Section 4.1b – Probability

Theoretical and Experimental Probability

- Both Theoretical and Experimental Probability are calculated the same way

$$P = \frac{\text{the number of times something happened}}{\text{the number of times we tried}}$$

So, to understand the difference between the two, let's look at the tossing of a coin.

Theoretical Probability: What we expect to happen (We can get heads or tails)

Experimental Probability: What actually happened (What happened over the course of flips)

Example 1: What are the odds of flipping heads theoretically and within the experiment?

Solution 1:

Theoretical Probability: tells us that it is: 50% (one or the other, over an infinite number of flips)

Experimental Probability: Requires some trials:

$Heads$	13
$Tails$	7
$Total$	20

Heads: $\frac{13}{20} = \frac{65}{100} = 65\%$

Tails: $\frac{7}{20} = \frac{35}{100} = 35\%$

The more we flip we will see that the outcomes will eventually even out, it may take an extremely large number of attempts!

For another example of **Experimental Probability**, let's look at dice.

Example 2: Use the data below to calculate the experimental probability of: rolling a five, an even number, a three or a 1

Solution 2: We need some trials:

Number	Outcome
1	4
2	5
3	8
4	1
5	3
6	4

Odds of rolling a 5: $\frac{3}{25} = \frac{12}{100} = 12\%$

Odds of rolling an even number: $\frac{10}{25} = \frac{40}{100} = 40\%$

Odds of rolling a 3 or 1: $\frac{12}{25} = \frac{48}{100} = 48\%$

Experimental Probability differs from the theoretical and **can have more interesting predictive capabilities** over **small sequences of attempts.**

Theoretical Probability

- The theoretical probability is the mathematical model considering all possible outcomes
- We use the equation:

$$P(E) = \frac{number\ of\ successes}{total\ number\ of\ outcomes}$$

Known as the **Sample Space**

$0 \leq P \leq 1$

Which is why we get percentages.

Example 3: What is the probability of getting a 5 or 6 when rolling a 6-sided die?

Solution 3: A die has six faces {1, 2, 3, 4, 5, 6} so the successful outcomes are {5, 6}.

$$P(E) = \frac{number\ of\ successes}{total\ number\ of\ outcomes} = \frac{2}{6} = \frac{1}{3} = 33.3\%$$

Example 4: What is the probability if a of a couple having three children, where one is a girl and two are boys? (Not including twins, considering sex at birth)

Solution 4: First consider the Sample Space

Let $B = Boy \quad G = Girl$

1st Born	B	B	B	G	G	G	B	G
2nd Born	B	B	G	B	G	B	G	G
3rd Born	B	G	B	B	B	G	G	G

So, we have 8 possible outcomes. A Sample Space of 8.

One girl and two boys occur 3 times. $P(E) = \frac{number\ of\ successes}{total\ number\ of\ outcomes} = \frac{3}{8} = 37.5\%$

Example 5: What is the probability of selecting a red Queen from a standard deck of cards?

Solution 5: A deck of cards has 52 cards. In those 52 cards, 4 are Queens, two of which are red.

$$P(E) = \frac{number\ of\ successes}{total\ number\ of\ outcomes} = \frac{2}{52} = \frac{1}{26} = 3.8\%$$

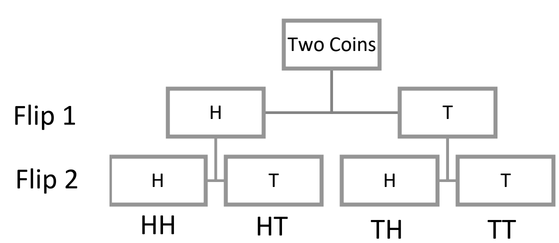
Using a Tree Diagram to Find the Sample Space

Example 6: Draw a Tree Diagram to visualize the potential outcomes of flipping a coin twice.

Solution 6:

Sample Space is: 4

Heads, Heads   Heads, Tails
Tails, Heads   Tails, Tails



Section 4. 1b – Practice Problems

1. Using the table provided below, what is the probability of:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

a) Rolling two dice with a sum of 8

b) Rolling doubles

c) Rolling two dice with a sum of 7 or 11

d) A sum less than 5

e) As sum that is odd

f) A sum that is even

2. The following table demonstrated experimental results, what is the experimental probability that:

Owner	Appearance	Performance	Total
Older Adult (Over 30)	110	120	230
Younger Adult (<30)	90	180	270
Both	200	300	500

a) A car owner selects their purchase based on appearance

b) An older adult car owner selects their vehicle based on appearance

c) A younger adult car owner selects their vehicle based on appearance

3. Given the data collected in the table below, what is the experimental probability that:

Athlete	Hockey	Running	Basketball	Soccer	Swimming	Total
Serious	102	58	32	63	25	280
Casual	96	47	41	24	12	220

a) The customer's favorite sport is soccer?

b) The customer is a serious athlete?

c) The customer is a casual athlete whose favorite sport is swimming?

d) Customer is a serious athlete whose favorite sport is basketball?

The following questions concern Theoretical Probability

4. A card is drawn from a standard deck of 52 cards. What is the probability that:

a) The card is a face card (Jack, Queen, King)

b) The card is a Spade

5. Two dice are thrown. What is the probability that you roll doubles?

6. If I placed three books on a shelf, what is the probability that I placed them in alphabetical order?

7. A card is drawn from a standard deck of 52 cards. What is the probability that it is a face card or a diamond?