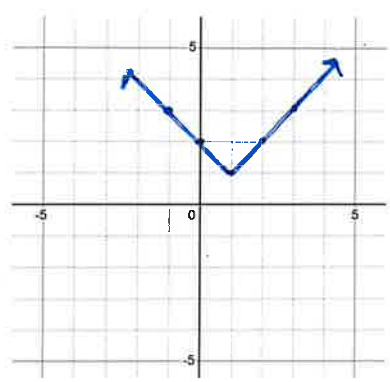
no max

abs/local min

$f(2) = -1$



h) $f(x) = \begin{cases} 2 - x & \text{if } x < 1 \\ x & \text{if } x \geq 1 \end{cases}$



no max

abs/local min

$f(1) = 1$

3. Find the critical numbers of the given functions.

a) $f(x) = 17 - 6x + 12x^2$

$f'(x) = -6 + 24x$

$0 = -6 + 24x$

$6 = 24x$

$\boxed{x = \frac{1}{4}}$

b) $f(x) = x^3 - 3x + 2$

$f'(x) = 3x^2 - 3$

$0 = 3x^2 - 3$

$3 = 3x^2$

$x^2 = 1$

$\boxed{x = \pm 1}$

c) $g(x) = x^4 - 4x^3 - 8x^2 - 1$

$g'(x) = 4x^3 - 12x^2 - 16x$

$\rightarrow 4x(x^2 - 3x - 4)$

$4x(x-4)(x+1)$

$\boxed{x = 0, -1, 4}$

d) $g(x) = 3x^4 - 16x^3 + 6x^2 + 72x + 8$

$g'(x) = 12x^3 - 48x^2 + 12x + 72$

$= 12(x^3 - 4x^2 + x + 6)$

$g'(-1) = 0$

$$\begin{array}{r|l} & x^2 - 5x + 6 \\ x+1 & x^3 - 4x^2 + x + 6 \\ & x^3 + x^2 \\ & -5x^2 + x \\ & -5x^2 - 5x \\ & 6x + 6 \end{array}$$

$x^2 - 5x + 6$

$(x-2)(x-3)$

$\boxed{x = -1, 2, 3}$

e) $y = 2x^3 + 3x^2 - 6x + 3$

$y' = 6x^2 + 6x - 6$

$6(x^2 + x - 1)$

Quad Eqn

$\frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$

$\frac{-1 \pm \sqrt{5}}{2}$

$\boxed{x = \frac{-1 \pm \sqrt{5}}{2}}$

f) $y = x^3 + x^2 + x + 1$

$y' = 3x^2 + 2x + 1$

Quad Eqn

$\frac{-2 \pm \sqrt{4 - 4(3)(1)}}{2}$

$\frac{-2 \pm \sqrt{-8}}{2} \leftarrow$ NOPE

$\boxed{\text{NONE}}$

g) $y = |x + 6|$

$$y = \begin{cases} x + 6 & x \geq -6 \\ -x - 6 & x < -6 \end{cases}$$

$\boxed{x = -6}$

$$y' = \begin{cases} 1 & x \geq -6 \\ -1 & x < -6 \end{cases}$$

$\leftarrow$ crit value

h) $y = \sqrt[3]{x}$

$y' = \frac{1}{3}x^{-2/3}$

$\boxed{x=0}$ is undefined but still a crit value

i) $y = x - \sqrt{x}$

$y' = 1 - \frac{1}{2x^{\frac{1}{2}}}$

$\boxed{x=0}$ undefined

$\frac{2x^{\frac{1}{2}} - 1}{2x^{\frac{1}{2}}} \rightarrow 2x^{\frac{1}{2}} - 1 = 0$

$2x^{\frac{1}{2}} = 1$

$x^{1/2} = \frac{1}{2}$

$\boxed{x = \frac{1}{4}}$

j) $y = x\sqrt{x-1}$

$y' = x\left(\frac{1}{2\sqrt{x-1}}\right) + \sqrt{x-1}$

$\frac{x}{2\sqrt{x-1}} + \frac{2(x-1)}{2\sqrt{x-1}}$

$\frac{x+2x-2}{2\sqrt{x-1}} = \frac{3x-2}{2\sqrt{x-1}}$

$\boxed{x=1}$ Und

$\boxed{x = 2/3}$

k) $y = \frac{t}{t+1}$

$y' = \frac{(t+1)(1) - t(1)}{(t+1)^2} = \frac{t+1-t}{(t+1)^2} = \frac{1}{(t+1)^2}$

$\boxed{t=-1}$ und

l) $y = \frac{t}{t^2+1}$

$y' = \frac{(t^2+1)1 - [2t(t)]}{(t^2+1)^2} \rightarrow \frac{t^2+1-2t^2}{(t^2+1)^2} \rightarrow \frac{1-t^2}{(t^2+1)^2}$

$\frac{(1-t)(1+t)}{(t^2+1)^2}$

$\boxed{t = \pm 1}$

4. Find the absolute maximum value and absolute minimum value of the function. NEED CRIT POINTS AND END POINTS

a) $f(x) = 2x^2 - 8x + 1, 0 \leq x \leq 3$

$f'(x) = 4x - 8$

$x = 2$

$f(2) = -7$ $f(3) = -5$

$f(0) = 1$

$\boxed{\begin{array}{l} f(0) = 1 \\ f(2) = -7 \end{array}}$

b) $f(x) = 3 + 2(x+1)^2, -3 \leq x \leq 2$

$f'(x) = 4(x+1)(1)$

$= 4x + 4$

$x = -1$

$f(-3) = 11$

$f(2) = 21$

$f(-1) = 3$

$\boxed{\begin{array}{l} f(2) = 21 \\ f(-1) = 3 \end{array}}$

c) $f(x) = 2x^3 - 3x^2, -2 \le x \le 2$

$f'(x) = 6x^2 - 6x$
$= 6x(x-1)$

$x = 0$
$x = 1$

$f(0) = 0$ $f(2) = 4$
$f(1) = -1$
$f(-2) = -28$

$\boxed{f(2) = 4 \quad f(-2) = -28}$

d) $f(x) = 2x^3 - 3x^2 - 36x + 62, -3 \le x \le 4$

$f'(x) = 6x^2 - 6x - 36$
$= 6(x^2 - x - 6) \rightarrow 6(x-3)(x+2)$

$x = 3$
$x = -2$

$f(-3) = 89$ $f(-2) = 106$
$f(4) = -2$
$f(3) = -19$

$\boxed{f(-2) = 106 \quad f(3) = -19}$

e) $f(x) = x^4 - 2x^2 + 16, -3 \le x \le 2$

$f'(x) = 4x^3 - 4x$
$= 4x(x^2 - 1)$

$x = \pm 1$
$x = 0$
$x = -3$
$x = 2$

$f(1) = 15$
$f(2) = 24$
$f(-3) = 79$
$f(0) = 16$ $f(-1) = 15$

$\boxed{f(1) = 15 \quad f(-3) = 79}$ $\boxed{f(-1) = 15}$

f) $f(x) = x^5 + 3x^3 + x, -1 \le x \le 2$

$f'(x) = 5x^4 + 9x^2 + 1$

never 0

$\boxed{f(-1) = -5 \quad f(2) = 58}$

g) $g(x) = x^2 + \frac{16}{x}, 1 \le x \le 4$

$g'(x) = 2x - \frac{16}{x^2} \rightarrow \frac{2x^3 - 16}{x}$

$x = 2$
$x = 0$
$x = 1$
$x = 4$

$g(0) =$ und
$g(1) = 17$
$g(2) = 12$
$g(4) = 16$

$\boxed{g(2) = 12 \quad g(1) = 17}$

h) $f(x) = 3x^{\frac{2}{3}} - 2x, 1 \le x \le 3$

$f'(x) = 2x^{-\frac{1}{3}} - 2$
$= \frac{2}{x^{\frac{1}{3}}} - 2 \rightarrow \frac{2 - 2x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$

$x = 1$
$x = 3$
$x = 0$

$f(0) = 0 \leftarrow$ not in interval

$\boxed{f(1) = 1 \quad f(3) \doteq 0.24}$

i) $f(x) = (x^2 - 9)^{\frac{2}{3}}, -6 \le x \le 6$

$f'(x) = \frac{2}{3}(x^2-9)^{-\frac{1}{3}} \cdot 2x \rightarrow \frac{2 \cdot 2x}{3\sqrt[3]{x^2-9}}$

$x = \pm 3$
$x = 0$
$x = 6$
$x = -6$

$\boxed{f(6) = 9 \quad f(-6) = 9 \quad f(3) = 0 \quad f(-3) = 0}$

$f(0) \doteq 4.32$

j) $f(x) = |2x - 1| - 1, 0 \le x \le 2$

$f(x) = \begin{cases} 2x - 1 - 1 & x \ge \frac{1}{2} \\ -2x & x < \frac{1}{2} \end{cases}$

$x = \frac{1}{2}$
$x = 0$
$x = 2$

$f(0) = 0$
$f(\frac{1}{2}) = -1$
$f(2) = 2$

$\boxed{f(2) = 2 \quad f(\frac{1}{2}) = -1}$

$f'(x) = 2 \quad x \ge \frac{1}{2}$
$-2 \quad x < \frac{1}{2}$

5. Show that the function $y = x^{21} + x^{11} + 13x$ does not have a local maximum or a local minimum.

$y' = \underbrace{21x^{20} + 11x^{10} + 13}$

never going to $= 0$ so no changes of direction

6. Find the value of $k$ if the function $y = x^2 + kx + 72$ has a local minimum at $x = 4$.

$y' = 2x + k$

at $x = 4$

$0 = 2(4) + k$

$0 = 8 + k$

$\boxed{k = -8}$

7. Find the values of $a$ and $b$ if the function $y = 2x^3 + ax^2 + bx + 36$ has a local maximum when $x = -4$ and a local minimum when $x = 5$.

$y' = 6x^2 + 2ax + b$

For local max $x = -4$

$0 = 6(-4)^2 + 2(-4)a + b$

$0 = 96 - 8a + b$

For local min $x = 5$

$0 = 6(5)^2 + 2a(5) + b$

$0 = 150 + 10a + b$

$96 - 8a + b = 150 + 10a + b$

$-54 = 18a$

$\boxed{a = -3}$

$0 = 150 + 10(-3) + b$

$0 = 150 - 30 + b$

$\boxed{-120 = b}$