Section 3.3 – Practice Problems

1. Compute the quotient using long division. Write answer in two ways:

i) $Dividend = (quotient)(Divisor) + Remainder$

ii) $\dfrac{dividend}{divisor} = (quotient)\dfrac{remainder}{divisor}$

a) $x - 3 \overline{)\, x^3 - 8x^2 - 3x + 2}$

Quotient: $x^2 - 5x - 18$

$x^3 - 3x^2$

$-5x^2 - 3x$

$-5x^2 + 15x$

$-18x + 2$

$-18x + 54$

-52

i) $x^3 - 8x^2 - 3x + 2 = (x^2 - 5x - 18)(x - 3) - 52$

ii) $\dfrac{x^3 - 8x^2 - 3x + 2}{(x-3)} = x^2 - 5x - 18 - \dfrac{52}{(x-3)}$

b) $\dfrac{8x^3 - 1}{2x - 1}$

$2x - 1 \overline{)\, 8x^3 + 0x^2 + 0x - 1}$

Quotient: $4x^2 + 2x + 1$

$8x^3 - 4x^2$

$4x^2 + 0x$

$4x^2 - 2x$

$2x - 1$

$2x - 1$

0

i) $8x^3 - 1 = (4x^2 + 2x + 1)(2x - 1)$

ii) $\dfrac{8x^3 - 1}{2x - 1} = 4x^2 + 2x + 1$

c) $x^2 + 1 \overline{)\, x^5 + 2x^4 - x^3 + x^2 - 3x + 4}$

Quotient: $x^3 + 2x^2 - 2x - 1$

$x^5 + x^3$

$2x^4 - 2x^3 + x^2$

$2x^4 + 2x^2$

$-2x^3 - x^2 - 3x$

$-2x^3 - 2x$

$-x^2 - x + 4$

$-x^2 - 1$

$-x + 5$

i) $x^5 + 2x^4 - x^3 + x^2 - 3x + 4 = (x^2 + 1)(x^3 + 2x^2 - 2x - 1) + (-x + 5)$

ii) $\dfrac{x^5 + 2x^4 - x^3 + x^2 - 3x + 4}{(x^2 + 1)} = (x^3 + 2x^2 - 2x - 1) + \dfrac{(-x + 5)}{(x^2 + 1)}$

d) $\dfrac{x^4 - 3x^2 + 8}{x^2 - 1}$

$x^2 - 1 \overline{)\, x^4 + 0x^3 - 3x^2 + 0x + 8}$

Quotient: $x^2 - 2$

$x^4 - x^2$

$-2x^2 + 0x + 8$

$-2x^2 + 2$

6

i) $x^4 - 3x^2 + 8 = (x^2 - 1)(x^2 - 2) + 6$

ii) $\dfrac{x^4 - 3x^2 + 8}{x^2 - 1} = (x^2 - 2) + \dfrac{6}{x^2 - 1}$

e) $x^2 - 4x - 12 \overline{)\, x^3 + 2x^2 - 13x + 10}$

Quotient: $x + 6$

$$- \left(x^3 - 4x^2 - 12x\right) \downarrow$$
$$6x^2 - x + 10$$
$$6x^2 - 24x - 72$$
$$23x + 82$$

i) $x^3 + 2x^2 - 13x + 10 = (x+6)(x^2 - 4x - 12) + (23x + 82)$

ii) $\dfrac{x^3 + 2x^2 - 13x + 10}{x^2 - 4x - 12} = (x+6) + \dfrac{(23x+82)}{x^2 - 4x - 12}$

f) $\dfrac{x^3 - 5x + 1}{x^2 - 2x}$

Quotient: $x + 2$

$$x^2 - 2x \overline{)\, x^3 + 0x^2 - 5x + 1}$$
$$x^3 - 2x^2 \downarrow$$
$$2x^2 - 5x$$
$$2x^2 - 4x$$
$$-x + 1$$

i) $x^3 - 5x + 1 = (x^2 - 2x)(x+2) + (-x+1)$

ii) $\dfrac{x^3 - 5x + 1}{x^2 - 2x} = (x+2) - (x+1)$

g) $x^3 + 3x + 2 \overline{)\, x^4 + 6x^3 + 11x^2 + 6x + 0}$

Quotient: $x + 6$

$$x^4 + 3x^2 + 2x$$
$$6x^3 + 8x^2 + 4x$$
$$6x^3 + 18x + 12$$
$$8x^2 - 14x - 12$$

i) $x^4 + 6x^3 + 11x^2 + 6x = (x^3 + 3x + 2)(x+6) + (8x^2 - 14x - 12)$

ii) $\dfrac{x^4 + 6x^3 + 11x^2 + 6x}{(x^3 + 3x + 2)} = (x+6) + \dfrac{(8x^2 - 14x - 12)}{(x^3 + 3x + 2)}$

h) $\dfrac{x^4 + 9x^3 - 5x^2 - 32x + 3}{x^2 - 1}$

Quotient: $x^2 + 9x - 4$

$$x^2 - 1 \overline{)\, x^4 + 9x^3 - 5x^2 - 32x + 3}$$
$$x^4 - x^2 \downarrow$$
$$9x^3 - 4x^2 - 32x$$
$$9x^3 - 9x \downarrow$$
$$-4x^2 - 23x + 3$$
$$-4x^2 + 4$$
$$-23x - 1$$

i) $x^4 + 9x^3 - 5x^2 - 32x + 3 = (x^2 + 9x - 4)(x^2 - 1) + (-23x - 1)$

ii) $\dfrac{x^4 + 9x^3 - 5x^2 - 32x + 3}{x^2 - 1} = (x^2 + 9x - 4) + \dfrac{(-23x - 1)}{(x^2 - 1)}$

2. Use synthetic division to find the quotient $Q(x)$ and the remainder R when the polynomial $P(x)$ is divided by the given binomial.

a) $P(x) = x^3 + 2x^2 - 3x + 1; x - 2$

$$\begin{array}{r|rrrr} 2 & 1 & 2 & -3 & 1 \\ & & 2 & 8 & 10 \\ \hline & 1 & 4 & 5 & 11 \end{array}$$

$Q(x) = x^2 + 4x + 5$

$R = 11$

b) $P(x) = x^3 - a^3; x - a$

Remember a is a constant

$$\begin{array}{r|rrrr} a & 1 & +0 & +0 & -a^3 \\ & & a & a^2 & a^3 \\ \hline & 1 & a & a^2 & 0 \end{array}$$

$Q(x) = x^2 + ax + a^2$

$R = 0$

c) $P(x) = 4x^3 + 5x - 3; x + 2$

$$\begin{array}{r|rrrr} -2 & 4 & 0 & 5 & -3 \\ & & -8 & 16 & -42 \\ \hline & 4 & -8 & 21 & -45 \end{array}$$

$Q(x) = 4x^2 - 8x + 21$

$R = -45$

d) $P(x) = x^5 - 5x^3 + 10; x - 1$

$$\begin{array}{r|rrrrrr} 1 & 1 & +0 & -5 & +0 & +0 & 10 \\ & & 1 & 1 & -4 & -4 & -4 \\ \hline & 1 & 1 & -4 & -4 & -4 & 6 \end{array}$$

$Q(x) = x^4 + x^3 - 4x^2 - 4x - 4$

$R = 6$

e) $P(x) = 0.1x^2 + 0.2; x - 2.1$

$$\begin{array}{r|rrr} 2.1 & 0.1 & +0 & +0.2 \\ & & 0.21 & +0.441 \\ \hline & 0.1 & 0.21 & 0.641 \end{array}$$

$Q(x) = 0.1x + 0.21$

$R = 0.641$

f) $P(x) = x^5 + 1; x + 1$

$$\begin{array}{r|rrrrrr} -1 & 1 & +0 & +0 & +0 & +0 & +1 \\ & & -1 & 1 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & -1 & 1 & 0 \end{array}$$

$Q(x) = x^4 - x^3 + x^2 - x + 1$

$R = 0$

g) $P(x) = 3x^4 + x^3 - 3x + 1; 3x + 1$

$$\begin{array}{r|rrrrr} -\frac{1}{3} & 3 & 1 & 0 & -3 & 1 \\ & & -1 & 0 & 0 & +1 \\ \hline & 3 & 0 & 0 & -3 & 2 \end{array}$$

$Q(x) = 3x^3 - 3$

but since we had $3x+1$

and used $x = -\frac{1}{3}$

multiply divisor by 3

divide quotient by 3

$Q(x) = x^3 - 1$

$R = 2$

h) $P(x) = 2x^4 - x^3 + 2x - 1; 2x - 1$

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -1 & +0 & +2 & -1 \\ & & 1 & 0 & 0 & 1 \\ \hline & 2 & 0 & 0 & 2 & 0 \end{array}$$

$Q(x) = 2x^3 + 2$

but divide by 2

$Q(x) = x^3 + 1$

$R = 0$

i) $P(x) = 3x^5 + 2x^4 + 5x^3 - 7x + 3;$
$x + 0.8$

$$\begin{array}{r|rrrrrr} -0.8 & 3 & 2 & 5 & 0 & -7 & 3 \\ & & -2.4 & 0.32 & -4.256 & 3.4048 & 2.87616 \\ \hline & 3 & -0.4 & 5.32 & -4.256 & -3.5952 & 5.87616 \end{array}$$

$Q(x) = 3x^4 - 0.4x^3 + 5.32x^2 - 4.256x - 3.5952$

$R = 5.87616$

j) $P(x) = 3x^4 - 3x^3 + 2x^2 - 3x + 1;$
$x - 0.4$

$$\begin{array}{r|rrrrr} 0.4 & 3 & -3 & 2 & -3 & +1 \\ & & 1.2 & -0.72 & 0.512 & -0.9952 \\ \hline & 3 & -1.8 & 1.28 & -2.488 & 0.0048 \end{array}$$

$Q(x) = 3x^3 - 1.8x^2 + 1.28x - 2.488$

$R = 0.0048$

k) $P(x) = x^4 - 5x^3 - 4x^2 + 5x + 3;$
$(x+1)(x-1) \leftarrow x^2 - 1$

$$\begin{array}{r|rrrrr} 1 & 1 & -5 & -4 & +5 & +3 \\ & & 1 & -4 & -8 & -3 \\ \hline -1 & 1 & -4 & -8 & -3 & 0 \\ & & -1 & 5 & 3 & \\ \hline & 1 & -5 & -3 & 0 & \end{array}$$

$Q(x) = x^2 - 5x - 3$

$R = 0$

l) $P(x) = x^5 - x^4 - 8x^3 + 7x^2 + 7x - 30;$
$x^2 - x - 6$

$(x-3)(x+2)$

$$\begin{array}{r|rrrrrr} 3 & 1 & -1 & -8 & +7 & +7 & -30 \\ & & 3 & 6 & -6 & 3 & 30 \\ \hline -2 & 1 & 2 & -2 & 1 & 10 & 0 \\ & & -2 & 0 & 4 & -10 & \\ \hline & 1 & 0 & -2 & 5 & 0 & \end{array}$$

$Q(x) = x^3 - 2x + 5$

$R = 0$

3. Divide by synthetic division. Write answers in the form $f(x) = c(x)g(x) + r$ where $f(x)$ is the given polynomial and $c(x)$ is the given factor.

a) $4x^3 - 7x^2 - 11x + 5; x + 2$

-2	4	-7	-11	$+5$
		-8	30	-38
	4	-15	19	-33

$4x^3 - 7x^2 - 11x + 5 = (x+2)(4x^2 - 15x + 19) + (-33)$

b) $6x^3 - 16x^2 + 17x - 6; 3x - 2$

$2/3$	6	-16	$+17$	-6
		4	-8	6
	6	-12	9	0

$6x^3 - 16x^2 + 17x - 6 = (x - \frac{2}{3})(6x^2 - 12x + 9)$

factor 3 out of here into here

$6x^3 - 16x^2 + 17x - 6$
$= (3x-2)(2x^2 - 4x + 3)$

c) $x^3 - 64; x - 2$

2	1	$+0$	0	-64
		2	4	8
	1	2	4	-56

$x^3 - 64 = (x-2)(x^2 + 2x + 4) + (-56)$

d) $4x^3 + 16x^2 - 23x + 15; 2x - 1$

$\frac{1}{2}$	4	16	-23	$+15$
		2	9	-7
	4	18	-14	8

$4x^3 + 16x^2 - 23x + 15 = (x - \frac{1}{2})(4x^2 + 18x - 14) + 8$

$4x^3 + 16x^2 - 23x + 15 = (2x-1)(2x^2 + 9x - 7) + 8$

$x-1+\sqrt{3} \rightarrow x-(1-\sqrt{3})$

$x-(2+\sqrt{2})$

e) $x^3 - 4x; x - 1 + \sqrt{3}$

$1-\sqrt{3}$	1	0	-4	0
		$1-\sqrt{3}$	$4-2\sqrt{3}$	$-2\sqrt{3}+6$
	1	$1-\sqrt{3}$	$-2\sqrt{3}$	$6-2\sqrt{3}$

$x^3-4x = (x-1+\sqrt{3})(x^2+(1-\sqrt{3})x-2\sqrt{3}) \underbrace{+(6-2\sqrt{3})}_{\text{Remainder}}$

$(1-\sqrt{3})(1-\sqrt{3})$

$1-2\sqrt{3}+3$

$4-2\sqrt{3}$

f) $-3x^3 + 8x^2 + 10x - 8; x - 2 - \sqrt{2}$

$2+\sqrt{2}$	-3	8	10	-8
		$-6-3\sqrt{2}$	$-2-4\sqrt{2}$	8
	-3	$2-3\sqrt{2}$	$8-4\sqrt{2}$	0

$-3x^3+8x^2+10x-8=$

$(x-2-\sqrt{2})(-3x^2+(2-3\sqrt{2})x+(8-4\sqrt{2}))$

$(2+\sqrt{2})(2-3\sqrt{2})$

$4-6\sqrt{2}+2\sqrt{2}-3(2)$

$4-4\sqrt{2}-6 \rightarrow -2-4\sqrt{2}$

$(2+\sqrt{2})(8-4\sqrt{2})$

$16-8\sqrt{2}+8\sqrt{2}-4(2)$

$16-8$

8

g) $x^4 - 4x^3 - 15x^2 + 58x - 40; x - 5$

5	1	-4	-15	$+58$	-40
		5	5	-50	40
	1	1	-10	8	0

$x^4-4x^3-15x^2+58x-40=$

$(x-5)(x^3+x^2-10x+8)$

h) $x^5 - 2x^4 + x^3 - 5; x - 2$

2	1	-2	$+1$	$+0$	$+0$	-5
		2	0	2	4	8
	1	0	1	2	4	3

$x^5-2x^4+x^3-5 = (x-2)(x^4+x^2+2x+4)+3$

i) $x^4 + 6x^3 + 11x^2 + 6x;\ x^2 + 3x + 2$

$(x+2)(x+1)$

$$\begin{array}{r|rrrrr} -2 & 1 & 6 & 11 & 6 & 0 \\ & & -2 & -8 & -6 & \\ \hline -1 & 1 & 4 & 3 & 0 & \\ & & -1 & -3 & & \\ \hline & 1 & 3 & 0 & & \end{array}$$

$x^4+6x^3+11x^2+6x = (x+2)(x+1)(x^2+3x)$

$x^4+6x^3+11x+6x = (x+2)(x+1)(x+3)x$

j) $x^4 + 9x^3 - 5x^2 - 36x + 4;\ \ x^2 - 4$

$(x+2)(x-2)$

$$\begin{array}{r|rrrrr} -2 & 1 & 9 & -5 & -36 & 4 \\ & & -2 & -14 & 38 & -4 \\ \hline 2 & 1 & 7 & -19 & 2 & 0 \\ & & 2 & 18 & -2 & \\ \hline & 1 & 9 & -1 & 0 & \end{array}$$

$x^4+9x^3-5x^2-36x+4 = (x+2)(x-2)(x^2+9x-1)$

k) $x^{3n} + 16x^{2n} + 64x^n + 64;\quad x^n + 4$

$(n\ is\ a\ positive\ integer)$

$$\begin{array}{r|rrrr} -4 & 1 & 16 & 64 & 64 \\ & & -4 & -48 & -64 \\ \hline & 1 & 12 & 16 & 0 \end{array}$$

$x^{3n}+16x^{2n}+64x^n+64 = (x^n+4)(x^{2n}+12x^n+16)$

l) $x^{3n} - 9x^{2n} + 27x^n - 27;\quad x^n - 3$

$$\begin{array}{r|rrrr} 3 & 1 & -9 & +27 & -27 \\ & & 3 & -18 & 27 \\ \hline & 1 & -6 & 9 & 0 \end{array}$$

$x^{3n}-9x^{2n}+27x^n-27 = (x^n-3)(x^{2n}-6x^n+9)$

4. Use synthetic division to solve for k and m

a) When $x^3 + kx + 1$ is divided by $x - 2$ the remainder is -3

$$\begin{array}{r|rrrr} 2 & 1 & 0 & k & 1 \\ & & 2 & 4 & 2(k+4) \\ \hline & 1 & 2 & k+4 & -3 \end{array}$$

I know this

$1 + 2k + 8 = -3$

$2k + 9 = -3$

$2k = -12$

$\boxed{k = -6}$

b) When $x^3 - x^2 + kx - 8$ is divided by $x - 4$ the remainder is 0

$$\begin{array}{r|rrrr} 4 & 1 & -1 & +k & -8 \\ & & 4 & 12 & 4(k+12) \\ \hline & 1 & 3 & k+12 & 0 \end{array}$$

I know this

$-8 + 4k + 48 = 0$

$4k + 40 = 0$

$4k = -40$

$\boxed{k = -10}$

c) When $2x^4 + kx^2 - 3x + 5$ is divided by $x - 2$ the remainder is 3

$$\begin{array}{r|rrrrr} 2 & 2 & 0 & k & -3 & 5 \\ & & 4 & 8 & 2k+16 & 4k+26 \\ \hline & 2 & 4 & k+8 & 2k+13 & 3 \end{array}$$

I know this

$5 + 4k + 26 = 3$

$4k + 31 = 3$

$4k = -28$

$\boxed{k = -7}$

d) When $x^3 + kx + 6$ is divided by $x + 2$ the remainder is 4

$$\begin{array}{r|rrrr} -2 & 1 & 0 & k & 6 \\ & & -2 & 4 & -2k-8 \\ \hline & 1 & -2 & k+4 & 4 \end{array}$$

I know this

$6 + (-2k - 8) = 4$

$-2k - 2 = 4$

$-2k = 6$

$\boxed{k = -3}$

e) When $x^3 + kx^2 - 2x - 7$ is divided by $x + 1$ the remainder is 5. What is the remainder when it is divided by $x - 1$?

$$\begin{array}{r|rrrr} -1 & 1 & k & -2 & -7 \\ & & -1 & -k+1 & k+1 \\ \hline & 1 & k-1 & -k-1 & k-6 \end{array}$$

$k-6=5 \rightarrow k=11$

$$\begin{array}{r|rrrr} 1 & 1 & 11 & -2 & -7 \\ & & 1 & 12 & 10 \\ \hline & 1 & 12 & 10 & 3 \end{array}$$

$\boxed{R=3}$

f) When $kx^3 + mx^2 + x - 2$ is divided by $x - 1$, the remainder is 6. When the polynomial is divided by $x + 2$, the remainder is 12.

$$\begin{array}{r|rrrr} 1 & k & m & 1 & -2 \\ & & k & m+k & m+k+1 \\ \hline & k & m+k & m+k+1 & 6 \end{array}$$

I know this

$-2+m+k+1=6$

$$\begin{array}{r|rrrr} -2 & k & m & 1 & -2 \\ & & -2k & 4k-2m & -8k+4m-2 \\ \hline & k & -2k+m & 4k-2m+1 & 12 \end{array}$$

$-2+-8k+4m-2=12$

$-8k+4m=16$

$\boxed{-2k+m=4}$

solve system

$m+k=7$

$-\ m-2k=4$

$3k=3$

$\boxed{\begin{array}{l} k=1 \\ m=6 \end{array}}$

$m+k=7$

$m+1=7$

g) $x^4 + kx^3 - mx + 15$ has no remainder when divided by $x - 1$ *and* $x + 3$

$$\begin{array}{r|rrrrr} 1 & 1 & k & 0 & -m & 15 \\ & & 1 & k+1 & k+1 & k-m+1 \\ \hline & 1 & k+1 & k+1 & k-m+1 & 0 \end{array}$$

$\boxed{k-m+16=0}$

$$\begin{array}{r|rrrrr} -3 & 1 & k & 0 & -m & 15 \\ & & -3 & -3k+9 & 9k-27 & -27k+3m+81 \\ \hline & 1 & k-3 & -3k+9 & 9k-27-m & 0 \end{array}$$

$\boxed{-27k+3m+96=0}$

both equal zero so equal to each other but need variable to cancel, so: $k-m+16=0$

$\downarrow$

$-3k+3m-48=0$

$-3k+3m-48=-27k+3m+96$

$-3k-48=-27k+96$

$24k=144$

$k=6$

$6-m+16=0$

$-m=-22$

$m=22$

$\boxed{\begin{array}{l} k=6 \\ m=22 \end{array}}$

h) When $P(x) = 3x^4 + kx^2 + 7$ is divided by $x - 1$, the remainder is the same as when $f(x) = x^4 + kx - 4$ is divided by $x - 2$.

$$\begin{array}{r|rrrrr} 1 & 3 & 0 & k & 0 & 7 \\ & & 3 & 3 & k+3 & k+3 \\ \hline & 3 & 3 & k+3 & k+3 & R \end{array}$$

$k+10=R$

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 0 & k & -4 \\ & & 2 & 4 & 8 & 2k+16 \\ \hline & 1 & 2 & 4 & k+8 & R \end{array}$$

$2k+12=R$

$R=R$ so;

$2k+12=k+10$

$\boxed{k=-2}$

$2(-2)+12=R$

$-4+12=R$

$\boxed{8=R}$