Section 3.5 – Practice Problems

1. If $xy^2 = 12$, and $\frac{dy}{dt} = 6$, find $\frac{dx}{dt}$ when $y = 2$

Differentiate with respect to t

Product Rule

$$x(2y)\frac{dy}{dt} + \frac{dx}{dt}y^2 = 0$$

$$2xy\frac{dy}{dt} + y^2\frac{dx}{dt} = 0 \rightarrow 2x(2)(6) + 2^2\frac{dx}{dt} = 0$$

$$24x + 4\frac{dx}{dt} = 0 \rightarrow -24x = 4\frac{dx}{dt}$$

$$-6x = \frac{dx}{dt}$$

$$xy^2 = 12 \rightarrow x = \frac{12}{y^2}$$

$$\frac{dx}{dt} = -6\left(\frac{12}{2^2}\right) = \boxed{-18}$$

2. If $x^3 + y^3 = 9$, and $\frac{dx}{dt} = 4$, find $\frac{dy}{dt}$ when $x = 2$

$$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt} = 0 \qquad y = \sqrt[3]{9 - x^3} \rightarrow \sqrt[3]{9-8} = \boxed{1}$$

$$3(2)^2(4) + 3(1)^2\frac{dy}{dt} = 0 \rightarrow 48 + 3\frac{dy}{dt} = 0 \qquad \frac{dy}{dt} = -\frac{48}{3} = \boxed{-16}$$

3. How fast is the area of a square increasing when the side is 3m in length and growing at a rate of 0.8m/min?

$$A = x^2 \qquad \frac{dA}{dl} = 2x \qquad \frac{dl}{dt} = 0.8\text{m/min}$$

$$\frac{dA}{dt} = \frac{dA}{dl}\cdot\frac{dl}{dt} \text{ at } x = 3 \rightarrow \frac{dA}{dt} = 6 \cdot 0.8 = \boxed{4.8\text{m}^2/\text{min}}$$

4. How fast is the edge length of a cube increasing when the volume of the cube is increasing at a rate of 144cm³/s and the edge length is 4cm.

$$V = x^3 \qquad \frac{dV}{dl} = 3x^2 \text{ at } x = 4 \qquad \frac{dV}{dl} = 48\text{cm}^3/\text{cm}$$

$$\frac{dV}{dt} = \frac{dV}{dl}\cdot\frac{dl}{dt} \qquad \frac{dV}{dt} = 144\text{cm}^3/\text{s}$$

$$144 = 48\frac{dl}{dt} \rightarrow \frac{144}{48} = \frac{dl}{dt}$$

$$\boxed{\frac{dl}{dt} = 3\text{cm/s}}$$

5. A stone is dropped into a lake, creating a circular ripple effect that travels outward at a speed of 25cm/s. Find the rate at which the area within the circle is increasing after 4s.

$A = \pi r^2 \qquad \frac{dA}{dr} = 2\pi r \qquad \text{need } \frac{dA}{dt} \qquad \frac{dr}{dt} = 25\text{cm/s}$

$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$

After 4 seconds
$r = 4(25) = 100$

$\frac{dA}{dt} = 2\pi(100) \cdot 25 = 5000\pi = \boxed{15707 \text{ cm}^2/\text{sec}}$

6. A spherical balloon is being inflated so that the volume is increasing at a rate of 8m³/min. How fast is the radius of the balloon increasing when the diameter is 2m? ← at $d = 2$ means $r = 1$

$V = \frac{4}{3}\pi r^3 \qquad \frac{dV}{dt} = 8 \text{ m}^3/\text{min} \qquad \frac{dr}{dt} = ?$

$\frac{dV}{dr} = 4\pi r^2$

$= 4\pi$

$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$

$8 = 4\pi \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{2}{\pi}$

$\boxed{\frac{dr}{dt} = 0.64 \text{ m/min}}$

7. A snowball melts so that its surface area decreases at a rate of 0.5cm²/min. Find the rate at which the radius decreases when the radius is 4cm.

$SA = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$ at $r = 4$

$\frac{dA}{dr} = 32\pi \text{ cm}^2/\text{cm}$

$\frac{dA}{dt} = -0.5\text{cm}^2/\text{min}$

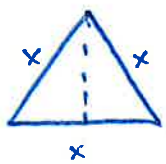
$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$ need $\frac{dr}{dt}$

$\frac{\frac{dA}{dt}}{\frac{dA}{dr}} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{-0.5}{32\pi}$

$\boxed{\frac{dr}{dt} = 0.005 \text{ cm/min}}$

8. The side of an equilateral triangle decrease at a rate of 2cm/s. At what rate is the area decreasing when the area is $100cm^2$?



$\frac{dx}{dt} = -2 cm/s$

$A = \frac{1}{2} x h$

$h^2 = x^2 - \left(\frac{1}{2}x\right)^2$

$h^2 = \frac{3}{4}x^2$

$h = \frac{\sqrt{3}}{2}x$

$A = \frac{1}{2} x \frac{\sqrt{3}}{2} x$

$A = \frac{\sqrt{3}}{4} x^2$

$100 = \frac{\sqrt{3}}{4} x^2$

$\frac{400}{\sqrt{3}} = x^2$

$\boxed{x = \frac{20}{\sqrt[4]{3}}}$

$\frac{dA}{dx} = \frac{\sqrt{3}}{2} x$

$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$

$\frac{dA}{dt} = \frac{\sqrt{3}}{2}\left(\frac{20}{\sqrt[4]{3}}\right)(-2)$

$\frac{3^{\frac{1}{2}} \cdot -20}{3^{\frac{1}{4}}}$

$= 3^{\frac{1}{4}} \cdot -20$

$= \boxed{-26.3 cm^2/s}$

9. The area of a triangle is increasing at a rate of $4cm^2$/min and its base is increasing at a rate of 1cm/min. At what rate is the altitude of the triangle increasing when the altitude is 20cm and the area is $80cm^2$?

$A = \frac{1}{2}bh$ ← implicit differation with respect to time

$\frac{dA}{dt} = 4 cm^2/min$

$\frac{db}{dt} = 1 cm/min$

$\frac{dh}{dt} = ?$

$\frac{dA}{dt} = \frac{1}{2}\left[b\frac{dh}{dt} + \frac{db}{dt}h\right]$

$\frac{dA}{dt} = \frac{1}{2}\left[8\frac{dh}{dt} + 1(20)\right]$

$4 = 4\frac{dh}{dt} + 10$

$-6 = 4\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = -\frac{3}{2} cm/min}$

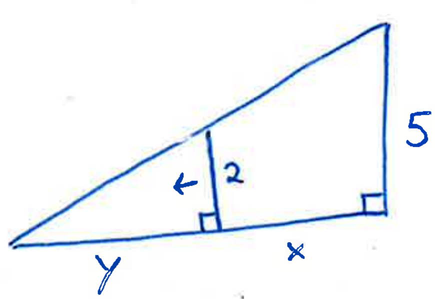
$A = \frac{1}{2}bh$ at $A = 80$, $h = 20$

$80 = \frac{1}{2}b(20)$

$80 = 10b$

$b = 8$

10. A man 2m tall walks away from a lamppost whose light is 5m above the ground. If he walks at a speed of 1.5m/s, at what rate is his shadow growing when he is 10m from the lamppost?



$\frac{5}{x+y} = \frac{2}{y}$

$\frac{dx}{dt} = 1.5 m/s$

$5y = 2x + 2y$

$3y = 2x \rightarrow y = \frac{2}{3}x$

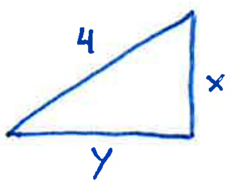
$\frac{dy}{dx} = \frac{2}{3}$

$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$= \frac{2}{3} \cdot \frac{3}{2}$

$= \boxed{1 m/s}$

11. A ladder 4m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 30cm/sec, how quickly is the top of the ladder sliding down the wall when the bottom of the ladder is 2m from the wall?



$x^2 + y^2 = 4^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = 30\text{cm/sec}$

when $y = 2\text{m}$ ↓ 200cm

$x = \sqrt{16 - y^2}$

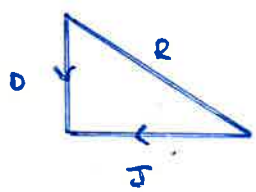
$x = \sqrt{16-4}$

$x = \sqrt{12} = 2\sqrt{3}\text{m}$ ↓ $200\sqrt{3}\text{cm}$

$2(200\sqrt{3})\frac{dx}{dt} + 2(200)(30) = 0 \rightarrow \frac{dx}{dt} = \frac{-12000}{400\sqrt{3}} = 17\text{cm/s}$

$= \boxed{0.17\text{m/s}}$

12. Jon is driving west at 60km/hr and Adrian is driving south at 70km/hr. Both cars are approaching the intersection of the two roads. At what rate is the distance between the cars decreasing when Jon's car is 0.4km and Adrian's 0.3km from the intersection?



$\frac{dA}{dt} = -70\text{km/hr}$ $\frac{dJ}{dt} = -60\text{km/hr}$

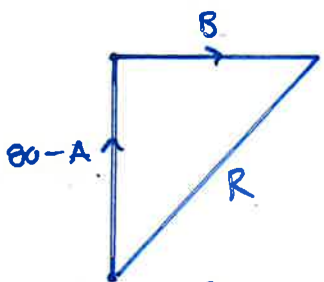
$\boxed{\text{Decreasing at a rate of } 90\text{km/h}}$

$R = \sqrt{A^2 + J^2}$

$\frac{dR}{dt} = \frac{1}{2\sqrt{A^2+J^2}} \cdot 2A\frac{dA}{dt} + 2J\frac{dJ}{dD}$

$= \frac{2(0.3)(-70) + 2(0.4)(-60)}{2\sqrt{0.3^2+0.4^2}} = \frac{-90}{1} = -90$

13. At 1:00pm ship A was 80km south of ship B. Ship A is sailing north at 30km/hr and ship B is sailing east at 40km/hr. How fast is the distance between them changing at 3:00pm?



$40\text{km/hr} \cdot 2\text{hr} = 80\text{km}$

$30\text{km/hr} \cdot 2\text{hr} = 60\text{km}$

$\frac{dA}{dt} = 30\text{km/hr}$

$\frac{dB}{dt} = 40\text{km/hr}$

$R^2 = (80-A)^2 + B^2$

$R^2 = 6400 - 160A + A^2 + B^2$

$A = 80 - 60 = 20$

$B = 80$

Implicit: $2R\frac{dR}{dt} = -160\frac{dA}{dt} + 2A\frac{dA}{dt} + 2B\frac{dB}{dt}$

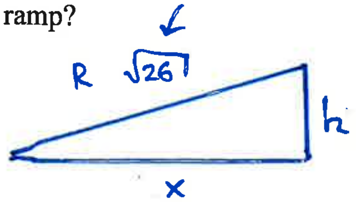
$R = \sqrt{(80-A)^2 + B^2}$

at $A = 60$, $B = 80$

$\frac{dR}{dt} = \frac{-160\frac{dA}{dt} + 2A\frac{dA}{dt} + 2B\frac{dB}{dt}}{2R} = \frac{-80\frac{dA}{dt} + A\frac{dA}{dt} + B\frac{dB}{dt}}{R}$

$\frac{-80(30) + 60(30) + 80(40)}{\sqrt{20^2+80^2}} \rightarrow \frac{2600}{82.5} = \boxed{31.5\text{km/h}}$

14. A water-skier skis over a ramp that stretched 5m horizontally and rises 1m off the water (Hint: You still need the length of the ramp) with a speed of 12m/s. How fast is she rising as she leaves the ramp?



$\frac{dR}{dt} = 12 \frac{m}{s}$

need $\frac{dh}{dt}$

$R^2 = h^2 + x^2$ have too many unknown

$\frac{h}{x} = \frac{1}{5}$ so $x = 5h$

$R^2 = h^2 + (5h)^2$

$R^2 = h^2 + 25h^2$

$R^2 = 26h^2$

$2R\frac{dR}{dt} = 52h\frac{dh}{dt}$

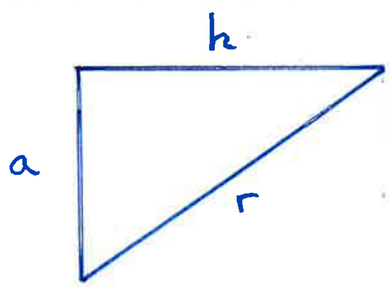
$R = \sqrt{26}$

$2\sqrt{26} \cdot 12 = 52h\frac{dh}{dt}$

at $h = 1$

$\frac{24\sqrt{26}}{52} = \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{6\sqrt{26}}{13} \doteq \boxed{2.35 \text{ m/s}}$

15. A plane flies horizontally with a speed of 600km/h at an altitude of 10km and passes directly over the town of Shawnigan. Find the rate at which the distance from the plane to Shawnigan is increasing when it is 20km away from Shawnigan.



$\frac{dh}{dt} = 600 \text{ km/h}$ $\quad \frac{dr}{dt} = ?$ $\quad \frac{da}{dt} = 0$

no change in altitude

$r^2 = a^2 + h^2$

$2r\frac{dr}{dt} = 2a\frac{da}{dt} + 2h\frac{dh}{dt}$

$r = \sqrt{a^2 + h^2}$

$r\frac{dr}{dt} = a\frac{da}{dt} + h\frac{dh}{dt}$

at $a = 10$, $h = 20$

$\frac{dr}{dt} = \frac{10(0) + 20(600)}{\sqrt{500}} = \frac{12000}{\sqrt{500}} \doteq \boxed{537 \text{ km/hr}}$

$r = \sqrt{100 + 400}$

$r = \sqrt{500}$