

Section 3.6 – Practice Problems

Unknown number problems

1. The sum of a number and its reciprocal is  $\frac{13}{6}$ . Find the number.

let  $x$  be the number

$$x + \frac{1}{x} = \frac{13}{6} \quad \text{LCD: } 6x$$

$$6x^2 + 6 = 13x$$

$$6x^2 - 13x + 6 = 0$$

$$x^2 - 13x + 36 = 0$$

$$(x - \frac{9}{6})(x - \frac{4}{6}) = 0$$

$$(x - \frac{3}{2})(x - \frac{2}{3}) = 0$$

$$(2x - 3)(3x - 2) = 0$$

$$x = \frac{3}{2}$$

$$x = \frac{2}{3}$$

3. Find two consecutive odd integers whose reciprocals add to  $\frac{8}{15}$

Let  $n$  be odd  $n+2$  is next odd

$$\frac{1}{n} + \frac{1}{n+2} = \frac{8}{15} \quad \text{LCD: } 15n(n+2)$$

$$15(n+2) + 15n = 8n(n+2) \rightarrow 8n^2 - 14n - 30 = 0$$

$$15n + 30 + 15n = 8n^2 + 16n$$

$$n^2 - 14n - 240 = 0$$

$n = 3$

$$(n - \frac{24}{8})(n + \frac{10}{8}) = 0$$

$3 \text{ and } 5$

$$(n - 3)(n + \frac{5}{4}) = 0$$

$$(n - 3)(4n + 5) = 0$$

$$n = 3 \quad n = -\frac{5}{4} \leftarrow \text{reject, not an integer}$$

2. Find two consecutive even integers whose reciprocals add to  $\frac{7}{24}$

$n, n+2$  consecutive even.

$$\frac{1}{n} + \frac{1}{n+2} = \frac{7}{24} \quad \text{LCD: } 24n(n+2)$$

$$24(n+2) + 24n = 7(n+2)n$$

$$24n + 48 + 24n = 7n^2 + 14n$$

$$7n^2 - 34n - 48 = 0$$

$$n^2 - 34n - 336 = 0$$

$$(n - \frac{42}{7})(n + \frac{8}{7}) = 0$$

$n = 6$

$6 \text{ and } 8$

$n = -\frac{8}{7} \leftarrow \text{reject, not an integer}$

$$336 \begin{matrix} \wedge \\ 3 \end{matrix} \begin{matrix} \wedge \\ 112 \end{matrix} \begin{matrix} \wedge \\ 2 \end{matrix} \begin{matrix} \wedge \\ 56 \end{matrix} \begin{matrix} \wedge \\ 28 \end{matrix} \begin{matrix} \wedge \\ 14 \end{matrix}$$

4. A number added to the product of 6 and the reciprocal of that number is  $-5$ . Find the number.

$$x + \left(6 \cdot \frac{1}{x}\right) = -5$$

$$x + \frac{6}{x} = -5 \rightarrow x^2 + 6 = -5x$$

$$x^2 + 5x + 6 = 0$$

$$(x + 3)(x + 2) = 0$$

$$x = -3$$

$x = -3 \text{ or } -2$

$$x = -2$$

Work Problems

5. It would take Sue 4 hours to paint a large room, and it would take Bob 5 hours to paint the same room. If they work together, how long would it take them to complete the job?

If  $x$  is the total time

$\frac{1}{x}$  is the portion done in 1 hour

In 1 hour Sue paints  $\frac{1}{4}$  of the room

Bob paints  $\frac{1}{5}$  of the room

Together: LCD =  $20x$

$$\frac{1}{4} + \frac{1}{5} = \frac{1}{x}$$

$$5x + 4x = 20$$

$$9x = 20$$

$$x = \frac{20}{9} = 2\frac{2}{9}$$

Together it takes  $2\frac{2}{9}$  hours

6. Jane works twice as fast as her daughter Anna. If it takes 15 minutes to clean the kitchen together, how long would it take Anna to clean the kitchen by herself?

Together takes 15 minutes

so  $\frac{1}{15}$  in one minute

Jane:  $\frac{2}{x}$  in one minute, Anna  $\frac{1}{x}$

Together:  $\frac{2}{x} + \frac{1}{x} = \frac{1}{15}$       LCD:  $15x$

$$30 + 15 = x$$

$$x = 45$$

Anna takes 45 minutes alone.

7. Ken takes 3 hours longer to assemble a motor than Hans. When working together, it takes them 2 hours to assemble the motor. How long would it take Ken to do the job alone?

Together  $\frac{1}{2}$  in one hour

Hans:  $\frac{1}{x}$  in an hour

Ken:  $\frac{1}{x+3}$

LCD:  $(x+3)(2x)$

$$\frac{1}{x+3} + \frac{1}{x} = \frac{1}{2} \rightarrow 2x + 2(x+3) = x(x+3)$$

$$2x + 2x + 6 = x^2 + 3x \rightarrow x^2 - x - 6$$

$$x = 3 \quad x = -2 \text{ reject} \quad (x-3)(x+2)$$

Ken:  $x+3 = 6$  hours alone

8. A cold water tap can fill a tub in 6 minutes, and a hot water tap can fill the tub in 8 minutes. A drain can empty the full tub in 10 minutes. If both the taps are on and the drain is open, how long will it take to fill the tub? (Complete waste of water...)

$x$  is time to fill  
 $\frac{1}{x}$  is one minute together

In one minute:

cw  $\frac{1}{6}$

$$\frac{1}{6} + \frac{1}{8} - \frac{1}{10} = \frac{1}{x}$$

hw  $\frac{1}{8}$

LCD:  $120x$

drain  $\frac{1}{10}$

$$20x + 15x - 12x = 120$$

$$23x = 120$$

$$x = \frac{120}{23}$$

Takes  $5\frac{5}{23}$  mins

Distance Problems

$$S = \frac{D}{t} \rightarrow t = \frac{D}{S}$$

9. A boat travels 40km downstream in the same time it takes to travel 30km upstream. If the current flows at 6km/h, what is the speed of the boat in still water?

Let  $x$  be the still water

Speed downstream	$x+6$	$t_1 = t_2$
upstream	$x-6$	

$$\frac{40}{x+6} = t_1 \quad \frac{40}{x+6} = \frac{30}{x-6}$$

$$\frac{30}{x-6} = t_2 \quad 40(x-6) = 30(x+6)$$

$$40x - 240 = 30x + 180$$

$$10x = 420$$

$$x = 42$$

boat speed  
42km/h

11. A woman drives to work at an average speed of 50mph. The average speed of the return trip home is 30mph. What is the average speed of the round trip?

Let  $x$  be the average speed in km/hr

$$t_{\text{going}} + t_{\text{back}} = t_{\text{total}} \quad d = ?$$

$$\frac{d}{50} + \frac{d}{30} = \frac{2d}{x}$$

$$\text{LCD: } \frac{150x}{d}$$

$$t = \frac{d}{S}$$

average speed  
be  $x$ .

$$3x + 5x = 300$$

$$8x = 300$$

$$x = \frac{300}{8}$$

$x = 37.5 \text{ km/hr}$

10. The speed of a boat in Stillwater is 10mph. The boat travels 24miles upstream and back downstream in a total of 5 hours. What is the speed of the current?

$$t_1 + t_2 = \text{total time} \quad \text{total} = 5 \text{ hrs}$$

$t_1 = \frac{24}{10-x}$	$t_2 = \frac{24}{10+x}$
upstream	downstream

$$\frac{24}{10-x} + \frac{24}{10+x} = 5 \quad \text{LCD: } (10-x)(10+x)$$

$$24(10+x) + 24(10-x) = 5(10-x)(10+x)$$

$$240 + 24x + 240 - 24x = 5(100 - x^2)$$

$$480 = 500 - 5x^2 \rightarrow -20 = -5x^2$$

$$x^2 = 4 \quad x = \pm 2 \quad \text{reject } -2$$

Speed of current = 2mi/hr

12. On a 100km round trip, Jessica averages 40km/h to her destination and 60km/h returning. What is the average speed for the entire trip? Same logic as last question

$d$  total is 100

$$d_1 = 50 \quad d_2 = 50 \quad t = \frac{D}{S}$$

$$\frac{50}{40} + \frac{50}{60} = \frac{100}{x} \quad \text{LCD: } 120x$$

$$150x + 100x = 12000$$

$$250x = 12000$$

$x = 48 \text{ km/hr}$