Section 3.3 – Writing Equations of Lines

This booklet belongs to: ________________ Block: ______

Equations of Parallel and Perpendicular Lines

- To write the equation of a line, two points are needed or a point and slope
- Sometimes this information is not given directly
- We have to problem solve to find the information we need
- Remember that parallel lines have equal slopes and perpendicular lines have slopes that are negative reciprocals to one another

Example: Write the General Form equation of a line parallel to $3x - 2y = 6$, and which goes through the point $A(4, -2)$.

Solution:

- First we need the Slope of the given line

$$3x - 2y = 6 \rightarrow -2y = -3x + 6 \rightarrow y = \frac{3}{2}x - 3, \quad m = \frac{3}{2}$$

- So the slope of the line parallel to $3x - 2y = 6$ also has a slope: $m = \frac{3}{2}$

- Substituting in the given point and slope to the Point-slope equation of a line gives us:

So, $y - y_1 = m(x - x_1) \rightarrow y - (-2) = \frac{3}{2}(x - 4)$ \rightarrow

$$y + 2 = \frac{3}{2}x - \frac{12}{2} \rightarrow \quad y + 2 = \frac{3}{2}x - 6 \rightarrow$$

$$y = \frac{3}{2}x - 8 \quad \text{(Slope-intercept Form)}$$

And, $y = \frac{3}{2}x - 8 \rightarrow 2y = 3x - 16 \rightarrow 3x - 2y - 16 = 0 \quad \text{(General Form)}$
Example: Write the General Form equation of a line perpendicular to $4x + 2y = 7$, and which goes through the point $A(-2, 5)$.

Solution:

- First we need to find the slope of the given line:

$$4x + 2y = 7 \quad \rightarrow \quad 2y = -4x + 7 \quad \rightarrow \quad y = \frac{-4}{2}x + \frac{7}{2} \quad m = -2$$

- So the slope of the line perpendicular to $4x + 2y = 7$ has slope:

$$m = \frac{1}{2}$$

- Substituting in the given point and slope to the Point-slope equation of a line gives us:

$$y - y_1 = m(x - x_1) \quad \rightarrow \quad y - 5 = \frac{1}{2}(x - (-2)) \quad \rightarrow$$

$$y - 5 = \frac{1}{2}x + \frac{2}{2} \quad \rightarrow \quad y - 5 = \frac{1}{2}x + 1 \quad \rightarrow$$

$$y = \frac{1}{2}x + 6 \quad \text{(Slope-intercept Form)}$$

And, $y = \frac{1}{2}x + 6 \quad \rightarrow \quad 2y = x + 12 \quad \rightarrow \quad x - 2y + 12 = 0 \quad \text{(General Form)}$
Section 3.3 – Practice Problems

Find the equation of the line in General Form, that passes through the given point and is parallel to the given line.

1. \( P(0,0); y = 2x - 5 \) 

2. \( P(0, 0); x = 2y + 5 \)

3. \( P(1, 3); 3x - y = 6 \)

4. \( P(-2, 0); 2x + 5y = 3 \)
5. \( P(-6, 3); y + 4x = -8 \)

6. \( P(5, -2); 3y + 1 = -4x \)

7. \( P(-5, 2); x = 3 \)

8. \( P(-5, 2); y = -4 \)
Find the equation of the line in General Form, that passes through the given point and is perpendicular to the given line.

9. $P(0,0); y = 2x - 5$

10. $P(0,0); x = 2y + 5$

11. $P(1,3); 3x - y = 6$

12. $P(-2,0); 2x + 5y = 3$
13. \( P(-6, 3); y + 4x = -8 \)

14. \( P(5, -2); 3y + 1 = -4x \)

15. \( P(-5, 2); x = 3 \)

16. \( P(-5, 2); y = -4 \)

17. Find the equation of a line parallel to \( 3x + 4y = 8 \) with the same \( y \)-intercept as \( 5x - 3y = 10 \)
18. Find the equation of a line **perpendicular** to $x - 3y = 8$ with the same $y$ - intercept as $3x + 2y = 6$

19. Find the equation of a line **parallel** to $2x + 7y = 10$ with the same $x$ - intercept as $3x - 4y = 5$
Answer Key

Section 3.3

1. \(2x - y = 0\)
2. \(x - 2y = 0\)
3. \(3x - y = 0\)
4. \(2x + 5y + 4 = 0\)
5. \(4x + y + 21 = 0\)
6. \(4x + 3y - 14 = 0\)
7. \(x = -5\)
8. \(y = 2\)
9. \(x + 2y = 0\)
10. \(2x + y = 0\)
11. \(x + 3y - 10 = 0\)
12. \(5x - 2y + 10 = 0\)
13. \(x - 4y + 18 = 0\)
14. \(3x - 4y - 23 = 0\)
15. \(y = 2\)
16. \(x = -5\)
17. \(y = \frac{-3}{4}x - \frac{10}{3}\)
   \(\text{or}\)
   \(9x + 12y = -40\)
18. \(y = -3x + 3\)
   \(\text{or}\)
   \(3x + y = 3\)
19. \(y = \frac{-2}{7}x + \frac{10}{21}\)
   \(\text{or}\)
   \(6x + 21y = 10\)
Extra Work Space