

Section 3.4 – Combined Operations with Rational Expressions

This booklet belongs to: _____ Block: _____

- Mixed Operations include the BEDMAS of Rational Expressions

B – Brackets
E – Exponents
D – Division
M – Multiplication
A – Addition
S – Subtraction

- Recall that both:
 - Multiplication and Division
 - Addition and Subtraction
 Can be done at the same time for
Left to Right

Example 1: Simplify $\frac{x+5}{x+6} + \frac{1}{x+4} \div \frac{x+6}{x^2-x-20}$

Solution 1: $\frac{x+5}{x+6} + \frac{1}{x+4} \div \frac{x+6}{x^2-x-20} =$

- Division 1st
- Flip and Multiply

$$= \frac{x+5}{x+6} + \frac{1}{x+4} \div \frac{x+6}{x^2-x-20} = \frac{x+5}{x+6} + \frac{1}{\cancel{x+4}} \cdot \frac{(x-5)\cancel{(x+4)}}{(x+6)}, \quad x \neq -6, -4, 5$$

$$= \frac{x+5}{x+6} + \frac{x-5}{x+6} = \frac{2x}{x+6}$$

Example 2: Simplify $\left(\frac{x-3}{x^2-9} + \frac{x+3}{x^2+6x+9}\right)\left(\frac{x+3}{x+1}\right)$

Solution 2: $\left(\frac{x-3}{x^2-9} + \frac{x+3}{x^2+6x+9}\right)\left(\frac{x+3}{x+1}\right) = \left(\frac{x-3}{(x-3)(x+3)} + \frac{x+3}{(x+3)(x+3)}\right)\left(\frac{x+3}{x+1}\right), \quad x \neq -3, -1, 3$

$$= \left(\frac{\cancel{x-3}}{(\cancel{x-3})(x+3)} + \frac{\cancel{x+3}}{(\cancel{x+3})(x+3)}\right)\left(\frac{x+3}{x+1}\right) = \left(\frac{1}{(x+3)} + \frac{1}{(x+3)}\right)\left(\frac{x+3}{x+1}\right)$$

$$= \left(\frac{2}{x+3}\right)\left(\frac{x+3}{x+1}\right) = \frac{2}{x+1}$$

Complex Fractions

- Fractional forms that contain fractions in the numerators and/or denominators are called complex fractions

Example: $\frac{\frac{2}{x} + \frac{3}{y}}{\frac{5}{x} - \frac{1}{y}}$

There are two methods for simplifying these:

Method 1: Get a common denominator for the numerator fractions and denominator fractions individually, then divide the resulting fractions

Method 2: Multiply the numerator and denominator in the complex fraction by the LCD of all the fractions, then simplify the fraction

Example 3: Simplify

$$\frac{\frac{2}{5x} - \frac{3}{x^2}}{\frac{7}{2x} + \frac{3}{4x^2}}$$

Solution 3:

Method 1

- The LCD of $5x$ and x^2 is $5x^2$
- The LCD of $2x$ and $4x^2$ is $4x^2$

$$\begin{aligned} \frac{\frac{2}{5x} - \frac{3}{x^2}}{\frac{7}{2x} + \frac{3}{4x^2}} &= \frac{\frac{2 \cdot x}{5x \cdot x} - \frac{3 \cdot 5}{x^2 \cdot 5}}{\frac{7 \cdot 2x}{2x \cdot 2x} + \frac{3}{4x^2}} \\ &= \frac{\frac{2x - 15}{5x^2}}{\frac{14x + 3}{4x^2}} = \frac{2x - 15}{5x^2} \cdot \frac{4x^2}{14x + 3} \\ &= \frac{4(2x - 15)}{5(14x + 3)} \end{aligned}$$

Method 2

- The LCD of $5x$, x^2 , $2x$, and $4x^2$ is $20x^2$

$$\begin{aligned} \frac{\frac{2}{5x} - \frac{3}{x^2}}{\frac{7}{2x} + \frac{3}{4x^2}} &= \frac{20x^2 \left(\frac{2}{5x} - \frac{3}{x^2} \right)}{20x^2 \left(\frac{7}{2x} + \frac{3}{4x^2} \right)} \\ &= \frac{\frac{40x^2}{5x} - \frac{60x^2}{x^2}}{\frac{140x^2}{2x} + \frac{60x^2}{4x^2}} = \frac{8x - 60}{70x + 15} \\ &= \frac{4(2x - 15)}{5(14x + 3)} \end{aligned}$$

Example 4: Simplify

$$\frac{\frac{1}{x-1} + \frac{2}{x+2}}{\frac{2}{x+2} - \frac{1}{x-3}}$$

Solution 4:

Method 1

The LCD of $(x-1)$ and $(x+2)$ is $(x-1)(x+2)$ The LCD of $(x+2)$ and $(x-3)$ is $(x+2)(x-3)$

$$\begin{aligned} \frac{\frac{1}{x-1} + \frac{2}{x+2}}{\frac{2}{x+2} - \frac{1}{x-3}} &= \frac{\frac{1(x+2)}{(x-1)(x+2)} + \frac{2(x-1)}{(x+2)(x-1)}}{\frac{2(x-3)}{(x+2)(x-3)} - \frac{1(x+2)}{(x-3)(x+2)}} = \frac{\frac{x+2+2x-2}{(x-1)(x+2)}}{\frac{2x-6-x-2}{(x-3)(x+2)}} = \frac{\frac{3x}{(x-1)(x+2)}}{\frac{x-8}{(x-3)(x+2)}} \\ &= \frac{3x}{(x-1)\cancel{(x+2)}} \cdot \frac{(x-3)\cancel{(x+2)}}{x-8} = \frac{3x(x-3)}{(x-1)(x-8)} \end{aligned}$$

Method 2

The LCD of $(x-1)$, $(x+2)$ and $(x-3)$ is $(x-1)(x+2)(x-3)$

$$\begin{aligned} \frac{\frac{1}{x-1} + \frac{2}{x+2}}{\frac{2}{x+2} - \frac{1}{x-3}} &= \frac{(x-1)(x+2)(x-3)\left(\frac{1}{x-1} + \frac{2}{x+2}\right)}{(x-1)(x+2)(x-3)\left(\frac{2}{x+2} - \frac{1}{x-3}\right)} = \frac{(x+2)(x-3) + 2(x-1)(x-3)}{2(x-1)(x-3) - 1(x-1)(x+2)} \\ &= \frac{x^2 + 2x - 3x - 6 + 2x^2 - 2x - 6x + 6}{2x^2 - 2x - 6x + 6 - 1(x^2 + 2x - x - 2)} = \frac{3x^2 - 9x}{2x^2 - 2x - 6x + 6 - 1x^2 - 2x + x + 2} \\ &= \frac{3x^2 - 9x}{x^2 - 9x + 8} = \frac{3x(x-3)}{(x-1)(x-8)} \end{aligned}$$

- Use either Method, but take your time and make sure you go Step by Step!

Section 3.4 – Practice Problems

Simplify

1. $\frac{5}{x} - \frac{3}{x^3} \div \frac{2}{x}$

2. $\frac{4}{2x^3} - \frac{5x+10}{x^8} \div \frac{x+2}{x^3}$

3. $\frac{2}{x} + \frac{x^2 - y^2}{4x + 4y} \cdot \frac{12x^2}{3y - 3x}$

4. $\left(\frac{x-7}{x^2-16} - \frac{x-1}{16-x^2} \right) \left(\frac{x^2-16}{2} \right)$

5.
$$\left(\frac{y+3}{y-5} + \frac{y-2}{y+4}\right)(y^2 - y - 20)$$

6.
$$\left(\frac{x}{x^2-16} - \frac{2}{3x+12}\right)\left(\frac{x-4}{6}\right)$$

7.
$$\frac{2}{b^2} + \frac{6ab}{4ab+4b^2} \div \frac{7a-7b}{a^2-b^2}$$

8.
$$\frac{x^3}{3} - \frac{2x^2+xy}{xy} \cdot \frac{y}{10x+5y}$$

Simplify

9. $\frac{\frac{2}{3}}{\frac{5}{6}}$

10. $\frac{\frac{x}{y^3}}{\frac{x^2}{y}}$

11. $\frac{\frac{8x^4y^3}{3x}}{\frac{4xy^4}{y^2}}$

12. $\frac{\frac{x+2}{5}}{\frac{x-3}{x}}$

$$13. \frac{\frac{1}{x} - 2}{\frac{x}{5} + 1}$$

$$14. \frac{1 + \frac{8}{y}}{y + 3 - \frac{40}{y}}$$

$$15. \frac{\frac{3}{x-2} + \frac{2}{x+2}}{\frac{4}{x+2} - \frac{5}{x-2}}$$

$$16. \frac{\frac{1}{x-1} - 2}{\frac{3}{x-1} + 4}$$

$$17. \frac{\frac{2}{x-3} + \frac{4}{x}}{\frac{3}{x-2} + \frac{1}{x}}$$

$$18. \frac{3 - \frac{4}{y-1}}{5 - \frac{3}{1-y}}$$

$$19. \frac{\frac{1}{x^2+x} - \frac{1}{xy+y}}{\frac{1}{xy+y} - \frac{1}{x^2+x}}$$

$$20. \frac{\frac{x}{x^2-1} - \frac{3x+3}{1-x}}{\frac{2x-1}{x-1} + \frac{x}{1-x}}$$

21. Solve the formula for R

$$i = \frac{E}{R + \frac{r}{2}}$$

Solve the formula for r

$$i = \frac{E}{R + \frac{r}{2}}$$

Answer Key – Section 3.4

| | |
|-----|--------------------------|
| 1. | $\frac{10x-3}{2x^2}$ |
| 2. | $\frac{2x^2-5}{x^5}$ |
| 3. | $\frac{2-x^3}{x}$ |
| 4. | $x - 4$ |
| 5. | $2(y^2 + 11)$ |
| 6. | $\frac{x+8}{18(x+4)}$ |
| 7. | $\frac{28+3ab^2}{14b^2}$ |
| 8. | $\frac{5x^3-3}{15}$ |
| 9. | $\frac{4}{5}$ |
| 10. | $\frac{1}{xy^2}$ |

| | |
|-----|--|
| 11. | $\frac{2x^2y}{3}$ |
| 12. | $\frac{x(x+2)}{5(x-3)}$ |
| 13. | $\frac{5(1-2x)}{x(x+5)}$ |
| 14. | $\frac{1}{(y-5)}$ |
| 15. | $-\frac{(5x+2)}{(x+18)}$ |
| 16. | $-\frac{(2x-3)}{(4x-1)}$ |
| 17. | $\frac{3(x-2)^2}{(x-3)(2x-1)}$ |
| 18. | $\frac{(3y-7)}{(5y-2)}$ |
| 19. | -1 |
| 20. | $\frac{3x^2+7x+3}{(x^2-1)}$ |
| 21. | $R = \frac{2E - ri}{2i}$ $r = \frac{2(E - Ri)}{i}$ |

Extra Work Space