Section 3.4 – Practice Problems

1. Find $P(k)$.

a) $P(x) = x^4 + 3x^3 - 7x + 2; k = -2$

$P(-2) = (-2)^4 + 3(-2)^3 - 7(-2) + 2$

$= 16 - 24 + 14 + 2$

$= 8$

$\boxed{P(-2) = 8}$

b) $P(x) = -2x^4 - 3x^2 - 2; k = \sqrt{2}$

$P(\sqrt{2}) = -2(\sqrt{2})^4 - 3(\sqrt{2})^2 - 2$

$= -2(4) - 3(2) - 2$

$= -8 - 6 - 2$

$= -16$

$\boxed{P(\sqrt{2}) = -16}$

c) $P(x) = -2x^2 + 4x + 3; k = 1 + \sqrt{2}$

FOIL

$P(1+\sqrt{2}) = -2(1+\sqrt{2})^2 + 4(1+\sqrt{2}) + 3$

$-2(3+2\sqrt{2}) + 4 + 4\sqrt{2} + 3$

$-6 - \cancel{4\sqrt{2}} + 4 + \cancel{4\sqrt{2}} + 3$

$\boxed{1}$

$(1+\sqrt{2})(1+\sqrt{2}) = 1 + 2\sqrt{2} + 2 = 3 + 2\sqrt{2}$

d) $P(x) = x^5 - 5a^4x + 4a^5; k = a$

$P(a) = a^5 - 5a^4a + 4a^5$

$= a^5 - 5a^5 + 4a^5$

$= 0$

$\boxed{P(a) = 0}$

2. Use the Remainder Theorem to solve for k and m.

a) When $x^3 + kx + 1$ is divided by $x - 2$, the remainder is -3

$P(2) = 3$

so

$2^3 + k(2) + 1 = -3$

$8 + 2k + 1 = -3$

$9 + 2k = -3$

$2k = -12$

$\boxed{k = -6}$

b) When $x^3 - x^2 + kx - 8$ is divided by $x - 4$, the remainder is 0

$P(4) = 0$

$0 = 4^3 - 4^2 + k(4) - 8$

$= 64 - 16 + 4k - 8$

$0 = 40 + 4k$

$0 = 10 + k$

$\boxed{k = -10}$

c) When $2x^4 + kx^2 - 3x + 5$ is divided by $x - 2$, the remainder is 3

$2(2)^4 + k(2)^2 - 3(2) + 5 = 3$

$32 + 4k - 6 + 5 = 3$

$31 + 4k = 3$

$4k = -28$

$\boxed{k = -7}$

d) When $x^3 + kx + 6$ is divided by $x + 2$, the remainder is 4

$(-2)^3 + k(-2) + 6 = 4$

$-8 - 2k + 6 = 4$

$-2k - 2 = 4$

$-2k = 6$

$\boxed{k = -3}$

e) When $x^3 + kx^2 - 2x - 7$ is divided by $x + 1$, the remainder is 5. What is the remainder when it is divided by $x - 1$?

solve for k 1st

$(-1)^3 + k(-1)^2 - 2(-1) - 7 = 5$

$-1 + k + 2 - 7 = 5 \rightarrow k - 6 = 5$

$k = 11$

$P(1) = ?$

$P(1) = 1^3 + 11(1)^2 - 2(1) - 7 \rightarrow \boxed{3}$

f) When $kx^3 + mx^2 + x - 2$ is divided by $x - 1$, the remainder is 6. When the it is divided by $x + 2$, the remainder is 12.

Solve systems

$kx^3 + mx^2 + x - 2 = 6$ when $x = 1$

$k + m(1)^2 - 1 = 6 \rightarrow \boxed{k + m = 7} \rightarrow 8k + 8m = 56$

$kx^3 + mx^2 + x - 2 = 12$ when $x = -2$

$-8k + 4m - 4 = 12 \rightarrow \boxed{-8k + 4m = 16}$

$8k + 8m = 56$
$-8k + 4m = 16$
$12m = 72$

$\boxed{m = 6}$ $\boxed{k = 1}$

g) When $x^4 + kx^3 - mx + 15$ has no remainder when divided by $x - 1$ and $x + 3$.

when $x = 1$: $1^4 + k(1)^3 - m(1) + 15 = 0$

$1 + k - m + 15 = 0$

$-3k + 3m - 48 = 0 \leftarrow \boxed{k - m + 16 = 0}$

* would same value for m or k

when $x = -3$; $(-3)^4 + k(-3)^3 - m(-3) + 15 = 0$

$81 - 27k + 3m + 15 = 0$

$-27k + 3m + 96 = 0$

both equal 0 so equal each other:

$-27k + 3m + 96 = -3k + 3m - 48$

$-24k = -144$

$\boxed{k = +6}$ $\boxed{m = 22}$

$k - m + 16 = 0$

$+6 - m + 16 = 0$

$-m = -22$

h) When $P(x) = 3x^4 + kx^2 + 7$ is divided by $x - 1$, the remainder is the same as when $f(x) = x^4 + kx - 4$ is divided by $x - 2$

$P(1) = 3(1)^4 + k(1)^2 + 7 = R$

$3 + k + 7 = R$

$\boxed{k + 10 = R}$

$f(2) = 2^4 + k(2) - 4 = R$

$16 + 2k - 4 = R$

$\boxed{2k + 12 = R}$

both equal R

$k + 10 = 2k + 12$

$\boxed{-2 = k}$

3. Use the Remainder Theorem to solve the following:

a) If a Polynomial equation $P(x)$ is divded by $x - a$, what is the value of its remainder?

By the Remainder Theorem if $P(x)$ is divided by $(x-a)$ the remainder is $P(a)$

b) Given $P(x) = x^3 - rx^2 + 3x + r^2$, find all possible values of r so that $P(3) = 18$

$P(3) = 18$ so

$18 = 3^3 - r(3)^2 + 3(3) + r^2$

$18 = 27 - 9r + 9 + r^2$

$0 = r^2 - 9r + 27 + 9 - 18$

$0 = r^2 - 9r + 18$

$0 = (r-6)(r-3)$

$\boxed{r=6}$ $\boxed{r=3}$

c) When the polynomial $x^n + x - 8$ is divided by $x - 2$, the remainder is 10. What is the value of n.

$P(2) = 2^n + 2 - 8$

$10 = 2^n + 2 - 8$

$10 = 2^n - 6$

$16 = 2^n$

$2^4 = 2^n$ so $\boxed{n=4}$

d) When $x^2 + 5x - 2$ is divided by $x + a$, the remainder is 8. Find all possible values of a

$P(-a) = 8$

$(-a)^2 + 5(-a) - 2 = 8$

$a^2 - 5a - 10 = 0$ Quadratic Equation

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{5 \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)} = \frac{5 \pm \sqrt{25 + 40}}{2}$

$\boxed{\frac{5 \pm \sqrt{65}}{2}}$

e) When the polynomial $P(x) = kx^{50} + 2x^{30} + 4x + 7$ is divided by $x + 1$, the remainder is 23. What is the value of k.

$P(-1) = 23$

$23 = k(-1)^{50} + 2(-1)^{30} + 4(-1) + 7$

$23 = k + 2 - 4 + 7$

$23 = k + 5$

$\boxed{k = 18}$

f) Solve for k and m if $P(x) = 2x^3 + 3x^2 + kx + m$ and $P(1) = 8$ and $P(-2) = -13$.

$\downarrow$

$8 = 2(1)^3 + 3(1)^2 + k(1) + m \rightarrow 8 = 5 + k + m$

$\boxed{k + m = 3}$

$-13 = 2(-2)^3 + 3(-2)^2 + k(-2) + m$

$-13 = -16 + 12 - 2k + m$

$\boxed{-9 = -2k + m}$

$k + m = 3$

$-\ -2k + m = -9$

$3k = 12$

$\boxed{k = 4}$

$4 + m = 3$

$\boxed{m = -1}$

4. Find the missing factors by using Synthetic Division

a) $2x^3 - 7x^2 + 2x + 3 = (x - 1)(\ 2x+1\ )(\ x-3\ )$

$$\begin{array}{r|rrrr} 1 & 2 & -7 & +2 & +3 \\ & & 2 & -5 & -3 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$

$(x-1)(2x^2 - 5x - 3)$

$\downarrow$

$(2x+1)(x-3)$

b) $x^3 - 3x^2 - 10x + 24 = (x-2)(x-4)(x+3)$

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -10 & +24 \\ & & 2 & -2 & -24 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$(x-2)(x^2-x-12)$

$\downarrow$

$(x-4)(x+3)$

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c) $x^4 + x^3 - 9x^2 - 9x = x(x+1)(x+3)(x-3)$

$x(x^3+x^2-9x-9)$

$\downarrow$

$$\begin{array}{r|rrrr} -1 & 1 & +1 & -9 & -9 \\ & & -1 & 0 & 9 \\ \hline & 1 & 0 & -9 & 0 \end{array}$$

$x(x+1)(x^2-9)$

$\downarrow$

$(x+3)(x-3)$

d) $2x^4 - 7x^3 + 9x^2 - 5x + 1 = (x-1)^3(\ 2x-1\ )(\quad)$

1	2	−7	+9	−5	+1
		2	−5	4	−1
1	2	−5	4	−1	0
		2	−3	1	
1	2	−3	1	0	
		2	−1		
	2	−1	0		

$2x-1$

e) $2x^4 + 5x^3 - 11x^2 - 20x + 12 = (x^2-4)(\ 2x-1\ )(\ x+3\ )$

$(x+2)(x-2)$

−2	2	5	−11	−20	12
		−4	−2	26	−12
2	2	1	−13	6	0
		4	10	−6	
	2	5	−3	0	

$(x+2)(x-2)(2x^2+5x-3)$

$\downarrow$

$(2x-1)(x+3)$

f) $x^5 - 8x^4 + 25x^3 - 38x^2 + 28x - 8 = (x^2 - 3x + 2)(\ x-1\ )(\ x-2\ )^2$

$(x-2)(x-1)$

$$\begin{array}{r|rrrrrr}
2 & 1 & -8 & 25 & -38 & 28 & -8 \\
 & & 2 & -12 & 26 & -24 & 8 \\
\hline
1 & 1 & -6 & 13 & -12 & 4 & 0 \\
 & & 1 & -5 & 8 & -4 & \\
\hline
 & 1 & -5 & 8 & -4 & 0 &
\end{array}$$

$(x^2-3x+2)\underbrace{(x^3-5x^2+8x-4)}_{P(x)}$ Try Remainder Theorem

$P(1) = 1^3 - 5(1) + 8(1) - 4 = 0$

so $x-1$ is a factor

$$\begin{array}{r|rrrr}
1 & 1 & -5 & 8 & -4 \\
 & & 1 & -4 & 4 \\
\hline
 & 1 & -4 & 4 & 0
\end{array}$$

$(x^2-3x+2)(x-1)(x^2-4x+4)$

$\downarrow$

$(x-2)(x-2)$

5. Solve the following using the Factor Theorem and Rational Root Theorem where necessary

a) If 2 is a root of the equation $3x^3 + x^2 - 20x + 12 = 0$, determine the other roots

$\downarrow$

means $(x-2)$ is factor

$$\begin{array}{r|rrrr}
2 & 3 & 1 & -20 & 12 \\
 & & 6 & 14 & -12 \\
\hline
 & 3 & 7 & -6 & 0
\end{array}$$

$(x-2)(3x^2+7x-6)$

use AC Method

Grouping

Quadratic Eqn

I just factored intuitively.

$\downarrow$

$(x-2)(3x-2)(x+3)$

$\downarrow \qquad \downarrow$

$\boxed{x = \frac{2}{3}}$ $\boxed{x = -3}$

$x = \frac{1}{2}$ gives remainder of 0

b) What are all values of k for which one-half is a zero of:

$$P(x) = -4x^3 + 2x^2 - 2kx + k^3$$

$0 = -4\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 - 2k\left(\frac{1}{2}\right) + k^3$

$0 = -4\left(\frac{1}{8}\right) + 2\left(\frac{1}{4}\right) - k + k^3$

$0 = -\frac{1}{2} + \frac{1}{2} - k + k^3 \rightarrow k^3 - k = 0$

$k(k^2-1) = 0$

$k(k+1)(k-1) = 0$

$\boxed{\begin{array}{l} k=0 \\ k=1 \\ k=-1 \end{array}}$

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c) If $x = c$ is a root of the polynomial equation $P(x) = 0$, then what must be a factor of $P(x)$.

Very important concept. $x=c$ means $(x-c)$ is a factor

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d) If $x - a$ is a factor of $2x^3 - ax^2 + (1 - a^2)x + 5$, what is a

means when $x=a$ remainder is 0

$2(a)^3 - a(a)^2 + (1-a^2)a + 5 = 0$

$\cancel{2a^3} - \cancel{a^3} + a - \cancel{a^3} + 5 = 0$

$a + 5 = 0$

$\boxed{a = -5}$

e) Determine k so that $x+1$ is a factor of: $2x^4+(k+1)x^2-6kx+11$

$x=-1$ Remainder is 0

$2(-1)^4+(k+1)(-1)^2-6k(-1)+11=0$

$2+k+1+6k+11=0$

$7k+14=0$

$7k=-14$

$\boxed{k=-2}$

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f) A polynomial has factors: $x^2-4, x^2-2x, and\ x^2+x-2$. What is the lowest possible degree of the polynomial?

Factors are: $\left.\begin{aligned} x^2-4 &\rightarrow (x+2)(x-2) \\ x^2-2x &\rightarrow x(x-2) \\ x^2+x-2 &\rightarrow (x+2)(x-1) \end{aligned}\right\}$ look for common factors

min is: $x(x-2)(x+2)(x-1)$

min degree is 4.

g) For what number k is k a zero of: $f(x) = 2x^3 - kx^2 + (3 - k^2)x - 6$?

means when x = k remainder is 0

$0 = 2k^3 - k(k)^2 + (3 - k^2)k - 6$

$0 = \cancel{2}k^3 - \cancel{k^3} + 3k - \cancel{k^3} - 6$

$0 = 3k - 6$

$6 = 3k$

$\boxed{k = 2}$

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h) Find the complete factored form of the following:

$$P(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$$

If $P(-2) = P(-1) = P(1) = 0$ ← given roots

Factor: $(x+2)$
$(x+1)$
$(x-1)$

−2	1	3	−5	−15	4	12
		−2	−2	14	2	−12
−1	1	1	−7	−1	6	0
		−1	0	7	−6	
1	1	0	−7	6	0	
		1	1	−6		
	1	1	−6	0		

$\boxed{(x+2)(x+1)(x-1)(x+3)(x-2)}$

$x^2 + x - 6 \rightarrow (x+3)(x-2)$

$* (x+\sqrt{3})(x-\sqrt{3})$ did not show up a potentials here because they are not Radical Numbers

i) Factor completely:

$$P(x) = x^5 - 3x^4 + 8x^2 - 9x + 3$$

If $x - 1$ is a factor

$$\begin{array}{c|cccccc} 1 & 1 & -3 & 0 & 8 & -9 & 3 \\ & & 1 & -2 & -2 & 6 & -3 \\ \hline & 1 & -2 & -2 & 6 & -3 & 0 \end{array}$$

$k(x) = x^4 - 2x^3 - 2x^2 + 6x - 3$

Potential Roots: $\dfrac{\pm 1, \pm 3}{\pm 1}$

$k(1) = 1 - 2 - 2 + 6 - 3$

$= 0 \longleftarrow$ Good!

Try $\pm 1, \pm 3$

$$\begin{array}{c|ccccc} 1 & 1 & -2 & -2 & 6 & -3 \\ & & 1 & -1 & -3 & 3 \\ \hline & 1 & -1 & -3 & 3 & 0 \end{array}$$

$\rightarrow j(x) = x^3 - x^2 - 3x + 3$

Try Factor by grouping

$(x^3 - x^2)(-3x + 3)$

$x^2(x-1) - 3(x-1)$

$(x^2 - 3)(x - 1)$

$P(x) = (x-1)(x-1)(x-1)(x^2-3)$

$$\boxed{P(x) = (x-1)^3(x+\sqrt{3})(x-\sqrt{3})}$$

j) Determine values for a and b such that $x - 1$ is a factor of both:

$x^3 + x^2 + ax + b$ and $x^3 - x^2 - ax + b$

$$\begin{array}{c|cccc} 1 & 1 & 1 & a & b \\ & & 1 & 2 & a+2 \\ \hline & 1 & 2 & a+2 & 0 \end{array}$$
$\leftarrow$ need this

$$\begin{array}{c|cccc} 1 & 1 & -1 & -a & +b \\ & & 1 & 0 & -a \\ \hline & 1 & 0 & -a & 0 \end{array}$$
$\leftarrow$ need this

$\underbrace{a + b + 2 = 0 \quad \text{and} \quad -a + b = 0}_{\text{set equal to each other, } b\text{'s will cancel}}$

$-a + b = 0$

$-(-1) + b = 0$

$1 + b = 0$

$b = -1$

$a + b + 2 = -a + b$

$a + 2 = -a \longrightarrow 2a = -2$

$\boxed{a = -1}$

$\boxed{b = -1}$

See Website for Detailed Answer Key

Extra Work Space