Section 4.2 – Graphing and Solving Radicals

- We solved Radicals in PC 11. It was necessary to use the Power Theorem: If $a = b$ then $a^2 = b^2$ but this created potential **extraneous solutions** that must be checked with the original equation and **rejected if they do not satisfy it**.

Solving a Radical Algebraically

1. Isolate the radical on one side of the equal sign
2. Raise both sides of the equation to the power that is equal to the index of the root
3. Solve the equation
4. Check if the solutions satisfy the initial equation. If not then reject the given solution.

Recall that we also **Checked the Domain Restrictions** to see if our solutions satisfied those. Domain restrictions help with the graphing of Radicals

Restrictions on the Domain (Allowable values for x)

- When we think about numbers that exist (Real Numbers), $\sqrt{x+2}$ has some restrictions, because…
- We **can't have negatives** under the square root symbol
- So, the restriction we are looking at is: $x + 2 \geq 0 \rightarrow x \geq -2$

Example: Determine the restrictions on $\sqrt{2x-3} = x - 3$

Solution: All that matters is that the radicand, $2x - 3 \geq 0$

$2x - 3 \geq 0 \rightarrow 2x \geq 3 \rightarrow x \geq \frac{3}{2}$ **So, the restriction is:** $\boldsymbol{x \geq \frac{3}{2}}$

Example: Determine the restrictions on $\sqrt{3x+4} - \sqrt{2x-4} = 2$

Solution: Since we have two radicands, we need to **check both**

$3x + 4 \geq 0$	$2x - 4 \geq 0$
$3x \geq -4$	$2x \geq 4$
$x \geq -\frac{4}{3}$	$x \geq 2$

Since the restrictions **starts in the negatives** but has **another value at 2**, we need to take the larger number as the start point.

So, the restriction is: $\boldsymbol{x \geq 2}$

Solving a Radical Graphically (Solving for roots, x when $y = 0$, unless otherwise stated)

1. Identify the Domain
2. Identify the Range, if possible (can be quite challenging)
3. Use a table of values to find points (strategically use points that provide perfect roots)
4. Set $x = 0$ to find the $y - intercept$ (if it exists)
5. Sketch the graph as accurately as possible

The Domain Restriction will create a vertical asymptote that the graph stops at.

Example 1 Part I: Solve $f(x) = \sqrt{5 - x} - 2$ Graphically

Solution 1: Identify Domain Restrictions, and intercepts

Domain: $5 - x \geq 0$

$\boldsymbol{x \leq 5}$

Range: Since $5 - x \geq 0$

$\boldsymbol{y \geq -2}$

$\boldsymbol{y - intercept\ (when\ x = 0)}$:

$y = \sqrt{5 - 0} - 2$

$y = \sqrt{5} - 2 = \mathbf{0.24}$

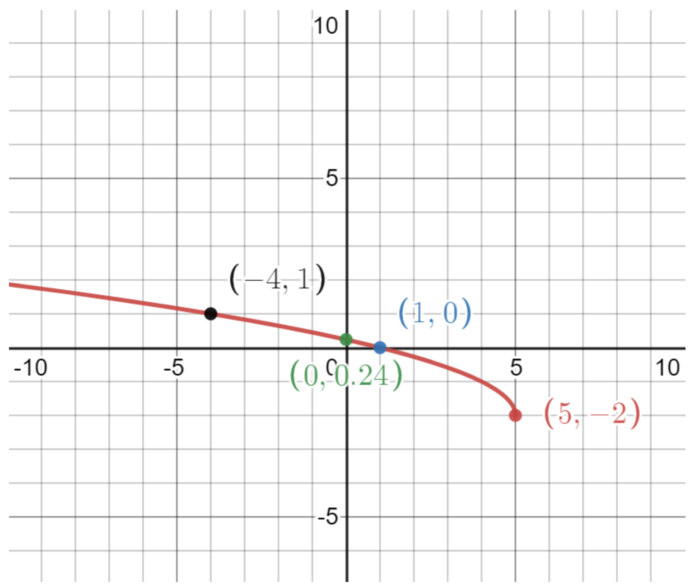


Table of Values

x	y
5	-2
1	0
-4	1

Example 1 Part II: Solve $f(x) = \sqrt{5 - x} - 2$ Algebraically

Solution 1:

$\boldsymbol{x - intecept\ (when\ y = 0)}$:

$0 = \sqrt{5 - x} - 2$

$2 = \sqrt{5 - x}$

$4 = 5 - x \quad \rightarrow \quad \boldsymbol{x = 1}$

$0 = \sqrt{5 - 1} - 2$

$0 = \sqrt{4} - 2$

$0 = 2 - 2$

$0 = 0$

Solution satisfies the equation, **it is not extraneous**

Example 2 Part I: Solve $f(x) = \sqrt{x+6} - x$ Graphically

Solution 2: Identify Domain Restrictions, and intercepts

Domain: $x + 6 \geq 0$

$\boldsymbol{x \geq -6}$

Range: Difficult to calculate

$\boldsymbol{y - intercept\ (when\ x = 0)}$:

$y = \sqrt{0+6} - 0$

$y = \sqrt{6} = \mathbf{2.45}$

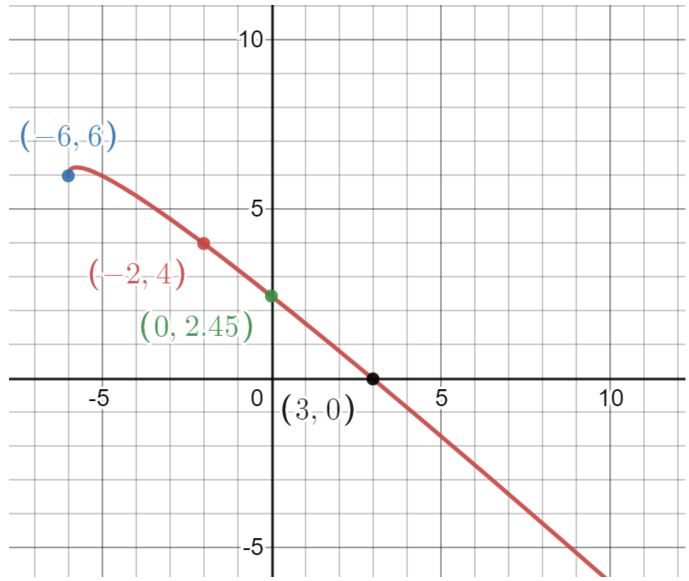


Table of Values

x	y
-6	6
-2	4
3	0

Example 2 Part II: Solve $f(x) = \sqrt{x+6} - x$ Algebraically

Solution 2:

$\boldsymbol{x - intecept\ (when\ y = 0)}$:

$0 = \sqrt{x+6} - x$

$x = \sqrt{x+6}$

$x^2 = x + 6 \quad \rightarrow \quad x^2 - x - 6 = 0$

$x^2 - x - 6 = 0 \quad \rightarrow \quad (x-3)(x+2) = 0$

$x = 3, -2$

Check when $x = 3$	Check when $x = -2$
$0 = \sqrt{3+6} - 3$	$0 = \sqrt{-2+6} - (-2)$
$0 = \sqrt{9} - 3$	$0 = \sqrt{4} + 2$
$0 = 3 - 3$	$0 = 2 + 2$
$0 = 0$	$0 = 4$
$Accept$	$Reject$

$x = 3$ is the only viable solution

<u>Example 3 Part I:</u> Solve $f(x) = \sqrt{4x+1} - 2$ Graphically

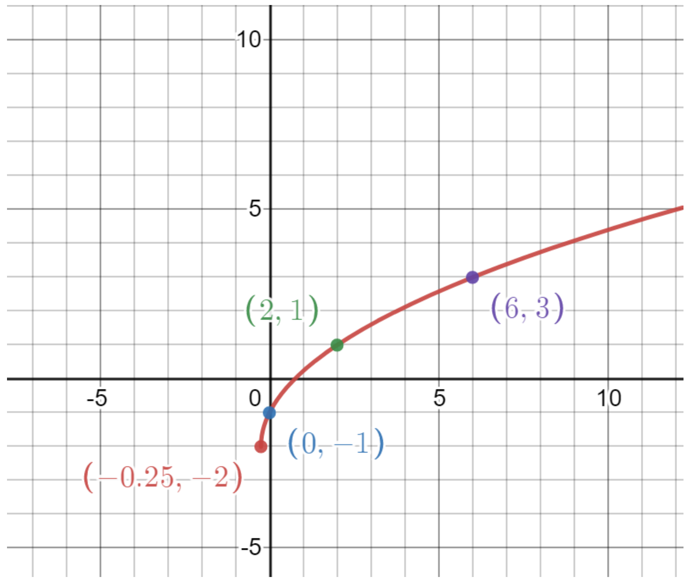
<u>Solution 3:</u> Identify Domain Restrictions, and intercepts

Domain: $4x + 1 \geq 0$

$$\boldsymbol{x \geq -\frac{1}{4}}$$

Range: Since $4x + 1 \geq 0$

$$\boldsymbol{y \geq -2}$$



$\boldsymbol{y - intercept\ (when\ x = 0)}$:

$$y = \sqrt{4(0)+1} - 2$$

$$y = \sqrt{1} - 2 = \boldsymbol{-1}$$

Table of Values

x	y
$-\frac{1}{4}$	-2
0	-1
2	1
6	3

Graph solutions are about identifying as many exact whole number points as you can, rational number points if necessary, and then keeping the graph as smooth and accurate as possible.

<u>Example 3 Part II:</u> Solve $f(x) = \sqrt{4x+1} - 2$ Algebraically

<u>Solution 3:</u>

$\boldsymbol{x - intecept\ (when\ y = 0)}$:

$$0 = \sqrt{4x+1} - 2$$

$$2 = \sqrt{4x+1}$$

$$4 = 4x + 1 \quad \rightarrow \quad 3 = 4x$$

$$\boldsymbol{x = \frac{3}{4}}$$

Check when $x = \frac{3}{4}$

$$0 = \sqrt{4(\frac{3}{4}) + 1 - 2}$$

$$0 = \sqrt{3+1} - 2$$

$$0 = \sqrt{4} - 2$$

$$0 = 2 - 2$$

$$0 = 0$$

Solution satisfies the equation, **it is not extraneous**

Example 4 Part I: Solve $f(x) = \sqrt{x+1} - x + 2$ Graphically

Solution 4: Identify Domain Restrictions, and intercepts

Domain: $x + 1 \geq 0$

$\boldsymbol{x \geq -1}$

Range: Difficult to calculate

$\boldsymbol{y - intercept\ (when\ x = 0)}$:

$y = \sqrt{0+1} - 0 + 2$

$y = \sqrt{1} + 2 = \mathbf{3}$

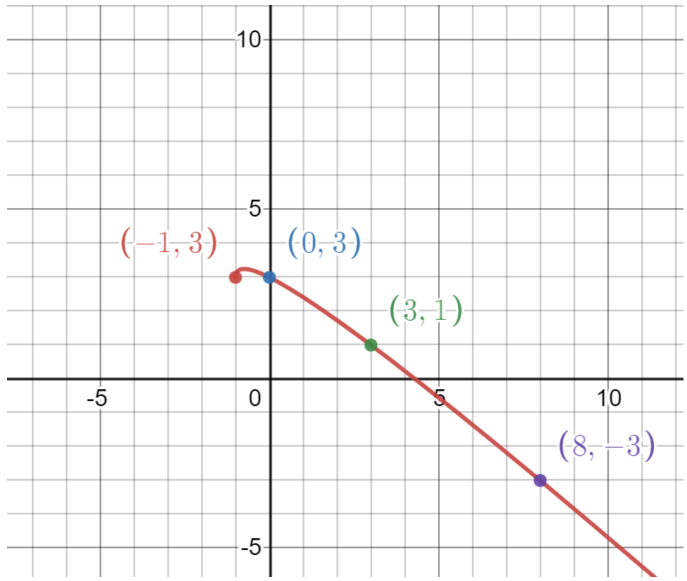


Table of Values

x	y
-1	3
0	3
3	1
8	-3

Example 4 Part II: Solve $f(x) = \sqrt{x+1} - x + 2$ Algebraically

Solution 4:

$\boldsymbol{x - intecept\ (when\ y = 0)}$:

$0 = \sqrt{x+1} - x + 2$

$x - 2 = \sqrt{x+1}$

$(x-2)^2 = x + 1 \quad \rightarrow \quad x^2 - 4x + 4 = x + 1$

$x^2 - 5x + 3 = 0$ Quadratic Equation Needed

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Check when $x = 0.697$	Check when $x = 4.303$
$0 = \sqrt{0.697+1} - 0.697 + 2$	$0 = \sqrt{4.303+1} - 4.303 + 2$
$0 = 2.606$	$0 = 0$
Reject	*Accept*

$$x = \frac{5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)} \rightarrow \frac{5 \pm \sqrt{25-12}}{2} = \frac{5 \pm \sqrt{13}}{2} \approx 0.697 \quad and \quad \mathbf{4.303}$$

Section 4.2 – Practice Problems

1. Answer the following questions to lockdown your vocabulary.

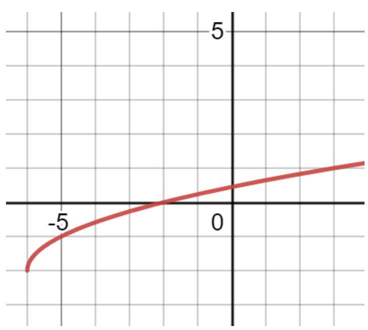
a) In a radical equation, there are variables in the ___________

b) The power system states that if $x = y$, then ______________

c) Solutions that does not satisfy the original equation are called __________ roots.

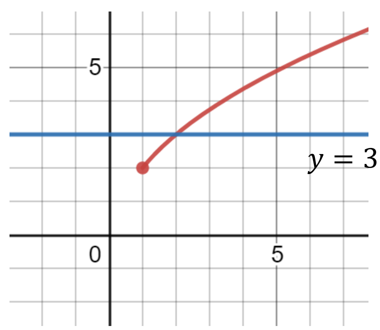
2. Solve the equation provided by interpreting the graph.

a) $\sqrt{x+6} - 2 = 0$



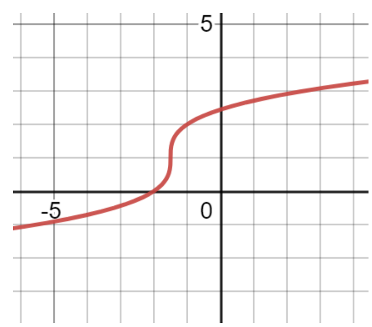
$x =$

b) $\sqrt{5x-1} = 3$



$x =$

c) $\sqrt[3]{2x+3} + 1 = 0$



$x =$

3. Find the $y - intercept$ and $x - intercept(s)$ of the following

a) $f(x) = \sqrt{2x} - 4$

b) $f(x) = \sqrt[3]{4x} + 2$

c) $f(x) = \sqrt{4x - 3} - 5$

d) $f(x) = \sqrt[3]{2x - 1} - 4$

e) $f(x) = \sqrt{2x} + 4$

f) $f(x) = \sqrt{4 - x} - 2$

g) $f(x) = \sqrt{x^2 + 1} - \sqrt{17}$

h) $f(x) = \sqrt{x^2 + 6x} - 4$

i) $f(x) = \sqrt[4]{\sqrt{x - 1}} - 2$

j) $f(x) = \sqrt{x^2 - 5x} - 6$

4. Find the roots of the following functions, get your solutions for extraneous roots.

a) $f(x) = \sqrt{13 - x} - x + 1$

b) $f(x) = \sqrt{2x - 3} + x - 3$

c) $f(x) = \sqrt{5 - 5x} + x - 1$

d) $f(x) = 2x - 8 + \sqrt{x + 1}$

e) $f(x) = \sqrt{x+3} - x - 3$

f) $f(x) = \sqrt{x+5} - x + 1$

5. Use a table of values and plot points to determine between which two integers the roots fall. Write your answer in the form $a < x < b$.

a) $f(x) = \sqrt{x+5} - x$

x	y

b) $f(x) = \sqrt{x+5} - x$

x	y

c) $f(x) = \sqrt{x+5} - x$

x	y

d) $f(x) = \sqrt{x+5} - x$

x	y

e) $f(x) = \sqrt{x+5} - x$

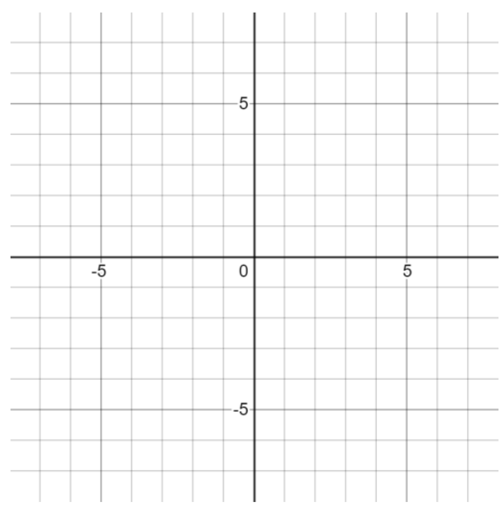
x	y

f) $f(x) = \sqrt{x+5} - x$

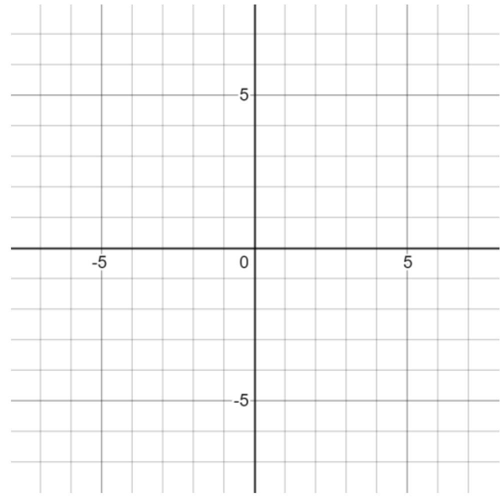
x	y

6. Solve the given radical equation to find the root(s) algebraically and graphically

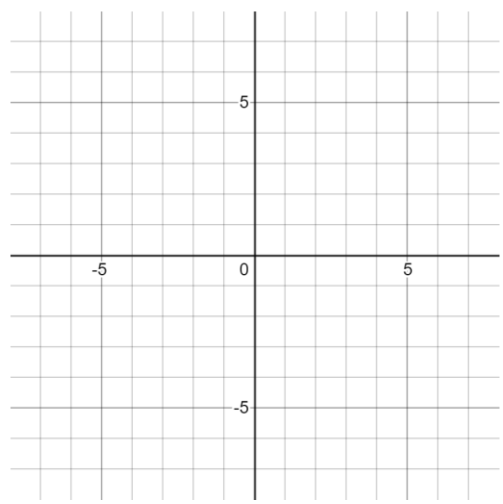
a) $f(x) = \sqrt{2x-3} - 3$



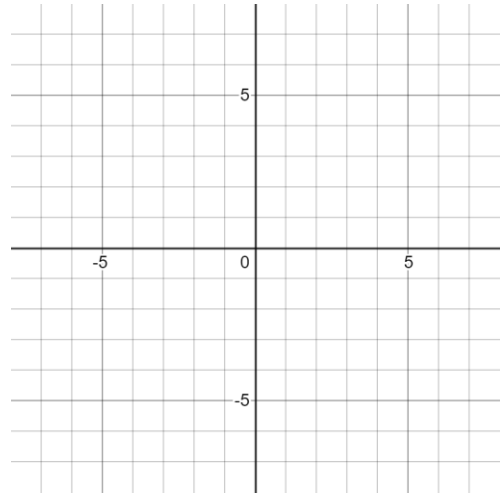
b) $f(x) = \sqrt[3]{x+4} + 1$



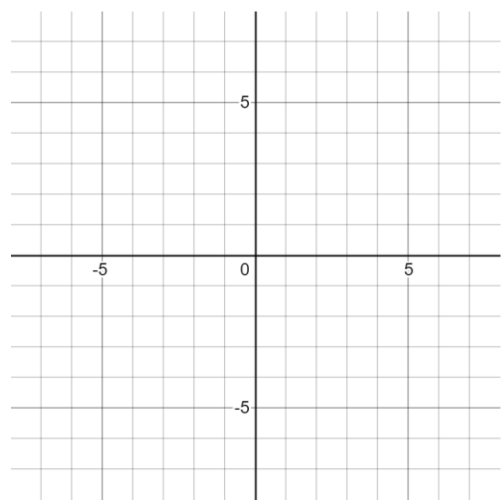
c) $f(x) = \sqrt{1-2x} + 3$



d) $f(x) = x + 8 - \sqrt{4 - 3x}$



e) $f(x) = \sqrt{x + 1} - x - 1$



See Website for Detailed Answer Key of the Remainder of the Questions

Extra Work Space