# Section 3.5 – Applications of Polynomial Equations

- The scenarios described in this section answer, 'when will I use this in life'
- I understand that you will probably never use the examples that follow, but the point is that when your mathematics becomes advanced enough, you will find yourself in scenarios where you do use it.
- It is very real that basic numeracy and an understanding of probability and statistics will be present more regularly, but do not pigeon-hole yourself just yet!

**Example 1:** A box is constructed such that the length is twice the width and the height is $2cm$ longer than the width, with a volume of $350cm^3$. Find the dimensions of the box.

**Solution 1:** $Volume\ of\ a\ Box = Length\ x\ Width\ x\ Height$

Solving for all three variables at once is not manageable. So, what we need is to express the variables with respect to one of them.

In this case, let width be $x$: That means: $Width = x, Length = 2x, Height = x + 2$

$V = (2x)(x)(x + 2)$

$350 = 2x(x)(x + 2)$

$350 = 2x^2(x + 2)$

$350 = 2x^3 + 4x^2$

$2x^3 + 4x^2 - 350 = 0$

$x^3 + 2x^2 - 175 = 0$

Rational Root Theorem give us possible roots of: $1, 5, 7, 25, 35, 175$

We only have to consider positive values (length can't be negative) and it is best to star with smaller values, as we are talking about a length of a side of a box.

Try $x = 1$

| | | | | |
|---|---|---|---|---|
| $1\rfloor$ | $1$ | $2$ | $0$ | $-175$ |
| | | $1$ | $3$ | $3$ |
| | $1$ | $3$ | $3$ | $-172$ |

Try $x = 5$

| | | | | |
|---|---|---|---|---|
| $5\rfloor$ | $1$ | $2$ | $0$ | $-175$ |
| | | $5$ | $35$ | $175$ |
| | $1$ | $7$ | $35$ | $0$ |

We have $x = 5$ as a factor, need to use Quadratic Equation to check

$x^2 + 7x + 35 = 0$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(35)}}{2(1)} = \frac{-7 \pm \sqrt{-91}}{2}$$

**No Solution**

**No More Roots**

$x = 5$ is our Solution.

So,

$Width = 5$

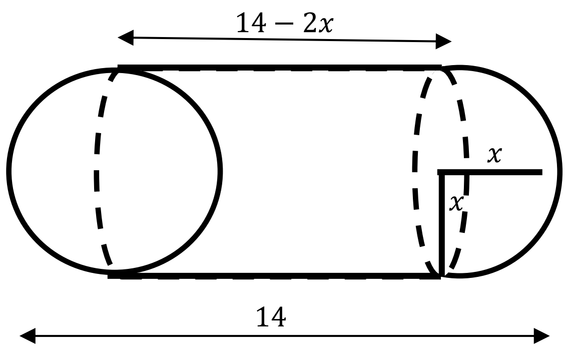
$Length = 5 \cdot 2 = 10$

$Height = 5 + 2 = 7$

$\mathbf{10cm\ x\ 5cm\ x\ 7cm}$

**Example 2:** A vitamin capsule has the shape of a right circular cylinder with hemispheres on each end. The total length of the capsule is $14mm$, and its volume is $108\pi\ mm^3$. Find the radius $x$ of the capsule.

**Solution 2:** First consider the shape of the capsule. Drawings are helpful!




$$Volume_{Sphere} = \frac{4}{3}\pi r^3 \quad = \quad \frac{4}{3}\pi x^3$$

$$Volume_{Cyl} = \pi r^2 h \quad = \pi x^2(14-2x)$$

$$Volume_{Total} = \frac{4}{3}\pi x^3 + \pi x^2(14-2x)$$

$$108\pi = \frac{4}{3}\pi x^3 + 4\pi x^2 - 2\pi x^3$$

$$108\pi = \pi\left(\frac{4}{3}x^3 + 14x^2 - 2x^3\right)$$

$$108 = \frac{4}{3}x^3 + 14x^2 - 2x^3$$

$$-\frac{2}{3}x^3 + 14x^2 - 108 = 0$$

$$x^3 - 21x^2 + 162 = 0$$

Possible roots are numerous, but we discard negatives and when roots get larger than 6, the length would become negative. That leaves us, 1, 2, 3, 6.

If $x = 1$;

$$(1)^3 - 21(1)^2 + 162 = 0$$

$$1 - 21 + 162 = 0 \quad \rightarrow \quad 142 = 0$$

If $x = 2$;

$$(2)^3 - 21(2)^2 + 162 = 0$$

$$8 - 84 + 162 = 0 \quad \rightarrow \quad 86 = 0$$

If $x = 3$;

$$(3)^3 - 21(3)^2 + 162 = 0$$

$$27 - 189 + 162 = 0 \quad \rightarrow \quad 0 = 0$$

So, $\boldsymbol{x = 3}$ **is a root**.

| 3 | 1 | −21 | 0 | 162 |
|---|---|---|---|---|
| | | 3 | −54 | −162 |
| | 1 | −18 | −54 | 0 |

$$x^3 - 21x^2 + 162 = (x-3)(x^2 - 18x - 54)$$

Quadratic Equation to factor: $(x^2 - 18x - 54)$

$$\frac{18 \pm \sqrt{(18)^2 - 4(1)(-54)}}{2(1)}$$

$$9 \pm 3\sqrt{15} = -2.62\ and\ 20.62$$

Reject: Negative

Reject: Too long, makes length negative

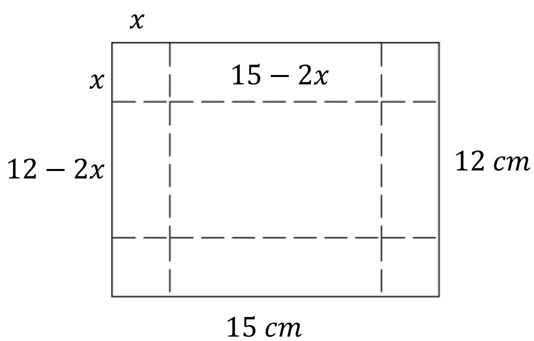
The only root hat satisfies the equation is:

$$x = 3$$

$$\boldsymbol{The\ Radius\ of\ the\ Capsule\ is\ 3mm}$$

**Example 3:** An open rectangular box is constructed by cutting a square of length $x$ from each corner of a $12cm\ by\ 15cm$ rectangular piece of cardboard, then folding up the sides. What is the length of the square that must be cut from each corner if the volume is $112cm^3$ ($x$ must be greater than $1$)?

**Solution 3:**




$$Volume = length\ x\ width\ x\ height$$

$$Volume = (15 - 2x)(12 - 2x)(x) = 112$$

$$4x^3 - 54x^2 + 180x - 112 = 0$$

$$2x^3 - 27x^2 + 90x - 56 = 0$$

Possible roots are $1, 2, 4, 7, 8$. Reject anything bigger than 6 because the width would become negative. And reject 1, because it has to be bigger.

If $x = 2$;

$$2(2)^3 - 27(2)^2 + 90(2) - 56 = 0$$

$$16 - 108 + 180 - 56 = 0 \quad \rightarrow \quad 32 = 0$$

If $x = 4$;

$$2(4)^3 - 27(4)^2 + 90(4) - 56 = 0$$

$$128 - 432 + 360 - 56 = 0 \quad \rightarrow \quad 0 = 0$$

So, $\boldsymbol{x = 4}$ **is a root**.

$$\begin{array}{c|cccc} 4 & 2 & -27 & 90 & -56 \\ & & 8 & -76 & 56 \\ \hline & 2 & -19 & 14 & 0 \end{array}$$

$$2x^3 - 27x^2 + 90x - 56 = (x - 4)(2x^2 - 19x + 14)$$

Quadratic Equation to factor: $(2x^2 - 19x + 14)$

$$\frac{19 \pm \sqrt{(19)^2 - 4(2)(14)}}{2(2)}$$

$$\frac{19 \pm \sqrt{249}}{4} = 0.81\ and\ 8.69$$

Reject: Less than one

Reject: Too long, makes width negative

The only root hat satisfies the equation is:

$$x = 4$$

$$\boldsymbol{The\ Volume\ is}: (15 - 2(4))(12 - 2(4))(4)$$

$$\boldsymbol{7cm\ x\ 4cm\ x\ 4cm = 112cm^3}$$

## Section 3.5 – Practice Problems

1. An open top rectangular box is constructed by cutting a square length $x$ from each corner of a $12cm\ by\ 15cm$ rectangle, and then folding up the sides with $x \geq 2cm$. What size square must be cut to have a volume of $162cm^3$?

2. What length must be cut if the volume of the box in question 1 is $150cm^3$.

Once Calculated, Plug the Function into DESMOS. Not Factorable using the Rational Root Theorem.

3. A silo is shaped like a cylinder topped by a hemisphere. The overall height of the silo is $12m$. Find the radius if the volume is $360\pi m^3$. $V_{Cylinder} = \pi r^2 h$ and $V_{Sphere} = \frac{4}{3}\pi r^3$

4. A right triangle has a hypotenuse $1cm$ longer than one of the sides. Find the length of the sides if the area of the triangle is $6cm^2$.
$Pythagorean\ Theorem = a^2 + b^2 = c^2$
$Area\ of\ a\ Triangle = \dfrac{1}{2}bh$

5. A box is $1m\ by\ 2m\ by\ 3m$. If each side is increased by the same amount, how much must you increase these sides to make the volume $10\ times$ larger.

6. A box measures $(1 \cdot 1 \cdot 2)m$. Each side is increased the same amount. How much is this increase if the volume is increased by six times the original volume.

Once Calculated, Plug the Function into DESMOS. Not Factorable using the Rational Root Theorem.

7. An open top box is made from a piece of cardboard measuring $5in\ x\ 8in$. Cutting out squares from each corner and folding the edges up makes a box with a volume of $14in^3$. How large a square must be cut from each corner?

8. The production of $x\ units$ produces revenue. $R(x) = 100x - x^2$ and costs of $C(x) = \frac{1}{3}x^3 - 6x^2 + 89x + 100$. At what point does the company make a profit? (Profit begins when $Revenue = Cost$)

Once Calculated, Plug the Function into DESMOS. Not Factorable using the Rational Root Theorem.

9. A shed is constructed in the shape of a cube with a triangular prism forming the roof. The total height of the shed is $6m$, with a volume of $80m^3$. Find the length of the sides of the shed. $Volume_{Cube} = x^3$
$Volume_{Triangular\ Prism} = \frac{1}{2}lwh$

10. An open box is made from a piece of cardboard $9in\ x\ 15in$, by cutting equal square corners and turning up the sides. Find the maximum volume of the box. (Think Max/Min for Parabolas)

Once Calculated, Plug the Function into DESMOS. Not Factorable using the Rational Root Theorem.

11. A box has a square base; the perimeter of the base plus the height is $120cm$. What length of the base yields a volume of $13\,500cm^3$?

12. Calculate the maximum volume of the box in the previous question. To achieve this maximum, what are the dimensions of the box?

**See Website for Detailed Answer Key**

**Extra Work Space**