Section 3.6 – Applications of Rational Equations

This booklet belongs to:_____Block: _____

- Problem solving is challenging because we don't have specific rules to follow
- There are some general guidelines that may help with this endeavor

Strategy for Solving Word Problems

1. Read the problem careful, sometimes a number of times. Identify what info is given
2. Let a variable represent the unknown information, and represent every else in terms of it
3. If possible/necessary make a diagram or drawing
4. Write an equation relating your unknown quantities to what you are given
5. Solve the equation
6. Check your solutions in terms of the original problem to make sure your answer makes sense

Example 1: The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number.

Solution 1: Let x be the number

$\frac{1}{x}$ is the reciprocal of the number so: $\frac{2}{x}$ is twice the reciprocal

$$x+\frac{2}{x}=\frac{9}{2}\ , \qquad x\neq 0$$

$$2x\left(x+\frac{2}{x}\right)=2x\left(\frac{9}{2}\right) \quad \rightarrow \quad 2x^2+4=9x \quad \rightarrow \quad 2x^2-9x+4=0$$

$$(x-4)(2x-1)=0 \qquad so: \qquad x=4 \quad or \quad x=\frac{1}{2}$$

Check:

$$4+\frac{2}{4}=\frac{9}{2} \qquad or \qquad \frac{1}{2}+\frac{2}{\frac{1}{2}}=\frac{9}{2}$$

$$\frac{9}{2}=\frac{9}{2} \qquad or \qquad \frac{1}{2}+4=\frac{9}{2} \quad \rightarrow \quad \frac{9}{2}=\frac{9}{2}$$

Therefore both $x=4$ or $x=\frac{1}{2}$ are solutions.

Example 2: Matt and Niki ride a bicycle a distance of $4km$ each morning. They both finish at the same time, but Niki starts $1\ minute$ before Matt, and Matt travels $1km/hr$ faster than Niki. At what speed are they travelling?

Solution 2: Niki's time minus Matt's time is $1\ minute$ or $\frac{1}{60}\ hour$.

$$Speed = \frac{Distance}{Time} \quad \text{so} \quad Time = \frac{Distance}{Speed}$$

	$Speed$	$Distance$	$Time$
$Niki$	x	$4km$	$\frac{4}{x}$
$Matt$	$x+1$	$4km$	$\frac{4}{x+1}$

$$\frac{4}{x} - \frac{4}{x+1} = \frac{1}{60} \quad \rightarrow \quad 60x(x+1)\left(\frac{4}{x} - \frac{4}{x+1} = \frac{1}{60}\right)$$

$$240(x+1) - 240x = x(x+1) \quad \rightarrow \quad 240x + 240 - 240x = x^2 + x$$

$$x^2 + x - 240 = 0$$

$$(x+16)(x-15) = 0$$

$$x = -16\ or\ 15 \quad (Reject\ x = -16km/hr)$$

So Niki travels $15km/hr$ and Matt travels $16km/hr$

Example 3: The cold water tap can fill a container two hours faster than the hot water tap. The two taps together can fill the container in $80\ minutes$. How long does it take each tap to fill the container on its own?

Solution 3: Let $x = \#\ of\ hours\ it\ takes\ for\ the\ cold\ water\ tap\ to\ fill\ the\ container\ alone$

Let $x + 2 = \#\ of\ hours\ it\ takes\ for\ the\ cold\ water\ tap\ to\ fill\ the\ container\ alone$

$$80\ minutes = \frac{4}{3}\ hours$$

	$Cold$	Hot	$Total$
$Time\ (hrs)$	x	$x+2$	$\frac{4}{3}$
$Rate\ (per\ hr)$	$\frac{1}{x}$	$\frac{1}{x+2}$	$\frac{3}{4}$

Cold Hot Together

$$\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4} \quad \rightarrow \quad 4x(x+2)\left(\frac{1}{x} + \frac{1}{x+2} = \frac{3}{4}\right)$$

$$4(x+2) + 4x = 3x(x+2) \quad \rightarrow \quad 4x + 8 + 4x = 3x^2 + 6x$$

$$3x^2 - 2x - 8 = 0$$

$$(3x+4)(x-2) = 0$$

$$x = -\frac{4}{3}\ or\ 2 \quad (Reject\ x = -\frac{4}{3})$$

So Cold Water Tap takes 2 hours alone, Hot Water Tap takes 4 hours alone

Example 4: A car travels from home to work at an average speed of $60km/hr$, and because of traffic returns from work at an average speed of $40km/hr$. What is the average speed for the entire trip?

$$Time = \frac{Distance}{Speed}$$

Solution 4: The time to travel $d\ km$ going to work is: $\frac{d}{60}$ hours

The time to travel $d\ km$ returning from work is: $\frac{d}{40}$ hours

At an average speed of $x\ km/hr$, the time to travel $2d\ km$ is: $\frac{2d}{x}$ hours

$Time_{going\ to\ work} + Time_{returning\ from\ work} = Time_{total\ time\ of\ trip}$

$$\frac{d}{60} + \frac{d}{40} = \frac{2d}{x} \quad (since\ d\ is\ constant\ the\ distnce\ is\ irrelevant, so\ let\ d = 1)$$

$$\frac{1}{60} + \frac{1}{40} = \frac{2}{x} \quad \rightarrow \quad 120x\left(\frac{1}{60} + \frac{1}{40}\right) = 120x\left(\frac{2}{x}\right)$$

$$2x + 3x = 240 \quad \rightarrow \quad 5x = 240 \quad \rightarrow \quad x = 48$$

Therefore the average speed is: $48km/hr$

Example 5: It takes Aaron $9\ hours$ longer than Chris to paint a house. Working together they can do the job in $20\ hours$. How long would it take each person, working alone, to paint the house?

Solution 5: Let $x = the\ number\ of\ hours\ Chris\ needs\ to\ cmoplete\ the\ job\ alone$

Let $x + 9 = the\ number\ of\ hours\ Aaron\ needs\ to\ complete\ the\ job\ alone$

Then Chris completes $\frac{1}{x}$ of the job in one hour and Aaron completes $\frac{1}{x+9}$ of the job in one hour

In 20 hours, Chris completes $\frac{20}{x}$ of the job and Aaron completes $\frac{20}{x+9}$ of the job

In 20 hours, Chris and Aaron together complete $\frac{20}{20}$ of the job $= 1$

$$\frac{20}{x} + \frac{20}{x+9} = 1 \quad \rightarrow \quad x(x+9)\left(\frac{20}{x} + \frac{20}{x+9}\right) = x(x+9) \quad \rightarrow \quad 20(x+9) + 20x = x(x+9)$$

$$20x + 180 + 20x = x^2 + 9x \quad \rightarrow \quad x^2 - 31x - 180 = 0 \quad \rightarrow \quad (x-36)(x+5) = 0$$

$$x = 36 \quad or \quad -5 \quad (Reject\ x = -5)$$

Therefore Chris takes $36\ hours$ and Aaron takes $45\ hours$ to finish the job alone.

Example 6: A speed boat can travel $108km$ downstream in the same time it can travel $78km$ upstream. If the current in the river is $10km/hr$ what is the speed of the boat in still water?

Solution 6: $Speed = \frac{Distance}{Time}$ so $Time = \frac{Distance}{Speed}$

	$Speed$	$Distance$	$Time$
$With\ Current$	$x + 10$	$108km$	$\frac{108}{x+10}$
$Against\ Current$	$x - 10$	$78km$	$\frac{78}{x-10}$

$Speed\ in\ still\ water\ is\ x\ km/hr$

$Speed\ with\ the\ current\ is\ (x + 10)km/hr$

$Speed\ against\ the\ current\ is\ \ (x - 10)km/hr$

$Time\ is\ equal\ is\ both\ sitautions\ so\colon\ \ Time_{with\ current} = Time_{against\ current}$

$$\frac{108}{x+10} = \frac{78}{x-10} \qquad \rightarrow \qquad 108(x-10) = 78(x+10)$$

$$108x - 1080 = 78x + 780 \qquad \rightarrow \qquad 30x = 1860 \qquad \rightarrow \qquad x = 62$$

Therefore the speed of the boat in still water is $62km/hr$

Section 3.6 – Practice Problems

Unknown number problems

1. The sum of a number and its reciprocal is $\frac{13}{6}$. Find the number.

2. Find two consecutive even integers whose reciprocals add to $\frac{7}{24}$

3. Find two consecutive odd integers whose reciprocals add to $\frac{8}{15}$

4. A number added to the product of 6 and the reciprocal of that number is -5. Find the number.

Work Problems

5. It would take Sue 4 hours to paint a large room, and it would take Bob 5 hours to paint the same room. If they work together, how long would it take them to complete the job?

6. Jane works twice as fast as her daughter Anna. If it takes 15 minutes to clean the kitchen together, how long would it take Anna to clean the kitchen by herself?

7. Ken takes 3 hours longer to assemble a motor than Hans. When working together, it takes them 2 hours to assemble the motor. How long would it take Ken to do the job alone?

8. A cold water tap can fill a tub in 6 minutes, and a hot water tap can fill the tub in 8 minutes. A drain can empty the full tub in 10 minutes. If both the taps are on and the drain is open, how long will it take to fill the tub? (Complete waste of water…)

Distance Problems

9. A boat travels $40km$ downstream in the same time it takes to travel $30km$ upstream. If the current flows at $6km/h$, what is the speed of the boat in still water?

10. The speed of a boat in Stillwater is $10mph$. The boat travels $24miles$ upstream and back downstream in a total of $5\ hours$. What is the speed of the current?

11. A woman drives to work at an average speed of $50mph$. The average speed of the return trip home is $30mph$. What is the average speed of the round trip?

12. On a $100km$ round trip, Jessica averages $40km/h$ to her destination and $60km/h$ returning. What is the average speed for the entire trip?

Answer Key – Section 3.6

1. $x = \frac{3}{2}\ and\ x = \frac{2}{3}$
2. $n = 6;\ \ numbers\ are\ 6\ and\ 8$
3. $n = 3;\ \ \ numbers\ are\ 3\ and\ 5$
4. $x = -3\ and\ -2$
5. $Together\ takes\ 2\frac{2}{9}\ hours$
6. $Anna\ takes\ 45\ minutes\ alone$
7. $Ken\ takes\ 6\ hours\ alone$
8. $5\frac{5}{23}\ minutes$
9. $Boat\ Speed: 42km/hr$
10. $Current\ Speed: 2miles/hr$
11. $x = 37.5km/hr$
12. $x = 48km/hr$

Extra Work Space