Section 3.4 – Practice Problems

1. A company determines that the cost, in dollars, of producing x items is:

$$C(x) = 55\,000 + 23x + 0.012x^2$$

a) Find the marginal cost function.

$\boxed{C'(x) = 23 + 0.024x}$

b) Find the marginal cost at a production level of 100 items.

$C'(100) = 23 + 0.024(100)$

$\boxed{= \$25.4/\text{item}}$

c) Find the cost of producing the 101[st] item.

$C(101) - C(100)$

$55000 + 23(101) + 0.012(101)^2 - [55000 + 23(100) + 0.012(100)^2] = \boxed{\$25.41}$

2. The cost, in dollars, for the production of x units of a commodity is:

$$C(x) = 1500 + \frac{x}{10} + \frac{x^2}{1000}$$

a) Find the marginal cost function.

$C'(x) = \frac{1}{10} + \frac{2x}{1000} = \boxed{\frac{1}{10} + \frac{x}{500}}$

b) Find the marginal cost at a production level of 800 items.

$C'(100) = \frac{1}{10} + \frac{800}{500} = \frac{1}{10} + \frac{8}{5} = \boxed{\$1.70/\text{item}}$

c) Find the cost of producing the 801[st] item.

$C(801) - C(800)$

$1500 + \frac{801}{10} + \frac{801^2}{1000} - \left[1500 + \frac{800}{10} + \frac{800^2}{1000}\right] = \boxed{\$1.70}$

3. A manufacturer determines that the revenue derived from selling x units of one of their products is:

$$R(x) = 8000x - 0.02x^3$$

a) Find the marginal revenue function.

$$\boxed{R'(x) = 8000 - 0.06x^2}$$

b) Find the marginal revenue when 300 units are sold.

$$R'(300) = 8000 - 0.06(300)^2$$

$$= \boxed{\$2600/\text{unit}}$$

c) Compare this to the actual gain in revenue when the 301st unit is sold.

$$R(301) - R(300)$$

$$8000(301) - 0.02(301)^3 - [8000(300) - 0.02(300)^3]$$

$$= \boxed{\$2581.98}$$

4. The Geehan-Herlaar Pen Company estimates the cost of manufacturing x pens is:

$$C(x) = 23\,000 + 0.24x + 0.0001x^2$$

and the revenue is:

$$R(x) = 0.98x - 0.0002x^2$$

a) Find the profit function.

$$P(x) = R(x) - C(x)$$

$$P(x) = 0.98x - 0.0002x^2 - [23000 + 0.24x + 0.0001x^2]$$

$$\boxed{P(x) = 0.74x - 0.0003x^2 - 23000}$$

b) Find the marginal profit function.

$$\boxed{P'(x) = 0.74 - 0.0006x}$$

c) Find the marginal profit when 1000 pens are sold.

$$P'(1000) = 0.74 - 0.0006(1000)$$

$$= \boxed{\$0.14/\text{pen}}$$

d) Compare this to the actual increase when the 1001st pen is sold.

$$P(1001) - P(1000)$$

$$0.74(1001) - 0.0003(1001)^2 - 23\,000 - [0.74(1000) - 0.0003(1000)^2 - 23\,000]$$

$$\boxed{\$0.1397}$$

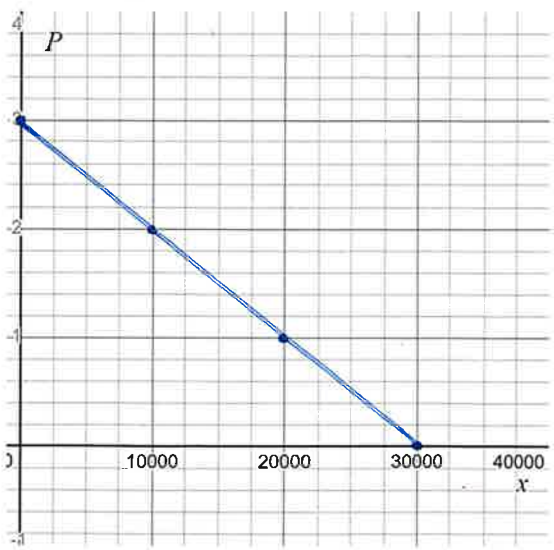
5. Greenwood's Mini Sandwiches has determined that the monthly demand for their sandwiches is given by:

$$P = \frac{30\,000 - x}{10\,000}$$

and the cost of making x sandwiches is:

$$C(x) = 6000 + 0.8x$$

a) Graph the demand function.



b) Fill in the following table to demonstrate the demand at the given prices.

P	\$0.00	\$0.50	\$1.00	\$1.50	\$2.00	\$2.50	\$3.00
x	30 000	25 000	20 000	15 000	10 000	5000	0

c) Find the revenue function.

$R(x) = xp(x)$

$= \dfrac{30\,000x - x^2}{10\,000} = \boxed{3x - \dfrac{x^2}{10000}}$

d) Find the marginal revenue function.

$R'(x) = 3 - \dfrac{2x}{10000} = \boxed{3 - \dfrac{x}{5000}}$

e) Find the marginal revenue when $x = 1000$.

$R'(1000) = 3 - \dfrac{1000}{5000} = \boxed{\$2.80}$

f) Find the profit function.

$P(x) = R(x) - C(x)$

$3x - \dfrac{x^2}{10000} - \left[6000 + 0.8x\right] \Rightarrow \boxed{P(x) = -\dfrac{x^2}{10000} + 2.2x - 6000}$

g) Find the marginal profit function.

$\boxed{P'(x) = -\dfrac{x}{5000} + 2.2}$

h) Find the marginal profit when $x = 10\,000$.

$P'(10000) = \dfrac{-10000}{5000} + 2.2 = \boxed{\$0.20}$

6. A company estimates that its production costs, in dollars, of producing x items is:

$$C(x) = 82\,000 + 23x + 0.001x^2$$

and the demand function for the product is given by:

$$P = 100 - 0.01x$$

a) Find the marginal cost function.

$\boxed{C'(x) = 23 + 0.002x}$

b) Find the marginal revenue function.

$R(x) = x(100 - 0.01x) = 100x - 0.01x^2$

$\boxed{R'(x) = 100 - 0.02x}$

c) Find the marginal profit function.

$P(x) = R(x) - C(x) \Rightarrow P(x) = 100x - 0.01x^2 - (82000 + 23x + 0.001x^2)$

$P(x) = -0.011x^2 + 77x - 82000$

$\boxed{P'(x) = -0.022x + 77}$

d) Find the marginal profit at a production level of 50 items.

$P'(50) = -0.022(50) + 77$

$= \boxed{\$75.90}$