Section 4.2 – Practice Problems

1. Answer the following questions to lockdown your vocabulary.

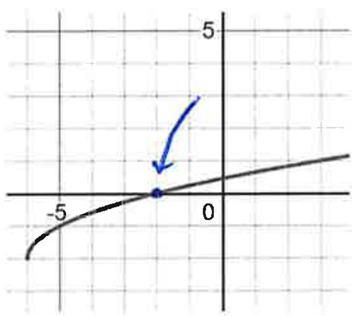
a) In a radical equation, there are variables in the Radicand

b) The power system states that if $x = y$, then $x^n = y^n$

c) Solutions that does not satisfy the original equation are called extraneous roots.

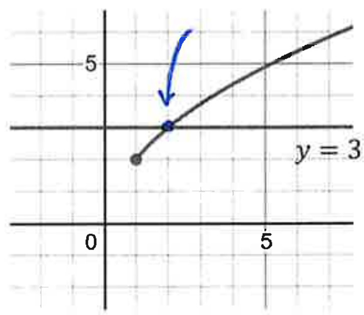
2. Solve the equation provided by interpreting the graph.

a) $\sqrt{x+6} - 2 = 0$



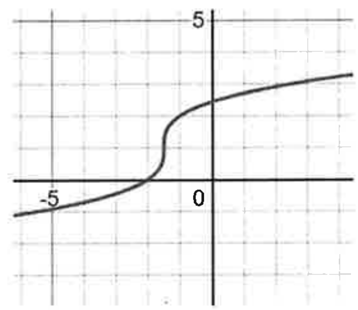
$x = -2$

b) $\sqrt{5x-1} = 3$



$x = 2$

c) $\sqrt[3]{2x+3} + 1 = 0$



$x = -2$

3. Find the $y - intercept$ and $x - intercept(s)$ of the following

x-int: $y=0$ y-int: $x=0$

a) $f(x) = \sqrt{2x} - 4$

$f(0) = \sqrt{0} - 4$

y-int: $(0,-4)$

$0 = \sqrt{2x} - 4$

x-int: $(8,0)$

$4 = \sqrt{2x}$

$16 = 2x$

$x = 8$

b) $f(x) = \sqrt[3]{4x} + 2$

$f(0) = 2$

y-int: $(0,2)$

x-int: $(-2,0)$

$0 = \sqrt[3]{4x} + 2$

$-2 = \sqrt[3]{4x}$

$-8 = 4x$

$x = -2$

c) $f(x) = \sqrt{4x-3} - 5$

$f(0) = \sqrt{-3} - 5$ ← not possible

y-int: None

x-int: $(7,0)$

$0 = \sqrt{4x-3} - 5$

$5 = \sqrt{4x-3}$

$25 = 4x-3$ → $28 = 4x$

$x = 7$

d) $f(x) = \sqrt[3]{2x-1} - 4$

y-int: $(0,-5)$

x-int: $(\frac{65}{2}, 0)$

$\sqrt[3]{2(0)-1} - 4$

$\sqrt[3]{-1} - 4$

$-1-4$

-5

$0 = \sqrt[3]{2x-1} - 4$

$4 = \sqrt[3]{2x-1}$

$64 = 2x-1$

$65 = 2x$

$x = \frac{65}{2}$

e) $f(x) = \sqrt{2x} + 4$

$f(0) = \sqrt{2(0)} + 4$

$= 0+4$

$= 4$

x-int: NONE

y-int: $(0,4)$

$0 = \sqrt{2x} + 4$

$-4 = \sqrt{2x}$

$16 = 2x$

$x = 8$ However $\sqrt{16} + 4 \neq 0$

f) $f(x) = \sqrt{4-x} - 2$

$0 = \sqrt{4-x} - 2$

$2 = \sqrt{4-x}$

$4 = 4-x$

$x = 0$

x-int: $(0,0)$

y-int: $(0,0)$

$f(0) = \sqrt{4-0} - 2$

$\sqrt{4} - 2$

$2-2$

0

g) $f(x) = \sqrt{x^2+1} - \sqrt{17}$

$f(0) = \sqrt{0^2+1} - \sqrt{17}$

$1 - \sqrt{17}$

$0 = \sqrt{x^2+1} - \sqrt{17}$

$\sqrt{17} = \sqrt{x^2+1} \rightarrow 17 = x^2 + 1$

$x^2 = 16$

$x = \pm 4$

y-int: $(0, 1-\sqrt{17})$

x-int: $(4,0)$

$(-4,0)$

h) $f(x) = \sqrt{x^2+6x} - 4$

$f(0) = -4$

$0 = \sqrt{x^2+6x} - 4$

$4 = \sqrt{x^2+6x}$

$16 = x^2 + 6x$

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

$x = -8$

$x = 2$

y-int: $(0,-4)$

x-int: $(-8,0)$

$(2,0)$

i) $f(x) = \sqrt[4]{x-1} - 2$

$f(0) = \sqrt[4]{x-1} - 2$

$= \sqrt[4]{-1} - 2$

not possible

$0 = \sqrt[4]{x-1} - 2$

$2 = \sqrt[4]{x-1}$

$16 = x - 1$

$16 = x - 1$

$x = 17$

y-int: (None)

x-int: $(17,0)$

j) $f(x) = \sqrt{x^2-5x} - 6$

$f(0) = \sqrt{0^2 - 5(0)} - 6$

-6

y-int: $(0,-6)$

x-int: $(-4,0)$

$(9,0)$

$0 = \sqrt{x^2-5x} - 6$

$6 = \sqrt{x^2-5x}$

$36 = x^2 - 5x$

$x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x = 9$

$x = -4$

Need $f(x)=0$

4. Find the roots of the following functions, get your solutions for extraneous roots.

a) $f(x) = \sqrt{13-x} - x + 1$

$0 = \sqrt{13-x} - x + 1$

$x - 1 = \sqrt{13-x}$

$(x-1)^2 = 13 - x \rightarrow x^2 - 2x + 1 = 13 - x$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$\boxed{x=4}$

$x = -3$

if $x = 4$ ✓

$0 = \sqrt{9} - 4 + 1$

$0 = 0$

if $x = -3$

$0 = \sqrt{16} - (-3) + 1$

$0 = 8$ Reject

b) $f(x) = \sqrt{2x-3} + x - 3$

$0 = \sqrt{2x-3} + x - 3$

$-x + 3 = \sqrt{2x-3}$

$(-x+3)^2 = \sqrt{2x-3}^2 \rightarrow (-x+3)^2 = 2x - 3$

$x^2 - 3x - 3x + 9 = 2x - 3$

$x^2 - 6x + 9 = 2x - 3$

$x^2 - 8x + 12 = 0$

$(x-2)(x-6) = 0$

$\boxed{x=2}$

$x = 6$

if $x = 2$

$0 = \sqrt{1} + 2 - 3$

$0 = 0$

if $x = 6$

$\sqrt{9} + 6 - 3$

$0 = 6$ ✗

Reject

c) $f(x) = \sqrt{5-5x} + x - 1$

$0 = \sqrt{5-5x} + x - 1$

$-x + 1 = \sqrt{5-5x}$

$(-x+1) = \sqrt{5-5x}$

$(-x+1)^2 = 5 - 5x$

$x^2 - 2x + 1 - 5 + 5x = 0$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$\boxed{\begin{array}{l} x = -4 \\ x = 1 \end{array}}$

if $x = -4$

$0 = \sqrt{25} + (-4) - 1$

$5 - 5$

$0 = 0$

if $x = 1$

$0 = \sqrt{0} + 1 - 1$

$0 = 0$

d) $f(x) = 2x - 8 + \sqrt{x+1}$

$0 = 2x - 8 + \sqrt{x+1}$

$-2x + 8 = \sqrt{x+1}$

$(-2x+8)^2 = \sqrt{x+1}^2$

$4x^2 - 32x + 64 = x + 1$

$4x^2 - 33x + 63 = 0$

AC Method

↓

$x^2 - 33x + 252 = 0$

$(x - \frac{21}{4})(x - \frac{12}{4}) = 0$

$(4x - 21)(x - 3) = 0$

$\boxed{x = 3}$

$x = 21/4$

Reject

if $x = 3$

$0 = 6 - 8 + \sqrt{4}$

$0 = -2 + 2$

$0 = 0$

if $x = 21/4$

$0 = 2\left(\frac{21}{4}\right) - 8 + \sqrt{21/4 + 1}$

$0 = \frac{21}{2} - \frac{16}{2} + \sqrt{25/4}$

$0 = \frac{5}{2} + \frac{5}{2}$

$0 = \frac{10}{2}$ $\quad 0 = 5$ ✗

e) $f(x) = \sqrt{x+3} - x - 3$

$0 = \sqrt{x+3} - x - 3$

$x+3 = \sqrt{x+3}$

$(x+3)^2 = x+3 \rightarrow x^2+6x+9 = x+3$

$x^2+5x+6=0$

$(x+2)(x+3)=0$

$\boxed{\begin{array}{c} x=-2 \\ x=-3 \end{array}}$

if $x=-2$

$0 = \sqrt{1} - (-2) - 3$

$1+2-3$

$0=0$ ✓

if $x=-3$

$0 = \sqrt{0} - (-3) - 3$

$0 = 0+3-3$

$0=0$ ✓

f) $f(x) = \sqrt{x+5} - x + 1$

$0 = \sqrt{x+5} - x + 1$

$x-1 = \sqrt{x+5}$

$(x-1)^2 = x+5 \rightarrow x^2-2x+1 = x+5$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$\boxed{x=4}$

$x=-1$

if $x=4$

$0 = \sqrt{9} - 4 + 1$

$0 = 3-4+1$

$0=0$ ✓

if $x=-1$

$0 = \sqrt{4} - (-1) + 1$

$= 2+1+1$

$0=4$

Reject

5. Use a table of values and plot points to determine between which two integers the roots fall. Write your answer in the form $a < x < b$.

Look for y-values going from positive to negative

a) $f(x) = \sqrt{x+5} - x$

x	y
-1	3
2	0.65
3	-0.17
4	-1

$\boxed{2 < x < 3}$

b) $f(x) = \sqrt[3]{2x+1} + 2$

x	y
0	3
-1	1
-2	0.56
-3	0.29
-4	0.087
-5	-0.08

$\boxed{-5 < x < -4}$

c) $f(x) = \sqrt{2x+6} - x$

x	y
-1	3
-3	3
1.5	1.5
3	0.46
4	-0.26

$\boxed{3 < x < 4}$

d) $f(x) = \sqrt{x+2} - 2x$

x	y
0	$\sqrt{2}$
1	-0.27

$\boxed{0 < x < 1}$

e) $f(x) = \sqrt{4-x} - x$

x	y
0	2
1	0.73
2	-0.59

$\boxed{1 < x < 2}$

f) $f(x) = \sqrt{10-x} - x - 1$

x	y
1	1
2	-0.17

$\boxed{1 < x < 2}$

6. Solve the given radical equation to find the root(s) algebraically and graphically

a) $f(x) = \sqrt{2x-3} - 3$

$x = 2$
$y = -2$

$x = \frac{7}{2}$
$y = -1$

$x = 6$
$y = 0$

D: $x \geq \frac{3}{2}$

R: $y \geq -3$

Algebraically

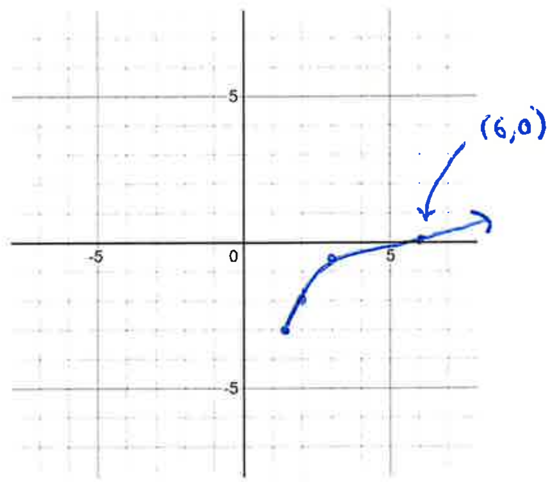
$0 = \sqrt{2x-3} - 3$

$3 = \sqrt{2x-3}$

$9 = 2x - 3$

$12 = 2x$

$\boxed{6 = x}$



b) $f(x) = \sqrt[3]{x+4} + 1$

$x = -4$ $y = 1$ $x = -3$ $x = 2$ $x = 0$ $y = 2.6$ $x = 4$ $y = 3$

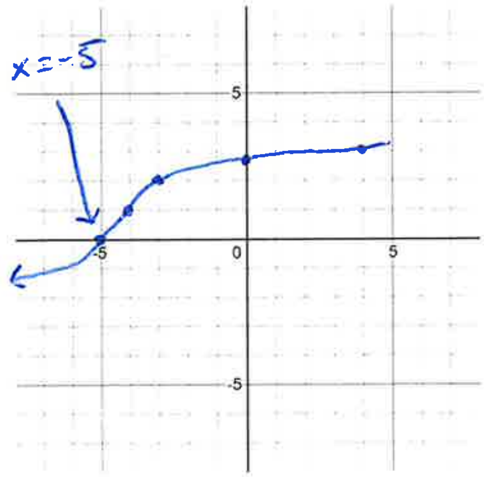
$x = -5$
$y = 0$

$0 = \sqrt[3]{x+4} + 1$

$-1 = \sqrt[3]{x+4}$

$-1 = x + 4$

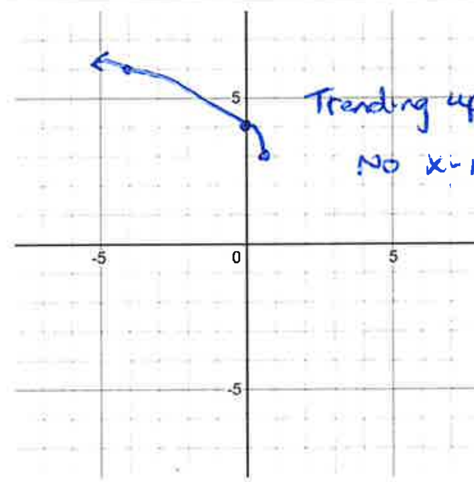
$\boxed{x = -5}$



c) $f(x) = \sqrt{1-2x} + 3$

$x = -4$
$y = 6$

D: $x \leq \frac{1}{2}$
R: $y \geq 3$



Trending up.
No x-intercept

Algebraically

$0 = \sqrt{1-2x} + 3$

$-3 = \sqrt{1-2x}$

Not possible so $\boxed{\text{no solution}}$

$1 - 2x \geq 0$

$1 \geq 2x$

$\frac{1}{2} \geq x$

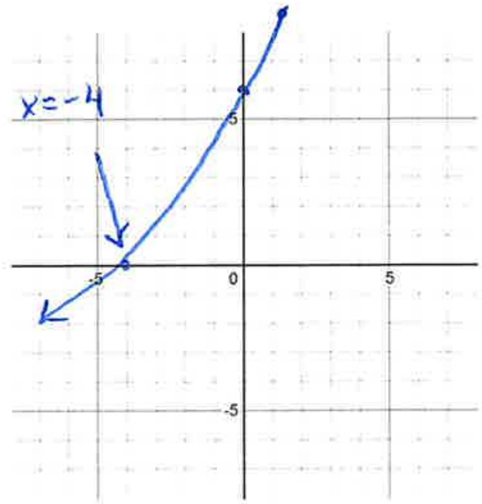
d) $f(x) = x + 8 - \sqrt{4 - 3x}$

if $x = 0$
$y = 6$

$4 - 3x \geq 0$
$4 \geq 3x$
$\frac{4}{3} \geq x$

$x = -4$
$y = 0$

$-4 + 8 - \sqrt{4 - 3(-4)}$
$-4 + 8 - 4$



Algebraically

$0 = x + 8 - \sqrt{4 - 3x}$

$\sqrt{4 - 3x} = x + 8$

$4 - 3x = (x+8)^2$

$4 - 3x = x^2 + 16x + 64$

$0 = x^2 + 19x + 60$

$(x + 15)(x + 4) = 0$

$x = -15$ ← reject

$\boxed{x = -4}$

$-15 + 8 - \sqrt{4 - 3(-15)}$
$-7 - \sqrt{49}$
$-7 - 7$
$0 = -14$
Reject

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e) $f(x) = \sqrt{x + 1} - x - 1$

$0 = \sqrt{x+1} - x - 1$

$x + 1 = \sqrt{x + 1}$

$(x+1)^2 = x + 1$

$x^2 + 2x + 1 = x + 1$

$x^2 + x = 0$

$x(x+1) = 0$

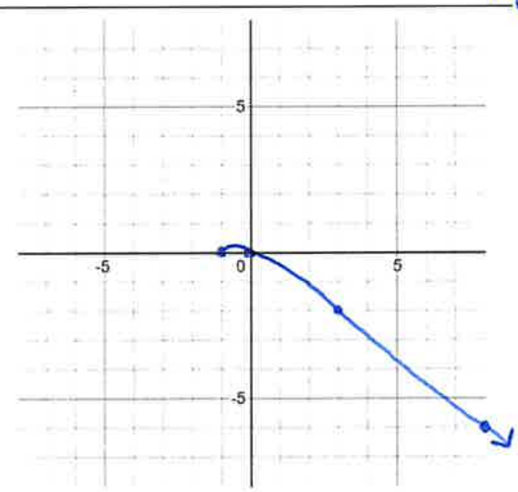
$\boxed{x = 0}$ $\boxed{x = -1}$

D: $x \geq -1$
if $x = -1$
$y = 0$

if $x = 0$
$y = 0$

$x = 3$
$y = -2$

$x = 8$
$y = -6$



See Website for Detailed Answer Key of the Remainder of the Questions

Extra Work Space