Section 3.3 – Practice Problems

1. Find the rate of change of the volume of a cube with respect to its edge length x when $x = 4$.

$V(x) = x^3$

$V'(x) = 3x^2$ $\quad V'(4) = 3(4)^2$

$= \boxed{48 \text{ units}^3/\text{unit}}$

2. Find the rate of change of the area of a circle with respect to its radius r when $r = 5\text{cm}$.

$A(x) = \pi r^2$

$A'(x) = 2\pi r$ at $5 = r$ $\quad A'(5) = 2\pi(5)$

$= \boxed{10\pi \text{ cm}^2/\text{cm}}$

3. If a tank holds 1000L of water, which takes an hour to drain from the bottom of the tank, then the volume V of water remaining in the tank after t minutes is given by:

$$V = 1000\left(1 - \frac{t}{60}\right)^2 \qquad 0 \leq t \leq 60$$

Find the rate at which the water is flowing out of the tank (instantaneous rate of change) after 10 minutes.

$V' = 1000\left[2\left(1 - \frac{t}{60}\right) \cdot \frac{-1}{60}\right]$

$1000\left[-\frac{2}{60} + \frac{t}{1800}\right]$

$\frac{-100}{3} + \frac{5t}{9} \rightarrow \frac{5t - 300}{9}$

at $t = 10$

$\boxed{-\frac{250}{9} \text{ L/min}}$

4. The mass of the part of a wire that lies between its left end and a point x metres to the right is $\sqrt{x}$ kilograms.

   a) Find an approximate value for the average density of the part of the wire from $x = 1\text{m}$ to $x = 1.1\text{m}$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{\sqrt{1.1} - \sqrt{1}}{1.1 - 1} = \frac{0.0488}{0.1} = \boxed{0.488 \text{ kg/m}}$$

   b) Find the linear density when $x = 1\text{m}$

$$f'(x) = \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2} \text{ kg/m}}$$

5. The mass of the left x centimeters of a string is $x + \frac{1}{2}x^2$ grams. Find the linear density when $x = 6\text{cm}$.

$$f'(x) = 1 + x$$

$$f'(6) = 7 \qquad \boxed{7 \text{ g/cm}}$$

6. The population of a bacteria colony after t hours is given by $n = 1000 + 180t + 25t^2 + 3t^3$. Find the growth rate after 3h.

$$n'(t) = 180 + 50t + 9t^2$$

$$n'(3) = 180 + 50(3) + 9(3)^2$$

$$= \boxed{411 \text{ bac/hr}}$$

7. The volume V of a substance kept at constant temperature will depend on the pressure P. The **isothermal compressibility β** is defined by

$$\beta = -\frac{1}{V}\frac{dV}{dP}$$

And measures how fast, per unit volume, the volume of a substance decreases as the pressure increases at constant temperature.

The volume V (in cubic meters) of a sample of air at 25°C was related to pressure P (in kilopascals) by the equation. Find compressibility when pressure is 40 kPa

$$V = \frac{5.3}{P}$$

$$\frac{dV}{dp} = -\frac{5.3}{p^2}\frac{m^3}{kPa} \qquad \beta = -\frac{1}{\frac{5.3}{p}}\cdot\left(-\frac{5.3}{p^2}\right) = -\frac{P}{5.3}\cdot\left(\frac{-5.3}{p^2}\right)$$

$$= \frac{1}{P} = \boxed{\frac{1}{40}\, m^3/kPa/m^3}$$

8. The concentration of dinitrogen pentoxide, N_2O_5, in the reaction $2N_2O_5 \rightarrow 4NO_2 + O_2$ were measured at one-minute intervals, as can be seen in the table below.

Time (min)	0	1	2	3	4
$[N_2O_5]$	0.160	0.113	0.080	0.056	0.040

Draw the graph of N_2O_5 as a function of time and use it to estimate the rate of reaction after two minutes.

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$\frac{0.03 - 0.13}{4 - 0}$$

$$\frac{-0.1}{4} = -0.025\ mol/L/min$$