

Name: KEY

Section 3.4 – Remainder, Factor, and Rational Root Theorems

1. What is the remainder, given the following criteria?

$$2x^4 - 3x^3 + 2x - 3 \text{ divided by } x - 2$$

$$\begin{aligned} P(2) &= 2(2)^4 - 3(2)^3 + 2(2) - 3 \\ &= 32 - 24 + 4 - 3 \\ &= 9 \end{aligned}$$

Remainder is: 9

2. What is the remainder, given the following criteria?

$$5x^3 + 2x^2 + x + 5 \text{ divided by } x + 3$$

$$\begin{aligned} P(-3) &= 5(-3)^3 + 2(-3)^2 + (-3) + 5 \\ &= -135 + 18 - 3 + 5 \\ &= -115 \end{aligned}$$

Remainder is: -115

3. Use the Rational Root Theorem to find potential roots, test them as factors using the Remainder Theorem, then use Synthetic Division initially factor, and then factor fully.

$$f(x) = 4x^3 + 12x^2 + 5x - 6$$

Test $f(1) = 27$

$f(-1) = 9$

$f(2) = 84$

$f(-2) = 0 \leftarrow x = -2 \text{ is a root}$

Possible roots: $\frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

$\left\{ \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{3}{4} \right\}$

$$\begin{array}{r|rrrr} -2 & 4 & 12 & 5 & -6 \\ & & -8 & -8 & 6 \\ \hline & 4 & 4 & -3 & 0 \end{array}$$

$\rightarrow (x-2)(4x^2+4x-3)$

$\rightarrow \boxed{(x-2)(2x+3)(2x-1)}$

$x^2 + 4x - 12$

$\left(\frac{x+6}{4}\right)\left(\frac{x-2}{4}\right)$

$\left(x + \frac{3}{2}\right)\left(x - \frac{1}{2}\right) \rightarrow (2x+3)(2x-1)$