Section 3.4 – Practice Problems

Simplify

1. $\frac{5}{x} - \frac{3}{x^3} \div \frac{2}{x}$

BEDMAS

$\frac{5}{x} - \frac{3}{x^3} \cdot \frac{x}{2}$

$\frac{5(2x)}{2x^2} - \frac{3}{2x^2}$ LCD: $2x^2$

$\frac{10x-3}{2x^2} \rightarrow \boxed{\frac{10x-3}{2x^2}}$

2. $\frac{4}{2x^3} - \frac{5x+10}{x^8} \div \frac{x+2}{x^3}$

$\frac{4}{2x^3} - \frac{5x+10}{x^8} \cdot \frac{x^3}{x+2}$

$\frac{4}{2x^3} - \frac{5(x+2)}{x^8} \cdot \frac{x^3}{(x+2)}$

$\frac{4}{2x^3} - \frac{5}{x^5}$ LCD: $2x^5$

$\frac{4x^2}{2x^5} - \frac{10}{2x^5} \rightarrow \frac{4x^2-10}{2x^5}$

$\boxed{\frac{2x^2-5}{x^5}}$

3. $\frac{2}{x} + \frac{x^2-y^2}{4x+4y} \cdot \frac{12x^2}{3y-3x}$

$\frac{2}{x} + \frac{(x+y)(x-y)}{4(x+y)} \cdot \frac{12x^2}{-3(x-y)}$

$\frac{2}{x} + (-x^2)$

$\frac{2}{x} - \frac{x^3}{x} \rightarrow \boxed{\frac{2-x^3}{x}}$

4. $\left(\frac{x-7}{x^2-16} - \frac{x-1}{16-x^2}\right)\left(\frac{x^2-16}{2}\right)$

$\left[\frac{x-7}{(x+4)(x-4)} - \frac{(x-1)}{(-1)(x+4)(x-4)}\right]\left(\frac{(x+4)(x-4)}{2}\right)$

$\left[\frac{x-7+x-1}{(x+4)(x-4)}\right]\left(\frac{(x+4)(x-4)}{2}\right)$

$(2x-8) \cdot \frac{1}{2}$

$\boxed{x-4}$

5. $\left(\frac{y+3}{y-5}+\frac{y-2}{y+4}\right)(y^2-y-20)$

$$\left[\frac{(y+3)(y+4)+(y-2)(y-5)}{(y-5)(y+4)}\right](y-5)(y+4)$$

$$(y+3)(y+4)+(y-2)(y-5)$$

$$y^2+7y+12+y^2-7y+10$$

$$2y^2+22$$

$$\boxed{2(y^2+11)}$$

6. $\left(\frac{x}{x^2-16}-\frac{2}{3x+12}\right)\left(\frac{x-4}{6}\right)$

$$\left[\frac{x}{(x-4)(x+4)}-\frac{2}{3(x+4)}\right]\left(\frac{x-4}{6}\right)$$

$$\left[\frac{3x-2(x-4)}{3(x+4)(x-4)}\right]\left(\frac{x-4}{6}\right)$$

$$\left(\frac{x+8}{3(x+4)(x-4)}\right)\left(\frac{x-4}{6}\right)$$

$$\boxed{\frac{x+8}{18(x+4)}}$$

7. $\frac{2}{b^2}+\frac{6ab}{4ab+4b^2}\div\frac{7a-7b}{a^2-b^2}$

$$\frac{2}{b^2}+\frac{6ab}{4b(a+b)}\cdot\frac{(a+b)(a-b)}{7(a-b)}$$

$$\frac{2}{b^2}+\frac{3a}{2\cdot 7}$$

$$\frac{28}{14b^2}+\frac{3ab^2}{14b^2}$$

$$\boxed{\frac{28+3ab^2}{14b^2}}$$

8. $\frac{x^3}{3}-\frac{2x^2+xy}{xy}\cdot\frac{y}{10x+5y}$

$$\frac{x^3}{3}-\frac{x(2x+y)}{xy}\cdot\frac{y}{5(2x+y)}$$

$$\frac{x^3}{3}-\frac{1}{5}$$

$$\boxed{\frac{5x^3-3}{15}}$$

Simplify

9. $\dfrac{\frac{2}{3}}{\frac{5}{6}}$

$\frac{2}{3} \div \frac{5}{6} \rightarrow \frac{2}{3} \cdot \frac{6}{5}$

$2 \cdot \frac{2}{5} = \boxed{\frac{4}{5}}$

10. $\dfrac{\frac{x}{y^3}}{\frac{x^2}{y}}$

$\frac{x}{y^3} \div \frac{x^2}{y}$

$\frac{x}{y^3} \cdot \frac{y}{x^2} = \frac{1}{y^2 x}$

$\boxed{\frac{1}{xy^2}}$

11. $\dfrac{\frac{8x^4y^3}{3x}}{\frac{4xy^4}{y^2}}$

$\frac{8x^4y^3}{3x} \div \frac{4xy^4}{y^2}$

$\frac{8x^4y^3}{3x} \cdot \frac{y^2}{4xy^4} = \frac{2x^4y^5}{3x^3y^4}$

$\boxed{\frac{2x^2y}{3}}$

12. $\dfrac{\frac{x+2}{5}}{\frac{x-3}{x}}$

$\frac{x+2}{5} \div \frac{x-3}{x}$

$\frac{x+2}{5} \cdot \frac{x}{x-3}$

$\boxed{\frac{x^2+2x}{5x-15} \text{ or } \frac{x(x+2)}{5(x-3)}}$

13. $\dfrac{\frac{1}{x}-2}{\frac{x}{5}+1}$

$\dfrac{\frac{1}{x}-\frac{2x}{x}}{\frac{x}{5}+\frac{5}{5}}$

$\dfrac{1-2x}{x} \div \dfrac{x+5}{5}$

$\dfrac{1-2x}{x} \cdot \dfrac{5}{x+5}$

$\boxed{\dfrac{5(1-2x)}{x(x+5)}}$

14. $\dfrac{1+\frac{8}{y}}{y+3-\frac{40}{y}}$

$\dfrac{\frac{y}{y}+\frac{8}{y}}{\frac{y^2}{y}+\frac{3y}{y}-\frac{40}{y}}$

$\dfrac{y+8}{y} \div \dfrac{y^2+3y-40}{y}$

$\dfrac{y+8}{y} \cdot \dfrac{y}{(y+8)(y-5)}$

$\boxed{\dfrac{1}{y-5}}$

15. $\dfrac{\frac{3}{x-2}+\frac{2}{x+2}}{\frac{4}{x+2}-\frac{5}{x-2}}$

$\dfrac{3(x+2)+2(x-2)}{(x-2)(x+2)}$

$\dfrac{4(x-2)-[5(x+2)]}{(x+2)(x-2)}$

$\dfrac{3x+6+2x-4}{(x+2)(x-2)} \cdot \dfrac{(x+2)(x-2)}{4x-8-5x-10}$

$\boxed{\dfrac{5x+2}{-x-18}}$

16. $\dfrac{\frac{1}{x-1}-2}{\frac{3}{x-1}+4}$

$\dfrac{1-2(x-1)}{x-1}$

$\dfrac{3+4(x-1)}{x-1}$

$\dfrac{1-2x+2}{3+4x-4}$

$\boxed{\dfrac{-2x+3}{4x-1}}$

when you have the same denominator they cancel out

17. $\dfrac{\frac{2}{x-3}+\frac{4}{x}}{\frac{3}{x-2}+\frac{1}{x}}$

$$\frac{\frac{2x+4(x-3)}{x(x-3)}}{\frac{3x+(x-2)}{x(x-2)}}$$

$$\frac{2x+4x-12}{x(x-3)} \cdot \frac{x(x-2)}{3x+x-2}$$

$$\frac{(6x-12)(x-2)}{(x-3)(4x-2)}$$

$$\frac{2(x-2)(x-2)}{(x-3)(2)(2x-1)}$$

$$\boxed{\frac{3(x-2)^2}{(x-3)(2x-1)}}$$

18. $\dfrac{3-\frac{4}{y-1}}{5-\frac{3}{1-y}}$

$$\frac{\frac{3(y-1)-4}{y-1}}{\frac{5(1-y)-3}{1-y}}$$

$$\frac{3y-3-4}{y-1} \cdot \frac{(-1)(y-1)}{5-5y-3}$$

$$\frac{(3y-7)\cdot(-1)}{-5y+2} \rightarrow \frac{(3y-7)(-1)}{(5y-2)(-1)}$$

$$\boxed{\frac{3y-7}{5y-2}}$$

19. $\dfrac{\frac{1}{x^2+x}-\frac{1}{xy+y}}{\frac{1}{xy+y}-\frac{1}{x^2+x}}$

$$\frac{\frac{1}{x(x+1)}-\frac{1}{y(x+1)}}{\frac{1}{y(x+1)}-\frac{1}{x(x+1)}}$$

$$\frac{\frac{y-x}{xy(x+1)}}{\frac{x-y}{xy(x+1)}} \rightarrow \frac{y-x}{x-y}$$

$$(-1)\frac{x-y}{x-y}$$

$$\boxed{-1}$$

20. $\dfrac{\frac{x}{x^2-1}-\frac{3x+3}{1-x}}{\frac{2x-1}{x-1}+\frac{x}{1-x}}$

$$\rightarrow \frac{\frac{x}{(x+1)(x-1)}-\frac{3(x+1)}{(-1)(x-1)}}{\frac{2x-1}{x-1}+\frac{x}{(-1)(x-1)}}$$

$$\frac{\frac{x}{(x+1)(x-1)}+\frac{3(x+1)}{(x-1)}}{\frac{2x-1-x}{x-1}} \rightarrow \frac{\frac{x+3(x+1)^2}{(x+1)(x-1)}}{\frac{x}{x-1}}$$

$$\frac{x+3(x^2+2x+1)}{x^2-1} \rightarrow \frac{x+3x^2+6x+3}{x^2-1}$$

$$\boxed{\frac{3x^2+7x+3}{x^2-1}}$$

21. Solve the formula for R

$$i = \frac{E}{R + \frac{r}{2}}$$

$$i = \frac{E}{R+\frac{r}{2}} \rightarrow i = \frac{E}{\frac{2R+r}{2}}$$

$$i = E \div \frac{2R+r}{2}$$

$$i = E \cdot \frac{2}{2R+r} \rightarrow i = \frac{2E}{2R+r}$$

$$(2R+r)i = 2E$$

$$2R+r = \frac{2E}{i}$$

$$\frac{1}{2} \cdot 2R = \left(\frac{2E}{i} - r\right)\frac{1}{2}$$

$$R = \frac{2E}{2i} - \frac{r}{2}$$

$$\boxed{R = \frac{2E - ri}{2i}}$$

Solve the formula for r

$$i = \frac{E}{R + \frac{r}{2}}$$

Same steps but from here

$$2R + r = \frac{2E}{i}$$

$$r = \frac{2E}{i} - 2R$$

$$r = \frac{2E - 2Ri}{i}$$

$$\boxed{r = \frac{2(E - Ri)}{i}}$$