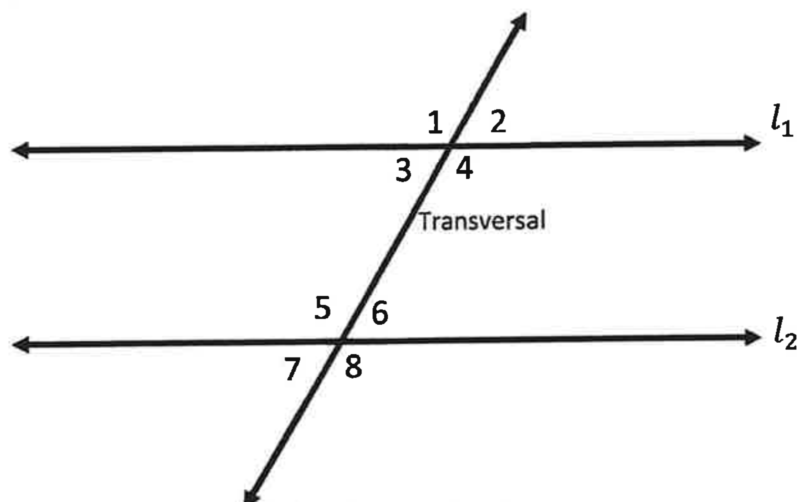
Section 4.1 – Geometry of Parallel Lines

First let's look at some vocabulary

a) Acute – an angle between 0 and 90 degrees
b) Obtuse – an angle between 90 and 180 degrees
c) Straight – angle exactly 180 degrees
d) Right – angle exactly 90 degrees
e) Complementary – two angles that add up to 90 degrees
f) Supplementary – two angles that add up to 180 degrees

When we look at angle relationships we can tell a lot about ANGLES FORMED BY A TRANSVERSAL

- When two **parallel lines** l_1 *and* l_2 are intersected by a third line, a **transversal**, eight angles are formed, 4 around each line.



- To study these relationships, we start with an assumption, or a...

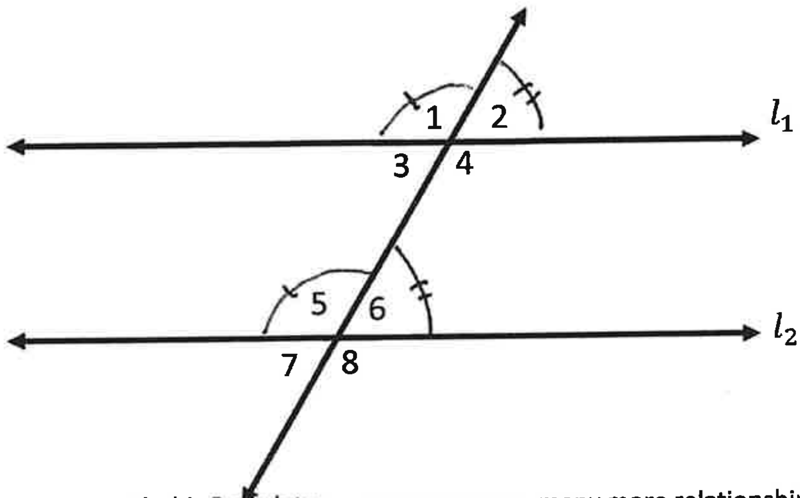
POSTULATE – accepted assumption without proof

- To devise our theorems we will use, postulates, inductive and deductive reasoning

❖ There are a series of rules named after letters of the alphabet, because they create that shape
❖ They all involve two parallel lines being intersected by a transversal

Corresponding Angles Postulate (F Rule)

- If two **parallel lines** are cut by a transversal, then the **corresponding angles are equal**
- If two lines are cut by a transversal, and the **corresponding angles are equal**, then the **lines are parallel**.



$\angle 1 = \angle 5$

$\angle 2 = \angle 6$

$\angle 3 = \angle 7$

$\angle 4 = \angle 8$

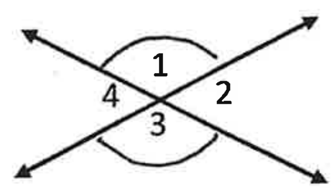
$l_1 \parallel l_2$

This means parallel

- With this Postulate we can now prove many more relationships between parallel lines and transversals
- Deductive reasoning will be used repeatedly for these proofs

Vertical Angles

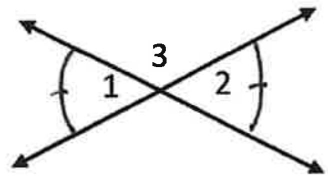
- When two lines intersect, they form two pairs of vertical angles
  - $\angle 1$ *and* $\angle 3$ are vertical angles
  - $\angle 2$ *and* $\angle 4$ are vertical angles



Proof – Vertical Angles are Equal

Given: $\angle 1$ *and* $\angle 2$ are vertical angles

Prove: $\angle 1 = \angle 2$



Proof Statement	Reason
$\angle 1 + \angle 3 = 180°$	Angles on a line add to 180° (supplementary)
$\angle 2 + \angle 3 = 180°$	Angles on a line add to 180° (supplementary)
$\angle 1 + \angle 3 = \angle 2 + \angle 3$	Both equal to 180° (substitution)
$\angle 1 = \angle 2$	Subtraction

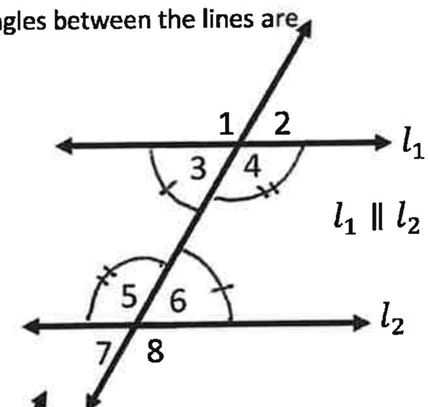
Vertical Angle Theorem

❖ If two angles are vertical angles, then the angles are equal.

Proved Statements are called **THEOREMS.**

Alternate Interior Angles (the Z rule)

- When two lines l_1 *and* l_2 are intersected by a transversal, the four angles between the lines are called **interior angles**

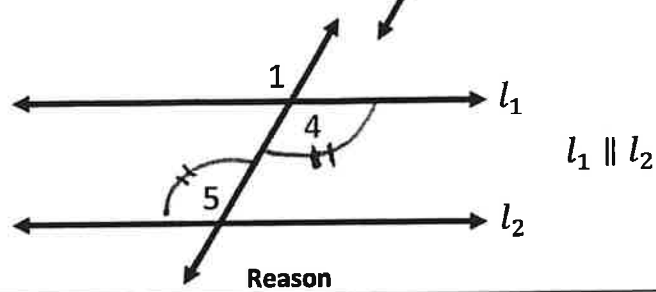


- $\angle 3, \angle 4, \angle 5,$ *and* $\angle 6$ are interior angles
- $\angle 3$ *and* $\angle 6$ are alternate interior angles
- $\angle 4$ *and* $\angle 5$ are alternate interior angles

Proof – Alternate Interior Angles of Parallel Lines are Equal

Given: $l_1 \parallel l_2$

Prove: $\angle 4 = \angle 5$



Proof Statement	Reason
$l_1 \parallel l_2$	Given
$\angle 1 = \angle 4$	Vertical Angles
$\angle 1 = \angle 5$	Corresponding Angles
$\angle 4 = \angle 5$	Substitution (both equal to $\angle 1$)

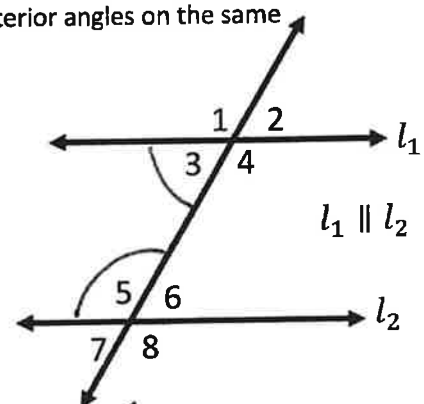
Alternate Interior Angle Theorem (Z Rule)

❖ If two parallel lines are cut by a transversal, then the alternate interior angles are equal.

❖ If two lines are cut by a transversal, and the alternate interior angles are equal, then the lines are parallel

Co-Interior Angles

- When two lines $l_1 and\ l_2$ are intersected by a transversal, then the interior angles on the same side of the transversal are called **co-interior angles**

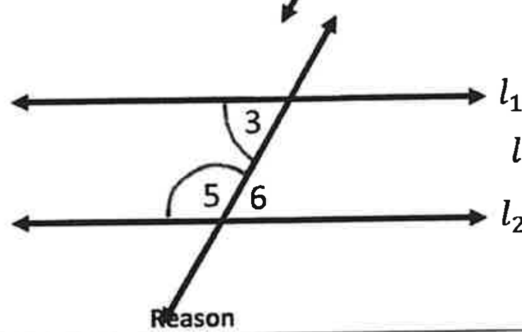


- $\angle 3, \angle 4, \angle 5, and\ \angle 6$ are interior angles
- $\angle 3\ and\ \angle 5$ are co-interior angles
- $\angle 4\ and\ \angle 6$ are co-interior angles

Proof – Co-Interior Angles of Parallel Lines are Supplementary

Given: $l_1 \parallel l_2$

Prove: $\angle 3 + \angle 5 = 180°$



Proof	Statement	Reason
	$l_1 \parallel l_2$	Given
	$\angle 3 = \angle 6$	Alternate interior Angles equal
	$\angle 5 + \angle 6 = 180°$	Angles on a line supplemental
	$\angle 5 + \angle 3 = 180°$	substitution
	$\angle 3 + \angle 5 = 180°$	Rewrite

Co-Interior Angle Theorem

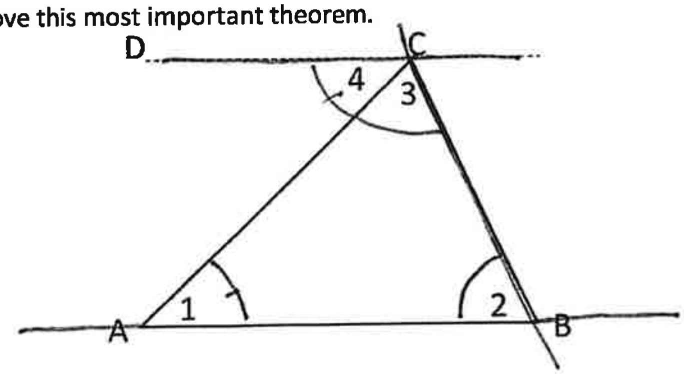
❖ If two parallel lines are cut by a transversal, then the co-interior angles are supplementary. (add to 180°)

❖ If two lines are cut by a transversal, and the co-interior angles are supplementary, then the lines are parallel.

Friday.

The Sum of Angles in a Triangle

- We will use our knowledge of parallel lines to prove this most important theorem.



Given: ΔABC

Prove: $\angle 1 + \angle 2 + \angle 3 = 180°$

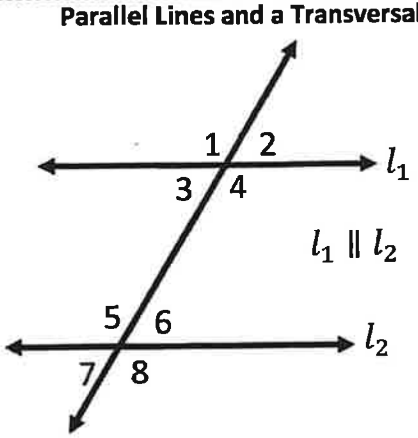
Proof	Statement	Reason
	Draw line DC parallel to AB	Construction
	$\angle 3 + \angle 4 = \angle DCB$	Angle Addition
	$\angle DCB + \angle 2 = 180°$	Co-Interior Angles
	$\angle 3 + \angle 4 + \angle 2 = 180°$	Substitution (From step 2)
	$\angle 1 = \angle 4$	Alternate Interior Angles
	$\angle 3 + \angle 1 + \angle 2 = 180°$	Substitution
	$\angle 1 + \angle 2 + \angle 3 = 180°$	Rewrite

Angle Sum of a Triangle Theorem

The Sum of angles in a triangle is 180°

Summary

Parallel Lines and a Transversal

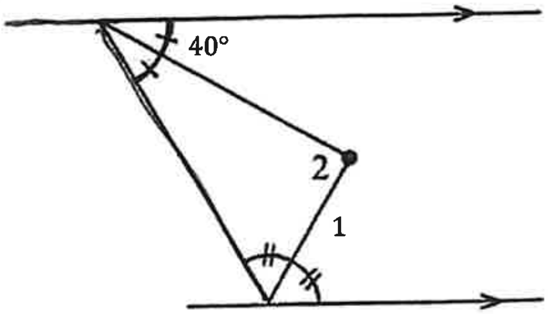


Vertical Angles	Corresponding Angles
$\angle 1 = \angle 4$	$\angle 1 = \angle 5$
$\angle 2 = \angle 3$	$\angle 2 = \angle 6$
$\angle 5 = \angle 8$	$\angle 3 = \angle 7$
$\angle 6 = \angle 7$	$\angle 4 = \angle 8$

Alternate Interior Angles	Co-Interior Angles
$\angle 3 = \angle 6$	$\angle 3 + \angle 5 = 180°$
$\angle 4 = \angle 5$	$\angle 4 + \angle 6 = 180°$

Find all the missing angles and state the reasons for each answer.

Example:



$\angle 1 = 50°$

$\angle 2 = 90°$

Solution:

$2(40°) + 2\angle 1 = 180°$ co-interior angles supplementary

$80° + 2\angle 1 = 180°$

$2\angle 1 = 100°$

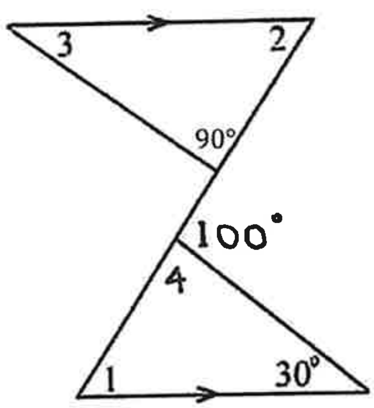
$\angle 1 = 50°$

$40° + \angle 2 + 50° = 180°$

angle sum of Δ

$\angle 2 = 90°$

Example:



$\angle 1 = 70°$

$\angle 2 = 70°$

$\angle 3 = 20°$

Solution:

$\angle 4 + 100° = 180°$ angles on a line supplementary

$\angle 4 = 80°$ algebra

$\angle 1 + 80° + 30° = 180°$ angle sum of Δ

$\angle 1 = 70°$ algebra

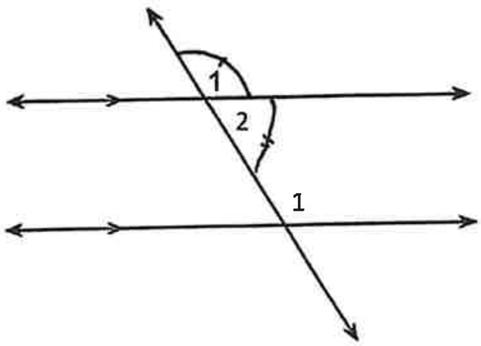
$\angle 2 = \angle 1$ alternate interior angles equal

$\angle 2 = 70°$

$90° + 70° + \angle 3 = 180°$ angle sum of Δ

$\angle 3 = 20°$ algebra

Example:



$\angle 1 = (x^2 - 25x)^\circ$

$\angle 2 = x^\circ$

Find the value of $\angle 1$.

Solution:

$\angle 1 + \angle 2 = 180^\circ$ co-interior angles supplementary

$x^2 - 25x + x = 180$ substitution

$x^2 - 24x - 180 = 0$

-180
6 -30
-24

1, 180
2, 90
3, 60
4, 45
5, 36
6, 30
9, 20
10, 18
12, 15

$(x+6)(x-30) = 0$

$x + 6 = 0$ or $x - 30 = 0$

$x = -6$ or $x = 30$

reject

$\angle 1 = x^2 - 25x$

$= (30)^2 - 25(30)$

$= 150^\circ$

$\angle 2 = x$

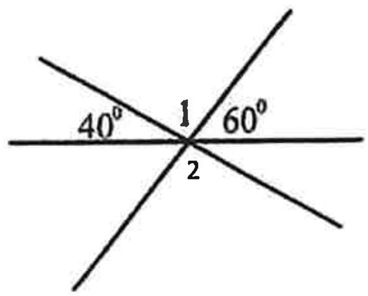
$= 30^\circ$

Practice Problems #1-8 Fri
then 9-16 Monday

Section 4.1 – Practice Problems

For the following questions, solve for the missing angles and give the reason.

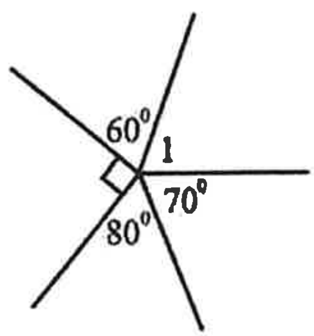
1.



$\angle 1 =$ ______, ______________________________

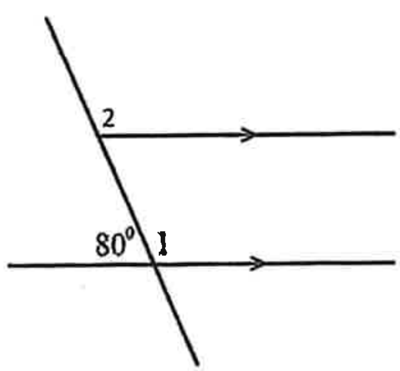
$\angle 2 =$ ______, ______________________________

2.



$\angle 1 =$ ______, ______________________________

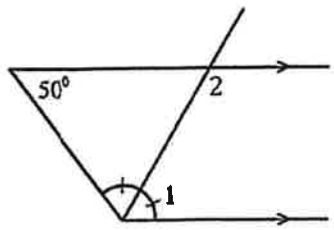
3.



$\angle 1 =$ ______, ______________________________

$\angle 2 =$ ______, ______________________________

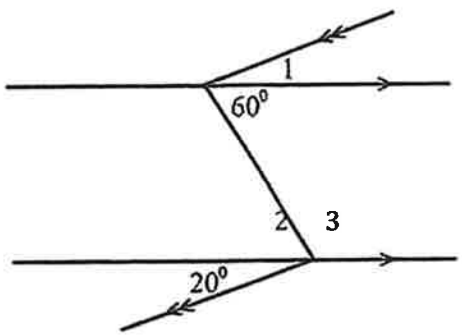
4.



∠1 = ______, ______________________________

∠2 = ______, ______________________________

5.

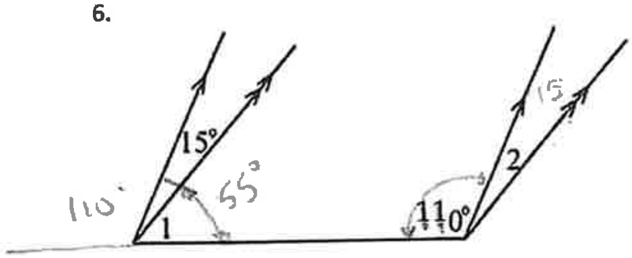


∠1 = ______, ______________________________

∠2 = ______, ______________________________

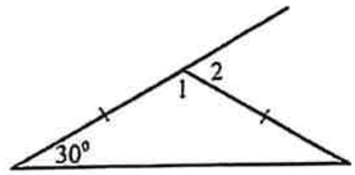
∠3 = ______, ______________________________

6.



∠1 = 55°, ______________________________

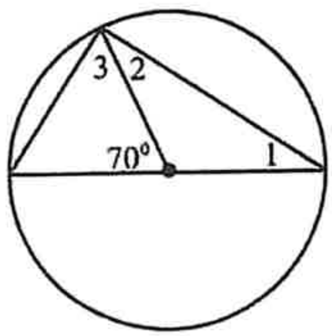
∠2 = 15°, ______________________________ 7.



∠1 = _____, ____________________

∠2 = _____, ____________________

8.

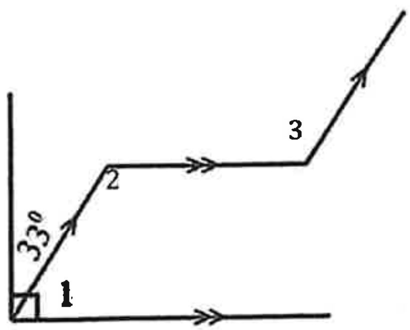


∠1 = _____, ____________________

∠2 = _____, ____________________

∠3 = _____, ____________________

9.

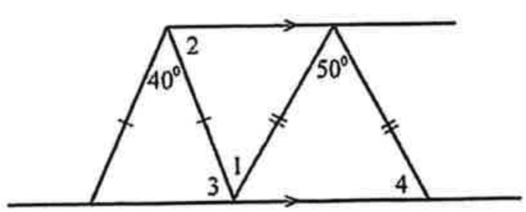


∠1 = _____, ____________________

∠2 = _____, ____________________

∠3 = _____, ____________________

10.



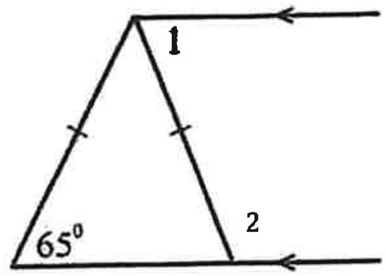
∠1 = _____, ____________________

∠2 = _____, ____________________

∠3 = _____, ____________________

∠4 = _____, ____________________

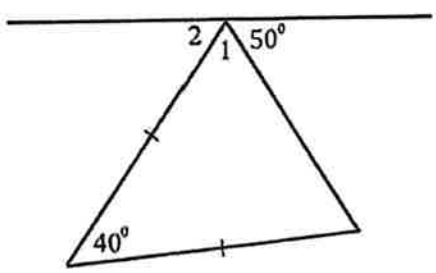
11.



$\angle 1 =$ _____, ____________________

$\angle 2 =$ _____, ____________________

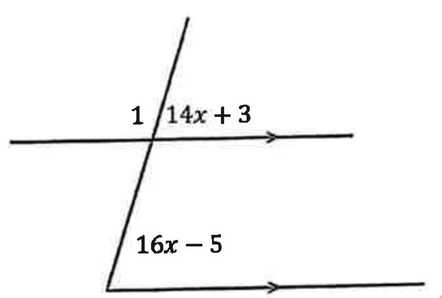
12.



$\angle 1 =$ _____, ____________________

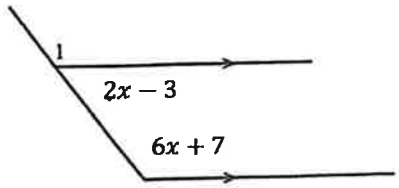
$\angle 2 =$ _____, ____________________

13.



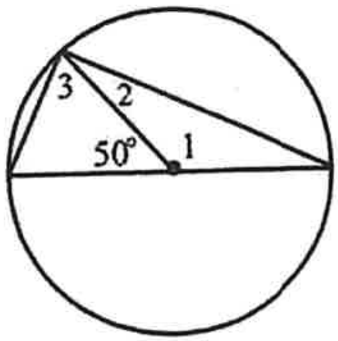
$\angle 1 =$ _____, ____________________

14.



$\angle 1 =$ ______, ______________________________

15.

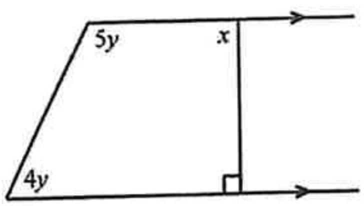


$\angle 1 =$ ______, ______________________________

$\angle 2 =$ ______, ______________________________

$\angle 3 =$ ______, ______________________________

16.



$\angle 1 =$ ______, ______________________________

$\angle 2 =$ ______, ______________________________