Section 3: Algebra, Elimination, and Logic

This book belongs to: ___________________________ Block: ______

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<thead>
<tr>
<th>Section</th>
<th>Due Date</th>
<th>Date Handed In</th>
<th>Level of Completion</th>
<th>Corrections Made and Understood</th>
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<tr>
<td>3.1</td>
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<td>3.2</td>
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<td>3.3</td>
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<td>3.4</td>
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Self-Assessment Rubric

<table>
<thead>
<tr>
<th>Category</th>
<th>Sub-Category</th>
<th>Description</th>
<th>Mark</th>
</tr>
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<tbody>
<tr>
<td>Expert (Extending)</td>
<td>4</td>
<td>Work meets the objectives; is clear, error free, and demonstrates a mastery of the Learning Targets</td>
<td>“You could teach this!”</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>Work meets the objectives; is clear, with some minor errors, and demonstrates a clear understanding of the Learning Targets</td>
<td>“Almost Perfect, one little error.”</td>
</tr>
<tr>
<td>Apprentice (Proficient)</td>
<td>3</td>
<td>Work almost meets the objectives; contains errors, and demonstrates sound reasoning and thought concerning the Learning Targets</td>
<td>“Good understanding with a few errors.”</td>
</tr>
<tr>
<td>Apprentice (Developing)</td>
<td>2</td>
<td>Work is in progress; contains errors, and demonstrates a partial understanding of the Learning Targets</td>
<td>“You are on the right track, but key concepts are missing.”</td>
</tr>
<tr>
<td>Novice (Emerging)</td>
<td>1.5</td>
<td>Work does not meet the objectives; frequent errors, and minimal understanding of the Learning Targets is demonstrated</td>
<td>“You have achieved the bare minimum to meet the learning outcome.”</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Work does not meet the objectives; there is no or minimal effort, and no understanding of the Learning Targets</td>
<td>“Learning Outcomes not met at this time.”</td>
</tr>
</tbody>
</table>

Learning Targets and Self-Evaluation

<table>
<thead>
<tr>
<th>L – T</th>
<th>Description</th>
<th>Mark</th>
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</thead>
<tbody>
<tr>
<td>3 – 1</td>
<td>• Understanding the balance of an equation, and maintaining equality</td>
<td></td>
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<tr>
<td></td>
<td>• 1 and 2 Step elimination and logic (Addition and Multiplication Principle)</td>
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<tr>
<td></td>
<td>• Grouping like terms</td>
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<tr>
<td>3 – 2</td>
<td>• Eliminating brackets (Distributive principle)</td>
<td></td>
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<td></td>
<td>• Eliminating fraction and decimals (LCM concepts of fractions)</td>
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<td></td>
<td>• Discovering equations from word problems (real life situations)</td>
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Comments: ____________________________________________________________________________
______________________________________________________________________________________
## Competency Evaluation

A valuable aspect to the learning process involves self-reflection and efficacy. Research has shown that authentic self-reflection helps improve performance and effort, and can have a direct impact on the growth mindset of the individual. In order to grow and be a life-long learner we need to develop the capacity to monitor, evaluate, and know what and where we need to focus on improvement. Read the following list of Core Competency Outcomes and reflect on your behaviour, attitude, effort, and actions throughout this unit.

- Rank yourself on the left of each column: 4 (Excellent), 3 (Good), 2 (Satisfactory), 1 (Needs Improvement)
- I will rank your Competency Evaluation on the right half of each column

<table>
<thead>
<tr>
<th>Personal Responsibility</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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</thead>
<tbody>
<tr>
<td>- I listen during instruction and come ready to ask questions</td>
<td></td>
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<tr>
<td>- I am on time for class</td>
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<tr>
<td>- I am fully prepared for the class, with all the required supplies</td>
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<tr>
<td>- I am fully prepared for Tests</td>
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<tr>
<td>- I follow instructions keep my Workbook organized and tidy</td>
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<td>- I am on task during work blocks</td>
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<tr>
<td>- I complete assignments on time</td>
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<table>
<thead>
<tr>
<th>Self-Regulation</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I keep track of my Learning Targets</td>
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<tr>
<td>- I take ownership over my goals, learning, and behaviour</td>
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<tr>
<td>- I can solve problems myself and know when to ask for help</td>
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<tr>
<td>- I can persevere in challenging tasks</td>
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<tr>
<td>- I am actively engaged in lessons and discussions</td>
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<td></td>
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<tr>
<td>- I only use my phone for school tasks</td>
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<table>
<thead>
<tr>
<th>Classroom Responsibility and Communication</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I am focused on the discussion and lessons</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- I ask questions during the lesson and class</td>
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<tr>
<td>- I give my best effort and encourage others to work well</td>
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<tr>
<td>- I am polite and communicate questions and concerns with my peers and teacher in a timely manner</td>
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<tr>
<td>- I clean up after myself and leave the classroom tidy when I leave</td>
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<table>
<thead>
<tr>
<th>Collaborative Actions</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
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<tbody>
<tr>
<td>- I can work with others to achieve a common goal</td>
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<tr>
<td>- I make contributions to my group</td>
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<tr>
<td>- I am kind to others, can work collaboratively and build relationships with my peers</td>
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<tr>
<td>- I can identify when others need support and provide it</td>
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<table>
<thead>
<tr>
<th>Communication Skills</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I present informative clearly, in an organized way</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>- I ask and respond to simple direct questions</td>
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<tr>
<td>- I am an active listener, I support and encourage the speaker</td>
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<tr>
<td>- I recognize that there are different points of view and can disagree respectfully</td>
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<tr>
<td>- I do not interrupt or speak over others</td>
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</tbody>
</table>

| Overall                                                      |    |   |   |   |

Goal for next Unit – refer to the above criteria. **Please select** (underline/highlight) **two areas** you want to focus on.
Section 3.1 – One and Two Step Equations

So it begins....

- When we think algebra, what comes to mind?
  - Headaches, moans & groans, anxiety...

- Don’t get yourself too riled up. Algebra is just the logical manipulation of an equation.
  - That’s where we start. With an equation.

- In order to be considered an equation you need a statement of inequality.

  \[ \text{Either: } = \quad \leq \quad \geq \]

- Whenever you have one of these in a statement it makes it an equation
- One side maintains equality with the other

In other words:

**BALANCE**

*Whatever we do from this point on in an equation, we have to use logical rules in order to maintain that balance, that equality.*

**Addition and Subtraction**

- It’s called the **ADDITION PRINCIPLE (ADDING TO MAKE 0)**

Consider this,

\[ 3 = 3 \quad \text{we have BALANCE} \]

- So if we **ADD** something to one side we have to **add it to both**:

  \[ 3 + 2 = 3 + 2 \]

- We use this concept to help **eliminate information** from one side of an equation
- This in turns adds it to the other side
**Example:** \( r - 4 = 7 \)

On the left we have an unknown. We need to get that unknown by itself on one side of the equals sign.

How do we do that?

- Well we have \(-4\), in order to eliminate it, we need it to be 0
- So what do we add to \(-4\) to make it 0, we need to add +4

So,

\[
r - 4 + 4 = 7 + 4
\]

Add +4 to both sides

And now, since \(-4 + 4 = 0\), we get \( r + 0 \) on the left, which is \( r \)

So after the elimination we get: \( r = 11 \) and we have solved for the unknown

- Now the previous example saw us subtracting from the unknown so we had to add a positive to both sides.
- When we add with the unknown, we have to add a negative (subtract) from both sides.

**Example:**

\( q + 5 = 15 \)

\[
q + 5 - 5 = 15 - 5
\]

\( q = 10 \)

Added a negative to both sides, in other words:

Subtracted

**Example:**

\[
\begin{align*}
  r - 4 &= 7 \\
  r - 4 + 4 &= 7 + 4 \\
  r &= 11
\end{align*}
\]

\[
\begin{align*}
  t + 5 &= 2 \\
  t + 5 - 5 &= 2 - 5 \\
  t &= -3
\end{align*}
\]
Example:

\[
\begin{align*}
q - 8 &= 10 \\
q - 8 + 8 &= 10 + 8 \\
q &= 18
\end{align*}
\]

\[
\begin{align*}
x + 4 &= -6 \\
x + 4 - 4 &= -6 - 4 \\
x &= -10
\end{align*}
\]

\[
\begin{align*}
a - 6 &= -13 \\
a - 6 + 6 &= -13 + 6 \\
a &= -7
\end{align*}
\]

\[
\begin{align*}
b + 8 &= -2 \\
b + 8 - 8 &= -2 - 8 \\
b &= -10
\end{align*}
\]

### Multiplication and Division

It’s called the **MULTIPLICATION PRINCIPLE (Multiplying to get 1)**

- **Multiplication and Division are inverses** of one another
- **Much like adding a negative** is the same as **subtraction**
- **Multiplying a fraction** is the same as **dividing**

Now for multiplication and division the number we want isn’t 0, it’s 1

- When we are **multiplying with the variable** we have to **divide** to end up with 1

**Example:**

\[
3x = 12
\]

- I don’t want 3\(x\), I want 1\(x\), so I’ll have to **divide by 3** (or multiply by \(\frac{1}{3}\))

\[
\frac{3x}{3} = 1x
\]

But don’t forget the whole **balance thing**. We need to **divide both sides**

\[
\frac{3x}{3} = \frac{12}{3}, \quad 1x = 4 \text{ or } x = 4
\]
- When we **multiply** with the variable, we do the **inverse**, division
- Then, if we **divide** with the variable, we do the **inverse**, multiplication

Consider this,

\[
\frac{1}{2} * 2 = 1
\]

- If you **multiply a fraction** by its **denominator** the cancel one another out, because the top and bottom divide to give you 1

\[
\frac{1}{2} * 2 = \frac{1 * 2}{2} = \frac{2}{2} = 1
\]

So,

\[
\frac{t}{5} = 10
\]

- Since we are **dividing** with the variable, we have to **multiply**

\[
\frac{t}{5} = 10, \quad 5 * \frac{t}{5} = 10 * 5, \quad \frac{5t}{5} = 50, \quad t = 50
\]

Multiply both sides by 5 \hspace{1cm} \text{Divide the left out}

---

**Examples:**

\[
\begin{align*}
5x &= 10 & -3r &= 27 \\
\frac{5x}{5} &= \frac{10}{5} & -\frac{3r}{3} &= \frac{27}{3} \\
x &= 2 & r &= -9
\end{align*}
\]
### Example:

<table>
<thead>
<tr>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4a = 3$</td>
<td>$8n = 2$</td>
</tr>
<tr>
<td>$\frac{4a}{4} = \frac{3}{4}$</td>
<td>$\frac{8n}{8} = \frac{2}{8}$</td>
</tr>
<tr>
<td>$a = \frac{3}{4}$</td>
<td>$n = \frac{1}{4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 3</th>
<th>Equation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{q}{5} = 2$</td>
<td>$\frac{d}{-4} = -8$</td>
</tr>
<tr>
<td>$5 \times \frac{q}{5} = 2 \times 5$</td>
<td>$-4 \times \frac{d}{-4} = -8 \times -4$</td>
</tr>
<tr>
<td>$q = 10$</td>
<td>$d = 32$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation 5</th>
<th>Equation 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{b}{7} = 2$</td>
<td>$\frac{v}{4} = -12$</td>
</tr>
<tr>
<td>$7 \times \frac{b}{7} = 2 \times 7$</td>
<td>$4 \times \frac{v}{4} = -12 \times 4$</td>
</tr>
<tr>
<td>$b = 14$</td>
<td>$v = -48$</td>
</tr>
</tbody>
</table>

- These are all 1 – **Step** equations
- They take 1 step to get your answer
- Addition, Subtraction, Multiplication, and Division

Next we will see examples that require **2 or more Steps**
**Two Steps**

If you are multiplying with a constant with a variable and adding or subtracting a number to it.

- We need **2 Steps**

**Example:**

\[2x + 5 = 11\]

- First get rid of the number that is being added or subtract, leaving a constant-variable product

\[2x + 5 - 5 = 11 - 5\]

- The 5’s cancel, leave us with:

\[2x = 6\]

- Then divide the 2, on both sides, to isolate the variable and solve

\[\frac{2x}{2} = \frac{6}{2} \rightarrow x = 3\]

**Example:**

\[3q - 8 = 10\]

\[3q - 8 + 8 = 10 + 8\]

\[3q = 18\]

\[\frac{3q}{3} = \frac{18}{3} \rightarrow q = 6\]

\[5x + 4 = -6\]

\[5x + 4 - 4 = -6 - 4\]

\[5x = -10\]

\[\frac{5x}{5} = \frac{-10}{5} \rightarrow x = -2\]

\[3a - 6 = -13\]

\[3a - 6 + 6 = -13 + 6\]

\[3a = -7\]

\[\frac{3a}{3} = \frac{-7}{3} \rightarrow a = -\frac{7}{3}\]

\[11b + 8 = -2\]

\[11b + 8 - 8 = -2 - 8\]

\[11b = -10\]

\[\frac{11b}{11} = \frac{-10}{11} \rightarrow b = -\frac{10}{11}\]
If you have **one fraction multiplying with a variable**?

- We need **2 Steps**

**Example:**

\[
\frac{2}{3}x = 6
\]

- **First Multiply** by the **Denominator** on **both** sides

\[
3 \times \frac{2}{3}x = 6 \times 3
\]

- The 3’s cancel on the left

\[
x = \frac{18}{2}
\]

- **Then Divide** by the **Numerator** on **both** sides

\[
x = 9
\]

**Example:**

\[
\frac{4}{5}x = 4 \rightarrow 5 \times \frac{4}{5}x = 4 \times 5 \rightarrow 4x = 20 \rightarrow \frac{4x}{4} = \frac{20}{4} \rightarrow x = 5
\]

- These can be done in 1 step by **multiplying by the reciprocal**.
- It only works in when you have **1 fraction**, a **variable** and the **answer**

\[
\frac{2}{3}x = 8 \rightarrow \frac{3}{2} \times \frac{2}{3}x = 8 \times \frac{3}{2} \rightarrow x = \frac{24}{2} \rightarrow x = 12
\]
# Section 3.1 – Practice Questions

Use the Addition and Subtraction Principle. ISOLATE THE VARIABLE, show steps.

<p>| | | |</p>
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<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>( w + 4 = 7 )</td>
<td>2.</td>
</tr>
<tr>
<td>4.</td>
<td>( t + 9 = -3 )</td>
<td>5.</td>
</tr>
<tr>
<td>7.</td>
<td>( z - (-2) = 5 )</td>
<td>8.</td>
</tr>
<tr>
<td>10.</td>
<td>( 12 - l = -4 )</td>
<td>11.</td>
</tr>
<tr>
<td>13.</td>
<td>( j + 7 = -4 )</td>
<td>14.</td>
</tr>
</tbody>
</table>
Use the Multiplication and Division Principle. ISOLATE THE VARIABLE, show steps.

16. \(3x = 12\)  
17. \(2x = 24\)  
18. \(4t = -13\)

19. \(-3t = -6\)  
20. \(-4r = 12\)  
21. \(-12m = 156\)

22. \(3t = 17\)  
23. \(-x = 4\)  
24. \(7h = 2\)

25. \(z \cdot \frac{1}{7} = 9\)  
26. \(\frac{k}{6} = -2\)  
27. \(t \cdot \frac{1}{8} = 4\)

28. \(\frac{r}{3} = -3\)  
29. \(\frac{j}{-4} = -6\)  
30. \(\frac{r}{6} = 35\)

31. \(\frac{t}{-2} = 5\)  
32. \(\frac{a}{7} = 0\)  
33. \(\frac{-w}{7} = -4\)

Use two-step processes to solve the following

34. \(3x - 5 = 10\)  
35. \(-2x + 5 = 7\)  
36. \(-4r - 4 = -4\)
<p>| | | |</p>
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<thead>
<tr>
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<tbody>
<tr>
<td>37.</td>
<td>$6x + 12 = -12$</td>
<td>38.</td>
</tr>
<tr>
<td>40.</td>
<td>$-2r - 5 = 9$</td>
<td>41.</td>
</tr>
<tr>
<td>43.</td>
<td>$-7q - 2 = -9$</td>
<td>44.</td>
</tr>
<tr>
<td>46.</td>
<td>$\frac{2}{3}x = 8$</td>
<td>47.</td>
</tr>
<tr>
<td>49.</td>
<td>$\frac{3}{-7}x = 5$</td>
<td>50.</td>
</tr>
</tbody>
</table>
Section 3.2 – Grouping Like Terms

- Now we find ourselves in the situation where we have more than just the 1 Step

Example: \[3t + 4 - 2t + 5 = -t - 3\]

- There is a lot going on here.
- 1st we can group the Like Terms
- Like Terms have the Same Variable to the Same Exponent

Example: \[2x \text{ and } 3x \hspace{1cm} 5t^2 \text{ and } 7t^2 \hspace{1cm} 18v^5 \text{ and } 19v^5\]

All of these have the same variable to the same exponent

Consider this:

\[
\begin{array}{c}
\text{2 apples} \\
\text{+ 3 apples} \\
\hline
5 \text{ apples}
\end{array} 
\quad 
\begin{array}{c}
8 T - \text{Shirts} \\
\text{– 3 T – Shirts} \\
\hline
5 T - \text{Shirts}
\end{array} 
\quad 
\begin{array}{c}
4t \\
\text{+ 7t} \\
\hline
11t
\end{array} 
\quad 
\begin{array}{c}
12d^2 \\
\text{– 7d}^2 \\
\hline
5d^2
\end{array}
\]

- We can’t combine apples and T – Shirts, much like we can’t combine t and d or x and \(x^2\), they aren’t the same things

- Once we have grouped the terms together, we can finish off the questions using the Steps we learned previously.
- It is very good form to combine everything on the individual sides of the equal sign first
- Remember that this is just grouping and the BALANCE is not affected.

Example:

\[3x + 4 - 2x + 5 = -x - 3\]

- 1st Combine the Like Terms on either side of the equals sign

\[\boxed{3x + 4 - 2x + 5 = -x - 3}\]

So we get:

\[x + 9 = -x - 3\]

- We have combined our variable terms and non-variable terms
Now we have to **combine across the equals sign**
- It doesn’t matter if the variables end up on the left or right
- General notation has the variable on the left

\[ x + 9 = -x - 3 \]

✓ 1\textsuperscript{st} Subtract 9 from both sides, to isolate the variable on the left

\[ x + 9 - 9 = -x - 3 - 9 \]

\[ x = -x - 12 \]

✓ 2\textsuperscript{nd} Add \( x \) to both sides, to cancel out the variable on the right

\[ x + x = -x - 12 + x \]

\[ 2x = -12 \]

✓ Last of all divide the variable by 2 to cancel out the 2

\[ \frac{2x}{2} = \frac{-12}{2} \rightarrow x = -6 \]

**Example:**

\[ 5t + 3 - 3t + 2t - 7 + 2t = t + 6 \]

\[ 5t + 3 - 3t + 2t - 7 + 2t = t + 6 \]

\[ 6t - 4 = t + 6 \]

\[ 6t - 4 - t = t + 6 - t \]

\[ 5t - 4 = 6 \]

\[ 5t - 4 + 4 = 6 + 4 \]

\[ 5t = 10 \]

\[ \frac{5t}{5} = \frac{10}{5} \]

\[ t = 2 \]
Section 3.2 – Practice Questions

When we combine terms, we can only combine ones that are the same.

1. $1\, \text{Apple} + 3\, \text{Apples} =$
2. $3\, \text{Bananas} - 2\, \text{Bananas} =$
3. $7\, \text{Cookies} + 5\, \text{Cookies} =$

It works the same with variables, they have to be the same to the same exponent!

4. $3x + 2x =$
5. $5x^2 - 2x^2 =$
6. $2x + 3y =$

7. $-2x + 7x =$
8. $-5x^2 - 7x^2 =$
9. $4x^2 + 5x - 3x =$

10. $-3xy - 4xy =$
11. $2t^2 - 3r^2 + t^2 - 5r^2 =$
12. $3r + 5r - 8r =$

Combine the LIKE TERMS and solve for the variable.

13. $3x - 4 - 2x + 8 = 7$
14. $4r + 6 - 3r - 5 = 2r + 8$
15. \[ 9k - 4 + 6 - 2k = 3k - 7 + 2k - 8 \]

16. \[ 8t - 4 + 3t + 5 = t - 7 \]

17. \[ z + 4 - 3z = -z + 4 \]

18. \[ 5 - 2s + 6 = 4s - 4 \]

19. \[ 2x + 3 + 4x = -3 - 2x \]

20. \[ 2t + 6 = 2t + 6 \]

21. \[ -4t + 7 = -4t + 8 \]

22. \[ 3f + 7 - 4f = -5 + 3f \]
Section 3.3 – Eliminating Brackets, Fractions, and Decimals

Eliminating Brackets

- In math we have a term called **Distributivity**

**Example:**

\[ a(b + c) = ab + ac \]

- \( a \) times \((b + c) = a \) times \( b \) plus \( a \) times \( c \)

- \( the \ a \) multiplies with \( both \ terms \ inside \ the \ brackets \)

- This is **DISTRIBUTIVITY**

- I use the term **WATERBOMB**

\[ a(b + c) = ab + ac \]

**Example:**

\[ 2(r + 6) = 2 \]

\[ 2(r + 6) = 2 \quad \text{Waterbomb} \]

\[ 2r + 12 = 2 \]

\[ 2r + 12 - 12 = 2 - 12 \quad \text{Subtract 12 from both sides} \]

\[ 2r = -10 \]

\[ \frac{2r}{2} = \frac{-10}{2} \quad \text{Divide both sides by 2} \]

\[ r = -5 \]

- Whenever there are **Brackets**, you **multiply in to them**

- **DISTRIBUTE, WATERBOMB**, whichever term you prefer
Example: \[4(s + 4) = 28\]
\[4s + 16 = 28\]
\[4s + 16 - 16 = 28 - 16\]
\[4s = 12\]
\[\frac{4s}{4} = \frac{12}{4}\]
\[s = 3\]

- Even if is just a negative symbol \(-\), this means \(-1\)

Example: \[-(r - 5) = 10\]
\[-r + 5 = 10\]
\[-r + 5 - 5 = 10 - 5\]
\[-r = 5\]
\[(-1)(-r) = 5(-1)\]
\[r = -5\]

- Every step is logical and maintains the balance
- Next we will look at dealing with Multiple Fractions
Eliminating Multiple Fractions

- How do you get rid of 1 denominator?

\( \frac{t}{2} = 4 \)

- We multiply that fraction by the denominator or any multiple of it.

- You see we multiply by a multiple because it gives us a whole number result, eliminating the fraction.

Watch:

<table>
<thead>
<tr>
<th>Multiple of 2</th>
<th>Multiple of 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 * ( \frac{1}{2} ) = 2</td>
<td>8 * ( \frac{1}{2} ) = 4</td>
</tr>
</tbody>
</table>

Whole Number Result  Whole Number Result

- So what if we have multiple denominators?

\[ \frac{1}{2}x + \frac{2}{3} = \frac{1}{4} \]

- To cancel out the 2, 3, and 4 respectively I would need to multiply everything by 2 and 3 and 4.

- But to get rid of all of them at once, I need to multiply them all by their lowest common multiple!

- Using the LCM will give me a whole number result for every fraction.
Example:

\[ \frac{1}{2}x - \frac{2}{3} = \frac{1}{4} \]

- 1st what is the LCM of 2, 3, and 4? It’s 12!
- Then multiply every term by it

\[
12 \cdot \frac{1}{2}x - \frac{2}{3} \cdot 12 = \frac{1}{4} \cdot 12
\]

- Multiply every fraction by the LCM

\[
\frac{12}{2}x - \frac{24}{3} = \frac{12}{4}
\]

- Remember the multiplication is with the numerator only

6x - 8 = 3

- Simplify the fractions

6x - 8 + 8 = 3 + 8

- Add 8 to both sides of the equation

6x = 11

- Divide both sides by 6

\[
6x = \frac{11}{6}
\]

\[
x = \frac{11}{6}
\]

- You MUST multiply every term to KEEP THAT BALANCE!!!

Eliminating Decimals

- It’s really quite simple; we just need to know what decimals are?
- Decimals are base 10 fractions

\[
0.1 = \frac{1}{10} \quad \text{tenth}
\]

\[
0.01 = \frac{1}{100} \quad \text{hundredth}
\]

\[
0.001 = \frac{1}{1000} \quad \text{thousandth}
\]

- So the LCM of fractions is always going to be: 10, 100, 1000, etc.
Example: \( 0.4x + 0.6 = 0.8 \)

\[
0.4x \times 10 + 0.6 \times 10 = 0.8 \times 10
\]

\( 4x + 6 = 8 \)

\( 4x + 6 - 6 = 8 - 6 \)

\( 4x = 2 \)

\[
\frac{4x}{4} = \frac{2}{4}
\]

\( x = \frac{2}{4} = \frac{1}{2} \)

- Multiply every term by the LCM: 10
- Simplify the Equation
- Subtract both sides by 6
- Divide both sides by 4
- Simplify your final answer

Example: \( 0.3x - 0.06 = 0.24 \)

\[
0.3x \times 100 - 0.06 \times 100 = 0.24 \times 100
\]

\( 30x - 6 = 24 \)

\( 30x - 6 + 6 = 24 + 6 \)

\( 30x = 30 \)

\[
\frac{30x}{30} = \frac{30}{30}
\]

\( x = 1 \)

- Multiply every term by the LCM: 100
- Simplify the Equation
- Add 6 to both sides
- Divide both sides by 30
- Simplify your final answer
Section 3.3 – Practice Questions

- Eliminate the **Brackets (WATERBOMB)** – Distributive Property
- Then solve for the unknown – these are **MULTI-STEP Equations**

1. \(2(x + 4) = 8\)
2. \(-3(s - 7) = -5\)

3. \(4(t + 2) = 2(t - 3)\)
4. \(-5(6 - z) = 3(z + 4)\)

5. \(-2(4t + 54) = 3(-t + 5)\)
6. \(3(3q - 4) = 2(4q + 5)\)

7. \(3(4r - 3) = 5(-2r + 6) + 2\)
8. \(8(3t - 12) = 12t\)
Eliminate the fractions, using LCM, then solve for the unknown.

9. \( \frac{t}{6} + \frac{1}{3} = \frac{1}{2} \)

10. \( \frac{7}{8}x - \frac{1}{16} + \frac{3}{4}x = \frac{1}{4} + x \)

11. \( \frac{2}{3}x - \frac{1}{4}x = \frac{1}{2}x + 1 \)

12. \( \frac{7}{2}q - 3q = -\frac{11}{2}q + \frac{3}{2} + \frac{5}{2}q \)

13. \( 1 + \frac{y}{5} = \frac{2}{3}y + \frac{12}{5} \)

14. \( \frac{4}{5}x - \frac{1}{2}x = \frac{3}{10}x + 4 \)
Eliminate the decimals, using factors of 10, solve for the unknown.

15. \(0.04k = 0.8\)  
16. \(0.2x + 0.22x = 0.84\)

17. \(2.1y - 2.8 = 5.6\)  
18. \(1.7w + 5 - 1.62w = 0.4w + 4.68\)

19. \(0.3 + 0.4x = 0.2 - 0.25x\)  
20. \(0.05 - 0.5x = 0.1 - 0.25x\)

21. \(0.6x - 0.01 = 0.02x + 0.29\)  
22. \(1.05 - 0.62x = 0.85 - 0.22x\)
The following questions are challenging MULTI-STEP questions. The Answer is provided below. SHOW THE STEPS that shows me you UNDERSTAND.

23. \( \frac{1}{3}(5x - 3) + \frac{1}{2}(1 - x) = \frac{1}{4}(x - 2) \)

\[
23. \quad \frac{1}{3}(5x - 3) + \frac{1}{2}(1 - x) = \frac{1}{4}(x - 2) \\
\]

\[
24. \quad \frac{2}{3} \left( \frac{7}{8} - \frac{x}{4} \right) - \frac{3}{8} = \frac{5}{8} \\
\]

Answer: \( x = 0 \)

Answer: \( x = \frac{-5}{2} \)
25. \( \frac{1}{4}(8x + 4) - 17 = -\frac{1}{2}(4x - 8) \)

Answer: \( x = 5 \)

26. \( 0.25(8y + 4) - 17 = -0.5(4y - 8) \)

Answer: \( y = 5 \)
Section 3.4 – Inequalities

- Inequalities are equations that give a range of possible answers
- The BALANCE of an inequality is the same as with an equal sign

Here is how we define them:

\[ x > 4 \quad \text{> mean strictly greater than 4} \]
\[ t < 2 \quad \text{< mean strictly less than 2} \]
\[ r \leq 3 \quad \text{\leq mean less than or equal to 3} \]
\[ q \geq -7 \quad \text{\geq mean greater than or equal to –7} \]

So we treat the inequality symbol exactly the same as an equals sign when applying our Algebraic Logic. There is one minor detail that changes; stay tuned.

**Example:** \( t + 7 > 4 \)

- Subtract 7 from both sides
  \[ t + 7 - 7 > 4 - 7 \]
  \[ t > -3 \]
- It graphs like this:
  \[ \text{Not filled in because it is STRCTLY GREATER} \]

**Example:** \( \frac{r}{4} \leq 7 \)

\[ 4 \cdot \frac{r}{4} \leq 7 \cdot 4 \]
\[ r \leq 28 \]
- Filled in because it is LESS THAN or EQUAL

Adrian Herlaar, School District 61

www.mrherlaar.weebly.com
Example: \(3t - 5 \geq 4\)

\[
3t - 5 + 5 \geq 4 + 5
\]

\[
3t \geq 9
\]

\[
\frac{3t}{3} \geq \frac{9}{3}
\]

\[
t \geq 3
\]

Now here is the BONUS....

\[\mathbf{\bullet} \quad \text{If at any point you MULTIPLY OR DIVIDE by a NEGATIVE, you FLIP the inequality}\]

\[\mathbf{\bullet} \quad \text{Manipulation of MULTI-STEP questions all works the same. Just don’t forget to FLIP the inequality when every time you multiply or divide by a NEGATIVE.}\]

Example: \(-3m \geq 12\)

\[
\frac{-3m}{-3} \geq \frac{12}{-3}
\]

\[
m \leq -4
\]

\[\mathbf{\bullet} \quad \text{Since we divided by a negative we have to FLIP the inequality}\]

Example: \(-\frac{1}{4}r - \frac{2}{3} \geq \frac{1}{6}\)

\[
12 * \left(-\frac{1}{4}r - \frac{2}{3}\right) \geq 12 * \frac{1}{6}
\]

\[
-\frac{12}{4}r - \frac{24}{3} \geq \frac{12}{6}
\]

\[
-3r - 8 \geq 2
\]

\[
-3r - 8 + 8 \geq 2 + 8
\]

\[
-3r \geq 10
\]

\[
\frac{-3r}{-3} \geq \frac{10}{-3} \rightarrow r \leq -\frac{10}{3}
\]

\[\mathbf{\bullet} \quad \text{Since we divided by a negative we have to FLIP the inequality}\]
**Word Problems**

- The hard part is **converting the words to variables and numbers**
- Start by **identifying your unknowns** and any other unknown with respect to your first one (you’ll see in an example)

**Example:**

A number multiplied by 2 and then adding 7 results in 23, what is the number?

- How do I write this?
- Follow along

<table>
<thead>
<tr>
<th>Let ( n ) be our number</th>
<th>Multiplied by two means</th>
<th>Adding 7</th>
<th>Result is 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>( 2n )</td>
<td>( 2n + 7 )</td>
<td>= 23</td>
</tr>
</tbody>
</table>

So,

\[
2n + 7 = 23
\]

\[
2n + 7 - 7 = 23 - 7
\]

\[
2n = 16
\]

\[
\frac{2n}{2} = \frac{16}{2}
\]

\[
n = 8
\]

**Example:**

The **sum** of 3 consecutive numbers is 48. What are the numbers?

- Consecutive means one right after another so, if my first number is \( n \): I have: \( n \), \( n + 1 \), \( n + 2 \)

Sum means addition:

\[
n + n + 1 + n + 2 = 48 \quad \rightarrow \quad 3n + 3 = 48
\]

\[
3n = 45
\]

\[
\frac{3n}{3} = \frac{45}{3} \quad \rightarrow \quad n = 15
\]

The Numbers are: \( 15, 16, \text{and } 17 \)
Section 3.4 – Practice Questions

- Inequalities represent an equation where one side is either greater than, lesser than or equal to the other side.
- The rules and logic of balance still work the same way.

1. \( x + 3 > 7 \)
2. \( t - 7 < -4 \)
3. \( 3x \leq 12 \)
4. \( -5q \geq 13 \)
5. \( \frac{x}{4} \leq 12 \)
6. \( 3(d + 4) \geq 15 \)
7. \( \frac{2}{3} t + \frac{1}{4} < 2 \)

8. \( 0.8z - 0.4z \geq -1 \)

9. \( \frac{2}{3} \left( \frac{1}{4} - \frac{1}{6} t \right) \geq \frac{1}{3} \)

10. \( \frac{1}{2} \left( \frac{2}{3} - \frac{4}{7} t \right) \geq \frac{1}{3} \)

**Word Problems**

- Discover the equation in the following sentences.

11. You open a book and the sum (addition) of the two pages is 111. What are the two pages?
12. In Victoria, a taxi charges $3.00 and then $0.60 for every kilometer you travel. How far can you go for $19.20?

13. The second angle of a triangle is four times as large as the first angle. The third angle is 30 degrees bigger than the first. What are the measurements of the three angles? (Angles in a triangle add to 180 degree)

14. If I have $2.05 and there are 3 dimes in the pile, how many quarters do I have?
15. If you double a number and add 36, you get five times the original number. What is the original number?

16. Three consecutive numbers such that first number plus twice the second, plus five less than the third is 27. What are the three numbers?
Extra Work Space
## Answer Key

### Section 3.1

<p>| | | | | |</p>
<table>
<thead>
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<td>1.</td>
<td>( w = 3 )</td>
<td>2.</td>
<td>( x = -20 )</td>
<td>3.</td>
</tr>
<tr>
<td>4.</td>
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<td>5.</td>
<td>( k = -2 )</td>
<td>6.</td>
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<td>9.</td>
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<tr>
<td>12.</td>
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<tr>
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<td>( f = -40 )</td>
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<td>16.</td>
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<tr>
<td>17.</td>
<td>( x = 12 )</td>
<td>18.</td>
<td>( t = -\frac{13}{4} )</td>
<td>19.</td>
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<tr>
<td>20.</td>
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<tr>
<td>21.</td>
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<td>22.</td>
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</tr>
<tr>
<td>24.</td>
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<td>( k = -12 )</td>
<td>27.</td>
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<tr>
<td>28.</td>
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<td>( r = 210 )</td>
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<td>35.</td>
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<td>( x = \frac{2}{3} )</td>
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<tr>
<td>40.</td>
<td>( r = -7 )</td>
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<td>( x = 12 )</td>
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<tr>
<td>49.</td>
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<td>( x = -\frac{7}{3} )</td>
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### Section 3.2

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<td>1.</td>
<td>4 Apples</td>
<td>2.</td>
<td>1 Banana</td>
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<td>4.</td>
<td>( 5x )</td>
<td>5.</td>
<td>( 3x^2 )</td>
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<td>15.</td>
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### Section 3.3

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### Section 3.4

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<tr>
<td>9.</td>
<td>$t \leq -\frac{3}{2}$</td>
<td>10.</td>
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<td>7, 8, 9</td>
<td></td>
</tr>
</tbody>
</table>