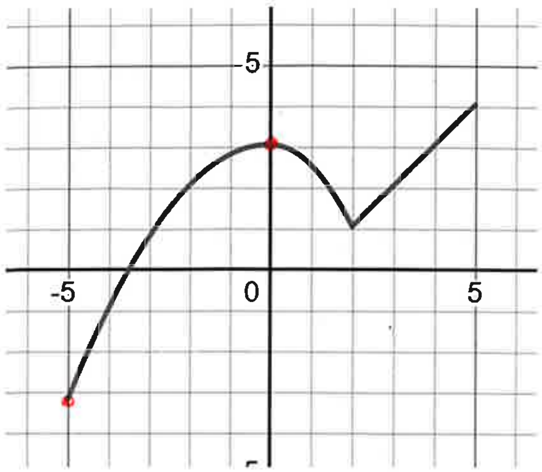
Section 4.1 – Practice Problems

1. State the intervals of increase and decrease for the given functions below.

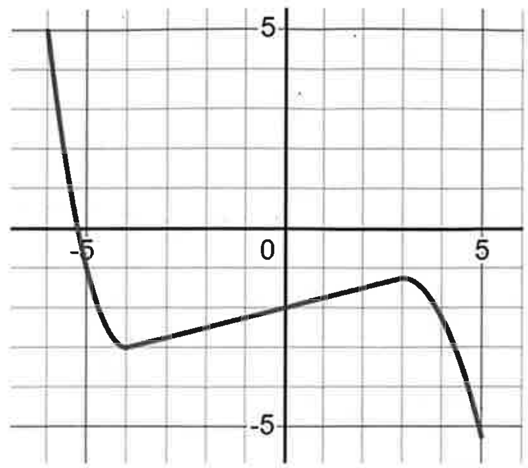
a)



Increasing: $(-5,0)$ and $(2,5)$

Decreasing: $(0,2)$

b)



Increasing: $(-4,3)$

Decreasing: $(-6,-4)$ and $(3,5)$

2. Find the intervals on which the following functions are increasing.

a) $f(x) = 12 + x - x^2$

$f'(x) = 1 - 2x$

$+ \quad \frac{1}{2} \quad -$

Increasing: $(-\infty, \frac{1}{2})$

Decreasing: $(\frac{1}{2}, \infty)$

b) $f(x) = x^4$

$f'(x) = 4x^3$

$- \quad 0 \quad +$

Increasing: $(0, \infty)$

c) $g(x) = x^3 - 3x + 2$

$g'(x) = 3x^2 - 3$

$= 3(x^2 - 1) \rightarrow 3(x+1)(x-1)$

$+ \quad -1 \quad - \quad 1 \quad +$

Increasing:

$(-\infty, -1)$ and $(1, \infty)$

d) $g(x) = 2x^3 - 3x^2$

$g'(x) = 6x^2 - 6x$

$= 6x(x-1)$

$+ \quad 0 \quad - \quad 1 \quad +$

Increasing: $(-\infty, 0)$ and $(1, \infty)$

e) $y = 3x^4 + 4x^3 - 12x^2 + 7$

$y' = 12x^3 - 12x^2 - 24x$

$= 12x(x^2 - x - 2)$

$12x(x-2)(x+1)$

$-$ $+$ $-$ $+$

-1 0 2

Increasing

$(-1, 0)$

$(2, \infty)$

f) $y = x^5 + 8x^3 + x$

$y' = 5x^4 + 24x^2 + 1$

always positive

always increasing

$(-\infty, \infty)$

3. Find the intervals on which the following functions are decreasing.

a) $f(x) = x^2 + x^3$

$f'(x) = 2x + 3x^2$

$f'(x) = x(2 + 3x)$

$+$ $-$ $+$

$-2/3$ 0

Decreasing: $(-2/3, 0)$

b) $g(x) = 2x^3 - 3x^2 - 36x + 62$

$g'(x) = 6x^2 - 6x - 36$

$= 6(x^2 - x - 6)$

$6(x-3)(x+2)$

$+$ $-$ $+$

-2 3

Decreasing: $(-2, 3)$

c) $h(x) = (1 - x^2)^2$

$h'(x) = 2(1-x^2) \cdot (-2x)$

$= -4x(1-x^2)$

$-$ $+$ $-$ $+$

-1 0 1

Decreasing: $(-\infty, -1)$ and $(0, 1)$

d) $F(x) = 4x + x^4$

$F'(x) = 4 + 4x^3$

$= 4(1 + x^3)$

$-$ | $+$ at -1

Decreasing: $(-\infty, -1)$

4. Find the intervals of increase and decrease on the following functions.

a) $f(x) = 3x^2 - 18x + 1$

$f'(x) = 6x - 18$

$= 6(x - 3)$

$-$ | $+$ at 3

Increasing: $(3, \infty)$

Decreasing $(-\infty, 3)$

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b) $f(x) = 2x^3 - 9x^2 - 60x + 82$

$f'(x) = 6x^2 - 18x - 60$

$= 6(x^2 - 3x - 10)$ $\rightarrow$ $= 6(x-5)(x+2)$

$+$ | $-$ | $+$ at -2, 5

Increasing: $(-\infty, -2.$ and $(5, \infty)$

Decreasing: $(-2, 5)$

c) $g(x) = x^4 - 2x^2 + 16$

$g'(x) = 4x^3 - 4x$

$= 4x(x^2 - 1)$

$= 4x(x+1)(x-1)$

$-\quad +\quad -\quad +$

$-1 \quad 0 \quad 1$

Dec : $(-\infty, -1)$ and $(0, 1)$

Inc : $(-1, 0)$ and $(1, \infty)$

d) $g(x) = 3x^4 - 16x^3 + 6x^2 + 72x + 8$

$(x+1)(x-2)(x-3)$

$g'(x) = 12x^3 - 48x^2 + 12x + 72$

$= 12(x^3 - 4x^2 + x + 6)$

$x^2 - 5x + 6$

$x+1 \,)\, x^3 - 4x^2 + x + 6$

$x^3 + x^2$

$-5x^2 + x \quad 6x+6$

$-5x^2 - 5x$

$-\quad +\quad -\quad +$

$-1 \quad 2 \quad 3$

dec : $(-\infty, -1)$ and $(2, 3)$

Inc : $(-1, 2)$ and $(3, \infty)$

e) $h(x) = x^3(x-1)^4$

$h'(x) = 3x^2(x-1)^4 + 4(x-1)^3(1)x^3$

$= 3x^2(x-1)^4 + 4x^3(x-1)^3$

$\Rightarrow x^2(x-1)^3[3(x-1) + 4x]$

$x^2(x-1)^3(7x-3)$

$3x - 3 + 4x = 0$

$7x = 3$

$x = 3/7$

$+\quad +\quad -\quad +$

$0 \quad 3/7 \quad 1$

Inc : $(-\infty, 3/7)$

Dec : $(1, \infty)$

f) $h(x) = \dfrac{x-1}{x+1}$

$+\quad +$

-1

$h'(x) = \dfrac{(x+1)(1) - (x-1)(1)}{(x+1)^2}$

$\dfrac{x+1-(x-1)}{(x+1)^2}$

$\dfrac{2}{(x+1)^2}$

Inc : $(-\infty, -1)$, $(-1, \infty)$

Inc: $(-\infty, 8/3)$

Dec: $(8/3, 4)$

g) $y = x\sqrt{4 - x}$

$y' = \frac{1}{2\sqrt{4-x}} \cdot (-1)x + \sqrt{4-x}$

+ | −
$8/3$ | 4

Domain $= \frac{-x}{2\sqrt{4-x}} + \sqrt{4-x}$

$\boxed{x \leq 4}$

$\Rightarrow \frac{-x+2(4-x)}{\sqrt{4-x}} = \frac{-x+8-2x}{\sqrt{4-x}} = \frac{-3x+8}{\sqrt{4-x}}$

$x = 8/3$

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h) $y = (x^2 - 9)^{\frac{2}{3}}$

$y' = \frac{2}{3}(x^2-9)^{-\frac{1}{3}}(2x)$

$= \frac{4x}{3(x^2-9)^{\frac{1}{3}}}$

− | + | − | +
-3 | 0 | 3

Inc: $(-3,0)$ and $(3,\infty)$

Dec: $(-\infty,-3)$ and $(0,3)$

5. Where is $y = 12x^5 + 15x^4 - 20x^3 + 27$ decreasing?

$y' = 60x^4 + 60x^3 - 60x^2$

$y' = 60x^2(x^2 + x - 1)$ use Quad Eqⁿ

$\sqrt{1^2 - 4(1)(-1)}$

+

$\frac{-1 \pm \sqrt{1^2 + 4}}{2}$

$\frac{-1 \pm \sqrt{5}}{2}$

-0.618 $\quad$ 0.618

+ | − | − | +
$\frac{-1-\sqrt{5}}{2}$ | 0 | $\frac{-1+\sqrt{5}}{2}$

$\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$