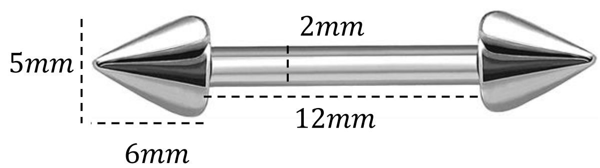
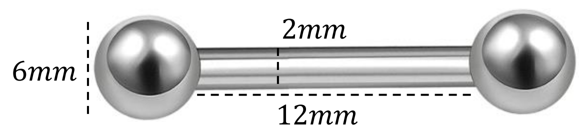
Section 3.5 – Volume of Composite Shapes and Dimension Changes

- Much like composite shapes (easily broken down into recognizable shapes) for area and surface area, we have a similar scenario for volume.
- Just like how we cut the area shapes into pieces like squares, triangles, or circles, we can cut the $3-D$ shape into cubes, prisms, spheres, and cylinders.

Example 1: A jewellery store sells different studs for piercings. Two of the most popular are shown below. What is the Volume of each of the studs?





Solution 1:

Volume of the Cylinder:

$$\pi r^2 \cdot h \quad \rightarrow \quad \pi(2)^2 \cdot 12 \quad \rightarrow \quad 150.8mm^3$$

Volume of the Cone End

$$\frac{1}{3}\pi r^2 \cdot h \quad \rightarrow \quad \frac{1}{3}\pi(2.5)^2 \cdot 6 \quad \rightarrow \quad 39.3mm^3$$

Volume of the Cylinder:

$$\pi r^2 \cdot h \quad \rightarrow \quad \pi(2)^2 \cdot 12 \quad \rightarrow \quad 150.8mm^3$$

Volume of the Sphere End

$$\frac{4}{3}\pi r^3 \quad \rightarrow \quad \frac{4}{3}\pi(2.5)^3 \quad \rightarrow \quad 65.4mm^3$$

Now just add them all together

Volume of the Cylinder:

$$150.8mm^3$$

Volume of the Cone Ends

$$39.3mm^3 \cdot 2 = 78.6mm^3$$

Total

$$150.8 + 78.6 = 229.4mm^3$$

Volume of the Cylinder:

$$150.8mm^3$$

Volume of the Sphere Ends

$$65.4mm^3 \cdot 2 = 130.8mm^3$$

Total

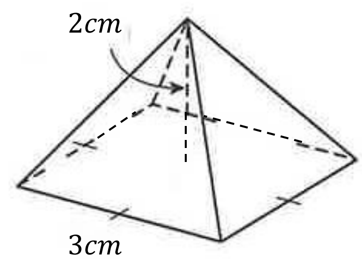
$$150.8 + 130.8 = 281.6mm^3$$

When working on these types of questions, it helps to be organized and show your step progressions!

Dimensions Changes

- Dimension changes for Volume are similar to those of Surface Area
- There is a ratio relationship at times, but it is often easiest to calculate the effect the change has on the volume and compare it to the original.

Example 2: Toblerone makes pyramidal chocolates. They small ones are the size shown below. They have to make a promotional size piece for a children's charity event. How does the volume change if they increase they double the base length and height?



Solution 2:

Volume of Original:

$$\frac{1}{3}b^2 \cdot h \quad \rightarrow \quad \frac{1}{3}(3)^2 \cdot (2) \quad \rightarrow \quad \frac{1}{3} \cdot 18 = 6cm^3$$

Volume of Increased Dimensions:

$$\frac{1}{3}b^2 \cdot h \quad \rightarrow \quad \frac{1}{3}(6)^2 \cdot (4) \quad \rightarrow \quad \frac{1}{3} \cdot 144 = 48cm^3$$

Use a ratio of the two sizes to determine the change in Volume.

$$\frac{New\ Volume}{Original\ Volume} = \frac{48}{6} = 8$$

The Volume increases by a factor of: 8

Since we square the base, we square the double. $(2)^2 = 4$. And then multiply that by the doubling of the height.

$$4 \cdot 2 = 8$$

Example 3: A kids soccer ball has a radius of 5 inches. An adult ball has a radius of 7 inches. How do their volumes differ?

Solution 3:

Kids Ball

$$Volume\ of\ a\ Sphere$$

$$\frac{4}{3}\pi r^3 \quad \rightarrow \quad \frac{4}{3}\pi(5)^3 \quad \rightarrow \quad \mathbf{523.6 in^3}$$

Adult Ball

$$Volume\ of\ a\ Sphere$$

$$\frac{4}{3}\pi r^3 \quad \rightarrow \quad \frac{4}{3}\pi(7)^3 \quad \rightarrow \quad \mathbf{1436.8 in^3}$$

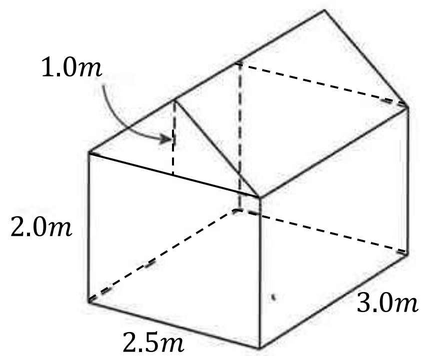
$$\frac{New\ Volume}{Original\ Volume} = \frac{1436.8}{523.6} = \mathbf{2.7}$$

The volume of the adult ball is approximately 2.7 times as much as the kids ball. This is because the radius increased by a factor of: $\frac{7}{5}$

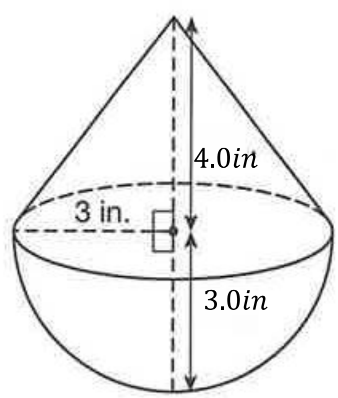
Since we cubed the radius, we cube the factor of the increase: $\left(\frac{7}{5}\right)^3 \quad 1.4^3 \cong \mathbf{2.7}$

Section 3.5 – Practice Problems

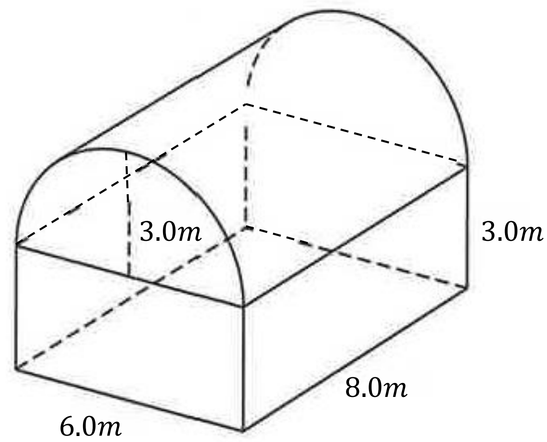
1. Determine the Volume of the following composite shape. Round to the nearest tenth if necessary.



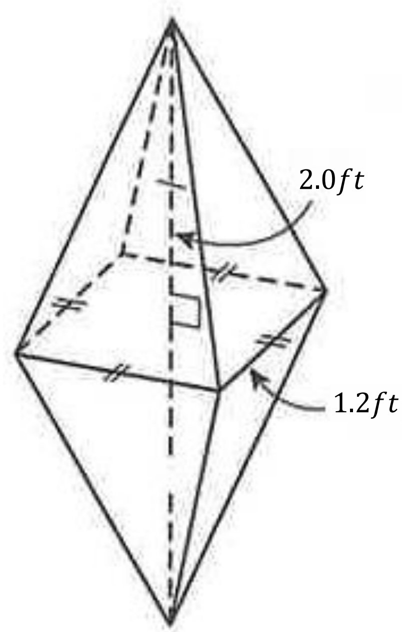
2. Determine the Volume of the following composite shapes. Round to the nearest tenth if necessary.



3. Determine the Volume of the following composite shapes. Round to the nearest tenth if necessary.



4. Determine the Volume of the following composite shapes. Round to the nearest tenth if necessary.



5. Daniella and her friend Ashley make jewellery. They are making pearl necklaces where each pearl has a diameter of $0.8cm$. A cylindrical is drilled into each pearl to allow the pearls to be strung. The hole has a radius of $0.9mm$. What is the volume of each pearl to the nearest millimeter (Watch you mm and cm, they need to be the same.

6. Complete the chart below.

Object	**Volume**	**Action Taken**	**New Volume**	**Ratio (Fraction)**
Square Pyramid $l = 3cm$ $h = 7cm$		Double the length and the width of the base		
Cone $r = 5cm$ $h = 6cm$		Double the radius		

Cube $l = 4cm$		Double the side length		

7. A granola bar has a length of $5in.$, a width of 1 and a half inches, and a height of $\frac{3}{4}$ of an inch. How can you change the dimensions to create a larger bar with four times the volume? Check your strategy. (Hint: Calculate the original volume, multiply it by 4 and then solve for one of the parameters as an unknown.)

Section 3.5 – Answer Key

1.	$18.75m^3$
2.	$94.2in^3$
3.	$257.1m^3$
4.	$1.9ft^3$
5.	$247.7mm^3$
6.	$Original: 21cm^3 \quad New: 168cm^3 \quad Ratio: 8$ $Original: 157.1cm^3 \quad New: 628.3cm^3 \quad Ratio: 4$ $Original: 64cm^3 \quad New: 512cm^3 \quad Ratio: 8$
7.	$Change\ length\ to: 20in$ $Change\ width\ to: 6in$ $Change\ height\ to: 3\ in$