Section 3.5 – Rational Equations

This booklet belongs to:_____**Block:** _____

To Solve Rational Equations:

1. Multiply each side of the equation by the LCD to clear the equation of fractions
2. Solve the equation for the given variable
3. Determine the restrictions on the variable, if any, and exclude solutions that are not allowed

Example 1: Solve $\frac{x}{2}+\frac{7}{3}=\frac{5}{6}$

Solution 1:

The LCM of $2, 3, 6, is\ 6$. Therefore the LCD is 6.

$$6\left(\frac{x}{2}+\frac{7}{3}\right)=6\left(\frac{5}{6}\right) \rightarrow \frac{6x}{2}+\frac{42}{3}=\frac{30}{6} \rightarrow 3x+14=5 \rightarrow 3x=-9 \rightarrow \boldsymbol{x=-3}$$

Example 2: Solve $3-\frac{6}{x}=x+8$

Solution 2:

The LCD is x. Zero cannot be a solution since dividing by zero is undefined.

$$x\left(3-\frac{6}{x}\right)=x(x+8) \rightarrow 3x-6=x^2+8x \rightarrow x^2+5x+6=0 \rightarrow (x+3)(x+2)=0$$

$$\boldsymbol{x=-3 \quad or \quad x=-2}$$

Example 3: Solve $\frac{2x}{x-4}=\frac{8}{x-4}+1$

Solution 3:

The LCD is $x-4$. In this equation $x=4$ Zero cannot be a solution since it results in dividing by zero.

$$(x-4)\left(\frac{2x}{x-4}\right)=(x-4)\left(\frac{8}{x-4}+1\right) \rightarrow 2x=8+x-4 \rightarrow x=4, \quad and\ since\ x\neq 4$$

$$\boldsymbol{There\ is\ No\ Solution}, \quad \boldsymbol{x=\emptyset}$$

Example 4: Solve $\frac{1}{x-4}-\frac{1}{x-2}=\frac{2x}{x^2-6x+8}$

Solution 4:

Since $x^2-6x+8=(x-2)(x-4)$, so the LCD is $(x-2)(x-4)$. In this equation $x=2\ and\ x=4$ cannot be solutions since they result in a denominator of zero.

$\frac{1}{x-4}-\frac{1}{x-2}=\frac{2x}{x^2-6x+8} \quad \rightarrow \quad \frac{1}{x-4}-\frac{1}{x-2}=\frac{2x}{(x-4)(x-2)}$

$(x-2)(x-4)\left(\frac{1}{x-4}-\frac{1}{x-2}\right)=(x-2)(x-4)\left(\frac{2x}{(x-4)(x-2)}\right)$

$(x-2)-(x-4)=2x \quad \rightarrow \quad x-2-x+4=2x \quad \rightarrow \quad 2=2x \quad \rightarrow \quad \boldsymbol{x=1}$

Example 5: Solve $\frac{x}{x-5}-\frac{3}{x+1}=\frac{30}{x^2-4x-5}$

Solution 5:

Since $x^2-4x-5=(x-5)(x+1)$, so the LCD is $(x-5)(x+1)$. In this equation $x=-1\ and\ x=5$ cannot be solutions since they result in a denominator of zero.

$\frac{x}{x-5}-\frac{3}{x+1}=\frac{30}{x^2-4x-5} \quad \rightarrow \quad \frac{x}{x-5}-\frac{3}{x+1}=\frac{30}{(x-5)(x+1)}$

$(x-5)(x+1)\left(\frac{x}{x-5}-\frac{3}{x+1}\right)=(x-5)(x+1)\left(\frac{30}{(x-5)(x+1)}\right)$

$x(x+1)-3(x-5)=30 \quad \rightarrow \quad x^2+x-3x+15=30 \quad \rightarrow \quad x^2-2x-15=0$

$(x-5)(x+3)=0 \quad \rightarrow \quad \boldsymbol{x=5 \ \ and \ \ x=-3}$

Since $x\neq 5$, only $x=-3$ is an acceptable solution

Example 6: Solve $\frac{4}{2x-1}=\frac{2}{x+3}$

Solution 6:

The LCD is $(2x-1)(x+3)$. In this equation $x=\frac{1}{2}$ and $x=-3$ cannot be solutions since they result in a denominator of zero.

$\frac{4}{2x-1}=\frac{2}{x+3} \quad \rightarrow \quad (2x-1)(x+3)\left(\frac{4}{2x-1}\right)=(2x-1)(x+3)\left(\frac{2}{x+3}\right)$

$4(x+3)=2(2x-1) \quad \rightarrow \quad 4x+12=4x-2 \quad \rightarrow \quad 12=-2$

Since This is a FALSE STATEMENT there is NO SOLUTION

Example 7: Solve $\frac{5}{x-7}-\frac{1}{2x}=\frac{9x+7}{2x^2-14x}$

Solution 7:

Since $2x^2-14x=2x(x-7)$. The LCD is $(2x)(x-7)$. In this equation $x=0$ and $x=7$ cannot be solutions since they result in a denominator of zero.

$$\frac{5}{x-7}-\frac{1}{2x}=\frac{9x+7}{2x^2-14x} \quad \rightarrow \quad 2x(x-7)\left(\frac{5}{x-7}-\frac{1}{2x}\right)=2x(x-7)\left(\frac{9x+7}{2x^2-14x}\right)$$

$$5(2x)-1(x-7)=(9x+7) \quad \rightarrow \quad 10x-x+7=9x+7 \quad \rightarrow \quad 9x+7=9x+7 \quad \rightarrow \quad 0=0$$

Since This is a TRUE STATEMENT there ARE INFINTE SOLUTIONS

Except $x=0$ and $x=7$

Example 8: Solve $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$ *for a.*

Solution 8:

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$$

$$\frac{1}{a}=\frac{1}{c}-\frac{1}{b} \quad \rightarrow \quad \frac{1}{a}=\frac{1}{c}\cdot\frac{b}{b}-\frac{1}{b}\cdot\frac{c}{c}$$

$$\frac{1}{a}=\frac{b}{bc}-\frac{c}{bc} \quad \rightarrow \quad \frac{1}{a}=\frac{b-c}{bc}$$

$$\boldsymbol{a=\frac{bc}{b-c}}$$

or

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$$

$$abc\left(\frac{1}{a}+\frac{1}{b}\right)=abc\left(\frac{1}{c}\right)$$

$$bc+ac=ab \quad \rightarrow \quad ac-ab=-bc$$

$$a(c-b)=-bc$$

$$\boldsymbol{a=\frac{-bc}{c-b}}$$

Section 3.5 – Practice Problems

Solve the equation

1. $\frac{x}{2}+\frac{x}{3}=5$

2. $\frac{y}{3}+5=\frac{3y}{4}$

3. $\frac{x}{4}-x-\frac{3}{2}=0$

4. $\frac{y}{2}+\frac{5y}{4}=\frac{y}{12}$

5. $\frac{3(x+1)}{4} = x + 1$

6. $\frac{2(y-1)}{3} + 5 = y$

7. $\frac{x-4}{3} - \frac{x-2}{2} = -\frac{5}{6}$

8. $\frac{3x}{5} - \frac{x-5}{7} = 3$

9. $\frac{x+2}{4} - \frac{x-1}{2} = \frac{2}{3}$

10. $\frac{4x+1}{5} = \frac{8x+2}{3} + 1$

Solve. Check your solutions.

11. $\frac{5}{3x} - \frac{1}{9} = \frac{4}{x}$

12. $\frac{7}{x+3} = \frac{5}{x-9}$

13. $\frac{y}{y+3} - 2 = \frac{-3}{y+3}$

14. $\frac{1}{2x} + \frac{4}{x} = \frac{9}{2x}$

15. $\frac{2}{z+5} + \frac{20}{z^2-25} = \frac{-3}{5-z}$

16. $\frac{3}{x-1} + \frac{1}{2x-2} = \frac{7}{4}$

17. $\frac{2y}{y^2-1} = \frac{2}{y+1} + \frac{1}{1-y}$

18. $\frac{z}{2z+2} + \frac{2z}{4z+4} = \frac{2z-3}{z+1}$

19. $\frac{2}{x} - \frac{x}{5x-12} = 0$

20. $\frac{y^2+3}{y-1} = \frac{4}{y-1}$

21. $\frac{y}{2y+2} + \frac{2y-16}{4y+4} = \frac{2y-3}{y+1}$

22. $\frac{5}{4z-2} - \frac{1}{1-2z} = \frac{7}{3z+6}$

23. $\frac{2x+3}{x-1} - \frac{2}{x+3} = \frac{5-6x}{x^2+2x-3}$

24. $\frac{x+1}{x+3} + \frac{x-3}{x-2} = \frac{x^2-11x}{x^2+x-6}$

25. $\frac{3-2x}{x+1} - \frac{10}{x^2-1} = \frac{2x+3}{1-x}$

26. $\frac{3y-7}{y^2-5y+6} + \frac{2y+8}{9-y^2} - \frac{y+2}{y^2+y-6} = 0$

Answer Key – Section 3.5

1. $x = 6$
2. $y = 12$
3. $x = -2$
4. $y = 0$
5. $x = -1$
6. $y = 13$
7. $x = 3$
8. $x = 5$
9. $x = \frac{4}{3}$
10. $x = -\frac{11}{14}$
11. $x = -21$
12. $x = 39$
13. $No\ Solution$

14. $x = Anything\ except\ 0$
15. $No\ Solution$
16. $x = 3$
17. $y = -3$
18. $z = 3$
19. $x = 6\ and\ x = 4$
20. $y = -1$
21. $No\ Solution$
22. $z = 8$
23. $x = -6\ and\ x = -\frac{1}{2}$
24. $x = -11\ and\ x = 1$
25. $No\ Solution$
26. $y = 1$

Extra Work Space