Section 3.3 - Practice Problems

- 1. Compute the quotient using long division. Write answer in two ways:
 - i) Dividend = (quotient)(Divisor) + Remainder
 - ii) $\frac{dividend}{divisor} = (quotient) \frac{remainder}{divisor}$

a)
$$x^{2} - 5x - 18$$

$$x - 3) x^{3} - 8x^{2} - 3x + 2$$

$$x^{3} - 3x^{2} \downarrow$$

$$-5x^{2} - 3x$$

$$-5x^{2} + 15x$$

$$-18x + 2$$

$$-18x + 54$$

$$-52$$

- i) $x^3 8x^2 3x + 2 = (x^2 5x 18)(x 3) 52$
- ii) $\frac{3}{x^2-8x^2-3x+2} = x^2-5x-18-\frac{52}{(x-3)}$
 - c) $x^{3} + 2x^{2} 2x 1$ $x^{5} + 2x^{4} - x^{3} + x^{2} - 3x + 4$ $x^{5} + x^{3} + x^{2}$ $2x^{4} - 2x^{3} + x^{2}$ $2x^{4} + 2x^{2}$ $-2x^{3} - x^{2} - 3x$ $-2x^{3} + -2x$ $-2x^{3} + -2x$ $-2x^{3} + -2x$
- i) $x^{5} + 2x^{4} x^{3} + x^{2} 3x + 4 = (x^{2} + 1)(x^{3} + 2x^{2} 2x 1) + (+x + 5)$

$$\frac{11)}{x^{5}+2x^{4}-x^{3}+x^{2}-3x+4} = (x^{3}+2x^{2}-2x-1)+(-x+5)$$
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- b) $\frac{8x^{3}-1}{2x-1}$ $2x-1) \frac{4x^{2}+2x+1}{8x^{3}+0x^{2}+0x-1}$ $\frac{6x^{3}-4x^{2}}{4x^{2}+0x}$ $\frac{4x^{2}+0x}{4x^{2}-2x}$ $\frac{2x-1}{2x-1}$
 - i) $8x^3 1 = (4x^2 + 2x + 1)(2x 1)$
 - $\frac{8x^3-1}{2x-1} = 4x^2 + 2x + 1$
 - d) $\frac{x^{4} 3x^{2} + 8}{x^{2} 1}$ $x^{2} 1$ $x^{2} + 0x^{3} 3x^{2} + 0x + 8$ $x^{4} x^{2} \downarrow \downarrow$ $-2x^{2} + 0x + 8$ $-2x^{2} \rightarrow 2$ 6
 - i) $x^4 3x^2 + 8 = (x^2 1)(x^2 2) + 6$
 - ii) $x^4 3x^2 + 8 = (x^2 2) + \frac{6}{x^2 1}$

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e)
$$x^{2}-4x-12 \overline{\smash)x^{3}+2x^{2}-13x+10}$$

 $- x^{3}-4x^{2}-12x \downarrow$
 $6x^{2}-x+10$
 $6x^{2}-24x-72$
 $23x+82$

i)
$$x^3 + 2x - 13x + 10 = (x+6)(x^2 + 4x - 12) + (23x + 6)$$

$$\frac{111}{x^{2}+2x^{2}-13x+10} = (x+6) + \frac{(23x+82)}{x^{2}-4x-12}$$

f)
$$\frac{x^3 - 5x + 1}{x^2 - 2x}$$

$$x + 2$$

$$x^2 - 2x)x^3 + 0x^2 - 5x + 1$$

$$x^3 - 2x^2$$

$$2x^2 - 5x$$

$$2x^2 - 4x$$

$$-x + 1$$

$$(x^{2}-5x+1)=(x^{2}-2x)(x+2)+(-x+1)$$

$$\frac{x^3-5x+1}{x^2-2x} = (x+2) - (x+1)$$

g)
$$x^3 + 3x + 2$$
 $5x^4 + 6x^3 + 11x^2 + 6x + 6$ h) $\frac{x^4 + 9x^3 - 5x^2 - 32x + 3}{x^2 - 1}$ $\frac{x^4 + 9x^3 - 5x^2 - 32x + 3}{x^2 - 1}$ $\frac{x^2 + 9x - 4}{6x^3 + 8x^2 + 4x}$ $\frac{x^2 - 1}{8x^2 - 14x - 12}$ $\frac{x^4 - x^2}{8x^2 - 14x - 12}$ $\frac{x^4 - x^2}{9x^3 - 4x^2 - 32x}$ $\frac{x^4 - x^2}{9x^3 - 9x}$

i)
$$x^4 + 6x^3 + 11x^2 + 6x = (x^3 + 3x + 2)(x + 6) + (9x^2 - 14x - 12)$$

(i)
$$\frac{x^4 + 6x^3 + 11x^2 + 6x}{(x^3 + 3x + 2)} = (x + 6) + \frac{(8x^2 - 14x - 12)}{(x^3 + 3x + 2)}$$

$$\frac{x^4 + 9x^3 - 5x^2 - 32x + 3}{x^2 - 1}$$

i)
$$x^4 + 9x^3 - 5x^2 - 32x + 3 = (x^2 + 9x - 4)(x^2 - 1) + (-23x - 1)$$

$$\frac{x^{4}+9x^{3}-5x^{2}-32x+3}{x^{2}-1} = (x^{2}+9x-4)^{4} + \frac{(-23x-1)}{(x^{2}-1)}$$

2. Use synthetic division to find the quotient Q(x) and the remainder R when the polynomial P(x) is divided by the given binomial.

a)
$$P(x) = x^3 + 2x^2 - 3x + 1; x - 2$$

b) $P(x) = x^3 + 2x^2 - 3x + 1; x - 2$
2 1 2 -3 1
2 8 10

$$Q(x) = x^2 + 4x + 5$$

 $R = 11$

$$P(x) = x^3 - a^3; x - a$$
Remember
$$a \text{ is a constant}$$

$$a + 0 + 0 - a^3$$

$$a a^2 a^3$$

$$a a^2 a^3$$

$$Q(x) = x^{2} + ax + a^{2}$$

$$R = 0$$

c)
$$P(x) = 4x^3 + 5x - 3; x + 2$$

 -2 | 4 0 5 - 3
 -8 | 16 - 42
 -8 | 21 - 45

$$Qxx = 4x^2 - 8x + 21$$
 $R = -45$

d)
$$P(x) = x^5 - 5x^3 + 10; x - 1$$

$$\begin{vmatrix} 1 & +0 & -5 & +0 & +0 & 10 \\ 1 & 1 & -4 & -4 & -4 \\ \hline 1 & 1 & -4 & -4 & 6 \end{vmatrix}$$

$$Q(x) = x^4 + x^3 - 4x^2 - 4x - 4$$

R= 6

e)
$$P(x) = 0.1x^2 + 0.2; x - 2.1$$

2.1 0.1 +0 + 6.2
0.21 +0.441

$$G(x) = 0.1x + 0.21$$

$$R = 0.641$$

Gen =
$$3x^3 - 3$$

but since we had $3x + 1$

and used $x = -\frac{1}{3}$

multiply divisor by 3

divide quadrent by 3

Qual = $x^3 - 1$
 $P = 2$

g) $P(x) = 3x^4 + x^3 - 3x + 1; 3x + 1$

h)
$$P(x) = 2x^4 - x^3 + 2x - 1; 2x - 1$$

$$\frac{1}{2} \begin{bmatrix} 2 & -1 & +0 & +2 & -1 \\ 1 & 0 & 0 & 1 \\ 2 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$$Q(x) = 2x^3 + 2$$
but divide by 2
$$Q(x) = x^3 + 1$$

$$R = 0$$

11

i)
$$P(x) = 3x^5 + 2x^4 + 5x^3 - 7x + 3$$
; j) $P(x) = 3x^4 - 3x^3 + 2x^2 - 3x + 1$; $x - 0.4$

$$-0.8 \quad 3 \quad 2 \quad 5 \quad 0 \quad -7 \quad 3$$

$$-2.4 \quad 6.32 - 4.266 \quad 3.4018 \quad 2.03416$$

$$3 \quad -6.4 \quad 5.32 \quad -4.266 \quad -35962 \quad 5.87616$$

$$3 - 3x^{4} - 3x^{3} + 2x^{2} - 3x + 1;$$

$$-0.4$$

$$0.4 \quad \begin{bmatrix} 3 - 3 & 2 & -3 & +1 \\ 1.2 & -0.72 & 0.5(2 & -0.9952 \\ \hline 3 - 1.8 & 1.28 & -2.489 & 0.0048 \end{bmatrix}$$

$$Q(x) = 3x^4 - 0.4x^3 + 5.32x^2 - 4.256x - 3.5952$$

$$R = 5.87616$$

Qw =
$$3x^3 - 1.9x^2 + 1.28x - 2.498$$

R = 0.0048

$$Q(x) = x^2 - 5x - 3$$

 $R = 0$

$$Q = x^3 - 2x + 5$$

 $R = 0$

3. Divide by synthetic division. Write answers in the form f(x) = c(x)g(x) + r where f(x) is the given polynomial and c(x) is the given factor.

a)
$$4x^3 - 7x^2 - 11x + 5$$
; $x + 2$
b) $6x^3 - 16x^2 + 17x - 6$; $3x - 2$
 -2
 -8
 30
 -38
 -8
 4
 -15
 19
 -33
b) $6x^3 - 16x^2 + 17x - 6$; $3x - 2$
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$$4x^{3}-7x^{2}-11x+5=(x+z)(4x^{2}-15x+19)+(-33)$$

$$6x^{3} - 16x^{2} + 17x - 6 = (x - \frac{7}{3})(6x^{2} - 12x + 9)$$

$$4x^{3} - 7x^{2} - 11x + 5 = (x + 2)(4x^{2} - 16x + 19) + (-33)$$

$$6x^{3} - 16x^{2} + 17x - 6 = (x - \frac{7}{3})(6x^{2} - 12x + 9)$$

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$$6x^{3} - 16x^{2} + 17x - 6 = (x - \frac{7}{3})(6x^{2} - 12x + 9)$$

$$6x^{3} - 16x^{2} + 17x - 6$$

$$6x^$$

c)
$$x^3 - 64$$
; $x - 2$
2 1 +0 0 -64
2 4 8
1 2 4 -56

$$x^{3}-64 = (x-2)(x^{2}+2x+4) + (-56)$$

$$4x^3+16x^2-23x+15=(x-\frac{1}{2})(4x^2+18x-14)+8$$

$$4x^3+16x^2-23x+15=(2x-1)(2x^2+9x-7)+8$$

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$$\times -1+\sqrt{3} \rightarrow \times -(1-\sqrt{3})$$

e)
$$x^3 - 4x$$
; $x - 1 + \sqrt{3}$

$$x^{3}-4x = (x-1+13!)(x^{2}+(1-13!)x-213!)$$

$$+(6-213!)$$

$$+(6-213!)$$
(x-2-12!)(-3x²+(2-312!)x+(19-212!))

$$-3x^{3}+8x^{2}+10x-8=$$

$$(x-2-2)(-3x^{2}+(2-32)x+(8-42))$$

$$(1-\sqrt{3})(1-\sqrt{3})$$
 $1-2(\sqrt{3})+3$
 $4-2(\sqrt{3})$

$$(2+\sqrt{2})(2-3\sqrt{2}) \qquad (2+\sqrt{2})(8-4\sqrt{2})$$

$$4-6\sqrt{2}+2\sqrt{2}-3(2) \qquad 16-8\sqrt{2}+8\sqrt{2}-4(2)$$

$$4-4\sqrt{2}-6 \Rightarrow -2-4\sqrt{2} \qquad 16-8$$

$$8$$

$$h) x^5-2x^4+x^3-5; x-2$$

g)
$$x^4 - 4x^3 - 15x^2 + 58x - 40$$
; $x - 5$ h) $x^4 - 4x^3 - 15x^2 + 58x - 40$
 $5 - 5 - 50 - 40$
 $1 - 10 - 8 - 0$

h)
$$x^5 - 2x^4 + x^3 - 5; x - 2$$

2 $1 - 2 + 1 + 0 + 0 - 5$
2 $0 - 2 + 4 + 8$

$$x^{4}-4x^{3}-15x^{2}+58x-40=$$
 $(x-5)(x^{3}+x^{2}-10x+8)$

$$x^{5} - 2x^{4} + x^{3} - 5 = (x-2)(x^{4} + x^{2} + 2x + 4) + 3$$

i)
$$x^4 + 6x^3 + 11x^2 + 6x$$
; $x^2 + 3x + 2$
 $(x+2)(x+1)$

-2 [1 6 |1 6 0

-2 -8 -6

-1 | 1 4 3 0

-1 -3

1 3 0

$$x^{4}+6x^{3}+11x^{2}+6x=(x+2)(x+1)(x^{2}+3x)$$

 $x^{4}+6x^{3}+11x+6x=(x+2)(x+1)(x+3)x$

j)
$$x^4 + 9x^3 - 5x^2 - 36x + 4$$
; $x^2 - 4$
 $(x+7)(x-2)$
 -2 | 1 | 9 | -5 | -36 | 4
 -2 | -14 | 38 | -4
2 | 1 | 7 | -19 | 2 | 0
2 | 18 | -2
1 | 9 | -1 | 0

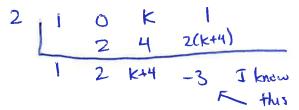
$$x^{4} + 9x^{3} - 5x^{2} - 36x + 4 = (x+2)(x-2)(x^{2}+9x-1)$$

$$x^{3n} + 16x^{2n} + 64x^{n} + 64 = (x^{n} + 4)(x^{2n} + 12x^{n} + 16)$$

1)
$$x^{3n} - 9x^{2n} + 27x^n - 27$$
; $x^n - 3$
3 | 1 - 9 + 27 - 27
3 -18 27
1 - 6 9 0

$$x^{2n} - 9x^{2n} + 27x^{n} - 27 = (x^{n} - 3)(x^{2n} - 6x^{n} + 9)$$

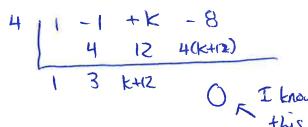
- 4. Use synthetic division to solve for k and m
 - a) When $x^3 + kx + 1$ is divided by x 2 the remainder is -3



$$1 + 2k + 8 = -3$$

 $2k + 9 = -3$
 $2k = -12$

b) When $x^3 - x^2 + kx - 8$ is divided by x - 4 the remainder is 0



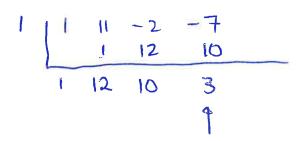
c) When $2x^4 + kx^2 - 3x + 5$ is divided by x - 2 the remainder is 3

$$4k = -28$$

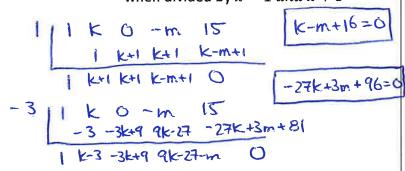
d) When $x^3 + kx + 6$ is divided by x + 2 the remainder is 4

e) When $x^3 + kx^2 - 2x - 7$ is divided by x + 1 the remainder is 5. What is the remainder when it is divided by x - 1?

$$-1$$
 | $K - 2$ -7 | $-1 - k + 1$ | $K + 1$ | $K - 6$



g) $x^4 + kx^3 - mx + 15$ has no remainder when divided by x - 1 and x + 3



both equal zero so equal to each other but need variable to concel, so: K-m+16=0

-3k+3m-48=0

$$-3k+3m-48=-27k+3m+96$$
 6-m+16=0

-m = -22

24k = 144

m = 22

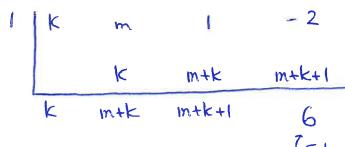
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See Website for Detailed Answer Key

k=6

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|c=6 m=22 f) When $kx^3 + mx^2 + x - 2$ is divided by x - 1, the remainder is 6. When the polynomial is divided by x + 2, the remainder is 12.



$$-2+m+k+1=6$$
 $m+k=7$ $-2/k$ m $1-2$

$$m+k=7$$
 $-2k=4$
 $-2k=4k-2m-8k+4m-3$
 $k=3$
 $k=2k+m-4k-2m+1/2$

this

$$m+k=7$$
 $m=6$ $m+1=7$ $m=6$ $m=16$ $m+1=7$ $m=16$ $m=16$ $m+1=7$ $m=16$ $m=16$

h) When $P(x) = 3x^4 + kx^2 + 7$ is divided by x - 1, the remainder is the same as when $f(x) = x^4 + kx - 4$ is divided by x - 2

$$2k+12=k+10$$
 $\rightarrow 2(-2)+12=R$
 $k=-2$ $-4+12=R$
 $8=R$

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