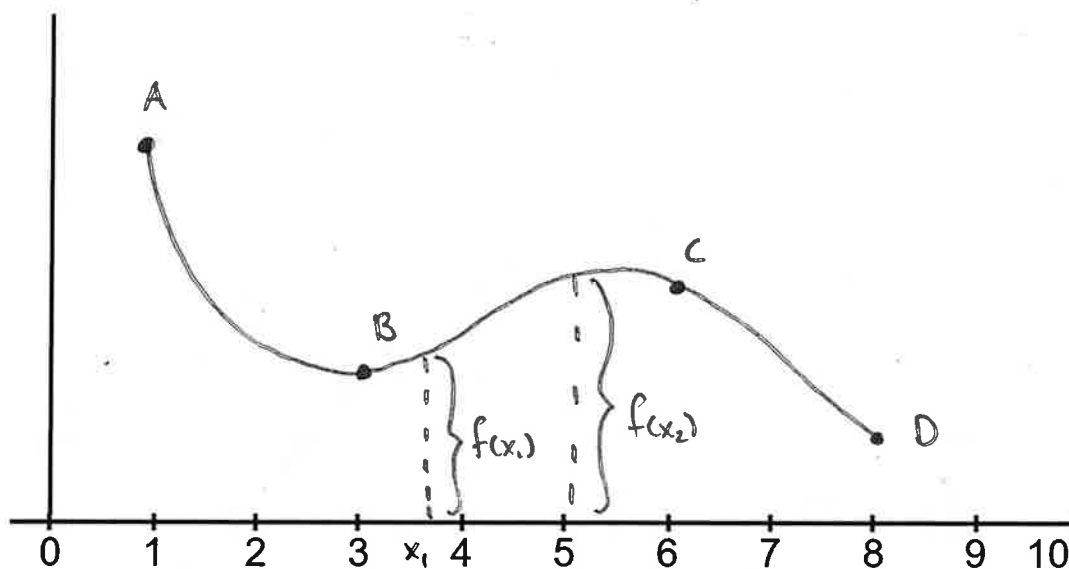


4.1 Increasing and Decreasing Functions

It is very helpful, when considering the behaviour of a function, to know **when it rises** and **when it falls**. The graph below of the function f , falls from A to B , rises from B to C , and then falls again from C to D .

We say that f is **decreasing** on the interval $(1, 3)$, **increasing** on the interval $(3, 6)$ and **decreasing** on the interval $(6, 8)$.

Notice that for any two numbers x_1 and x_2 , between 3 and 6 with $x_1 < x_2$, we have $f(x_1) < f(x_2)$



In general, a function f is called **increasing on an interval** if:

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2$$

And a function f is called **decreasing on an interval** if:

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2$$

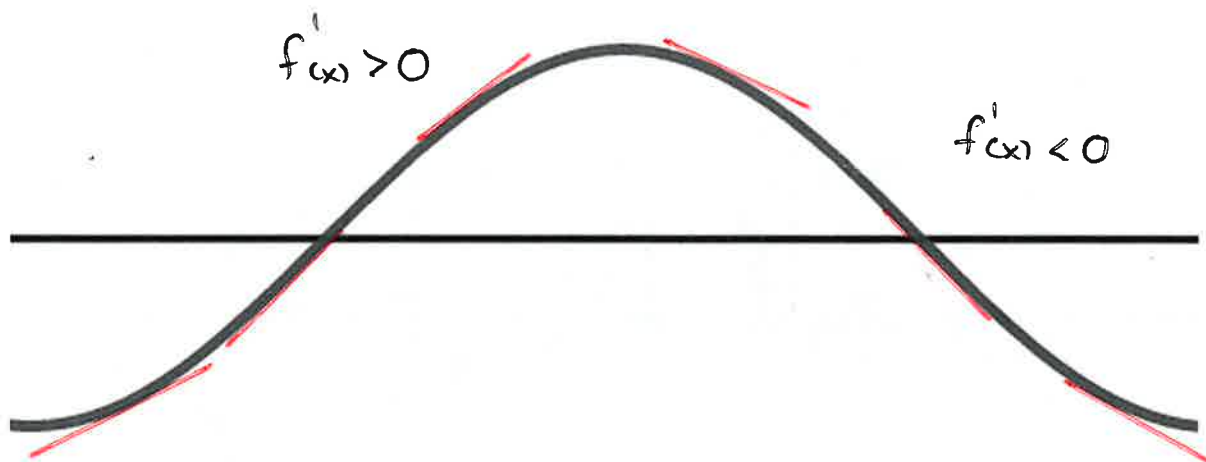
Connection to the Derivative

The graph below shows how the **derivative of a function** can tell us where the function is increasing or decreasing.

When $f'(x) > 0$, the **tangents have a positive slope**; thus, they are **increasing from left to right**, for this course we can infer that a positive derivative indicates an increasing function.

When $f'(x) < 0$, the **tangents have a negative slope**; thus, they are **decreasing from left to right**, for this course we can infer that a negative derivative indicates a decreasing function.

These two inferences are proved in more advanced course..

**Test for Increasing or Decreasing Functions**

1. If $f'(x) > 0$ for all x in an interval I , the f is increasing on I
2. If $f'(x) < 0$ for all x in an interval I , the f is decreasing on I

Ex 1: Find the interval on which the function $f(x) = 1 - 5x + 4x^2$ is increasing and decreasing. Draw a rough sketch of the graph

First find the derivative

$$f'(x) = -5 + 8x$$

increasing when $f'(x) > 0$

$$-5 + 8x > 0 \rightarrow 8x > 5$$

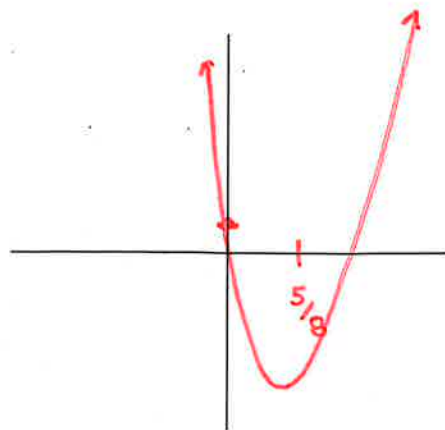
$$x > 5/8$$

decreasing when $f'(x) < 0$

$$-5 + 8x < 0$$

$$8x < 5$$

$$x < 5/8$$



Ex 2: Where is the function $y = x^3 + 6x^2 + 9x + 2$ increasing?

Derivative 1st

$$y' = 3x^2 + 12x + 9 \rightarrow 3(x^2 + 4x + 3)$$

$$3(x+3)(x+1)$$

$$3(x+3)(x+1) > 0 \rightarrow (x+3)(x+1) > 0$$

Interval	$(x+1)$	$(x+3)$	$f'(x)$	f
$x < -3$	-	-	+	increasing
$-3 < x < -1$	-	+	-	decreasing
$x > -1$	+	+	+	increasing

Increasing on: $(-\infty, -3) \cup (-1, \infty)$

Ex 3: Find the intervals of increase and decrease for the function:

$$g(x) = x^4 - 4x^3 - 8x^2 - 1$$

$$g'(x) = 4x^3 - 12x^2 - 16x$$

$$= 4x(x^2 - 3x - 4)$$

$$= 4x(x-4)(x+1)$$

Interval	$4x$	$(x-4)$	$(x+1)$	$f'(x)$	f
$(-\infty, -1)$	-	-	-	-	decreasing
$(-1, 0)$	-	-	+	+	increasing
$(0, 4)$	+	-	+	-	decreasing
$(4, \infty)$	+	+	+	+	increasing

Increasing: $(-1, 0) \cup (4, \infty)$

Decreasing: $(-\infty, -1) \cup (0, 4)$

Homework Problems

- Section 4.1: #1 - 5