

Section 3.3 – Adding and Subtracting Rational Expressions

This booklet belongs to: _____ Block: _____

- Adding and Subtracting Rational Expressions is exactly the same as with Rational Numbers
- You need to have a COMMON DENOMINATOR

Adding and Subtracting Expressions with Like Denominators

If $\frac{a}{b}$ and $\frac{c}{b}$ are rational numbers, then: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ and $\frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$, $b \neq 0$

Example 1: Add $\frac{x}{6} + \frac{2x}{6}$ **Solution 1:** $\frac{x}{6} + \frac{2x}{6} = \frac{3x}{6} = \frac{x}{2}$

Example 2: Subtract: $\frac{x}{x^2 - 1} - \frac{1}{x^2 - 1}$

Solution 2: $\frac{x}{x^2 - 1} - \frac{1}{x^2 - 1} = \frac{x - 1}{x^2 - 1} = \frac{\cancel{x-1}}{(\cancel{x-1})(x+1)} = \frac{1}{x+1}$

Adding and Subtracting Expressions with Unlike Denominators

- You need to determine the **Lowest Common Denominator (LCD)** (Multiple of the Denominators)
- Factor the denominators you have to identify what factors of the LCD are missing
- Multiply the numerator and denominator by the missing factor(s)
- Perform the addition and subtraction once the LCD have been established

Example 3: Subtract $\frac{7}{6x^2} - \frac{3}{8x^3}$

Solution 3: No need to factor. LCD is $24x^3$

$$\frac{7}{6x^2} - \frac{3}{8x^3} = \frac{7}{6x^2} \cdot \frac{4x}{4x} - \frac{3}{8x^3} \cdot \frac{3}{3}$$

$$\frac{28x}{24x^3} - \frac{9}{24x^3} = \frac{28x - 9}{24x^3}$$

Example 4: Subtract $\frac{1}{(x^2 - 1)} - \frac{2}{x^2 + x}$

Solution 4: LCD is: $x(x+1)(x-1)$

$$\frac{1}{(x-1)(x+1)} \cdot \frac{x}{x} - \frac{2}{x(x+1)} \cdot \frac{(x-1)}{(x-1)}$$

$$\frac{x - 2(x-1)}{x(x-1)(x+1)} = \frac{x - 2x + 2}{x(x+1)(x-1)} = \frac{-x + 2}{x(x+1)(x-1)}$$

Example 5: Add: $\frac{3x}{3x+6} + \frac{1}{x+2}$

Solution 5: Since $3x+6 = 3(x+2)$ the LCD is: $3(x+2)$

$$\frac{3x}{3x+6} + \frac{1}{x+2} = \frac{3x}{3(x+2)} + \frac{1}{x+2} = \frac{x+1}{x+2}$$

Example 6: Add: $\frac{7}{x-2} + \frac{4}{2-x}$

Solution 6: Since $(2-x)$ can be changed to $-(x-2)$, therefore the LCD is $(x-2)$

$$\frac{7}{x-2} + \frac{4}{2-x} = \frac{7}{x-2} - \frac{4}{x-2} = \frac{3}{x-2}$$

Example 7: Add: $\frac{3x+9}{x^2+7x+10} + \frac{14}{x^2+3x-10}$

Solution 7: Since $x^2+7x+10 = (x+2)(x+5)$ and $x^2+3x-10 = (x-2)(x+5)$, therefore the LCD is $(x-2)(x+2)(x+5)$

$$\frac{3x+9}{x^2+7x+10} + \frac{14}{x^2+3x-10} = \frac{3x+9}{(x+2)(x+5)} \cdot \frac{(x-2)}{(x-2)} + \frac{14}{(x-2)(x+5)} \cdot \frac{(x+2)}{(x+2)}$$

$$\frac{(3x+9)(x-2) + 14(x+2)}{(x+2)(x+5)(x-2)} = \frac{3x^2 + 9x - 6x - 18 + 14x + 28}{(x+2)(x+5)(x-2)} = \frac{3x^2 + 17x + 10}{(x+2)(x+5)(x-2)}$$

$$\frac{(3x+2)\cancel{(x+5)}}{(x-2)(x+2)\cancel{(x+5)}} = \frac{3x+2}{(x+2)(x-2)}$$

Section 3.3 – Practice Problems

Add or Subtract, Simplify if possible and assume the denominators do not equal zero

1.
$$\frac{3x}{x+1} + \frac{3}{x+1}$$

2.
$$\frac{6y}{y-2} - \frac{12}{y-2}$$

3.
$$\frac{14x}{2x+3y} + \frac{21y}{2x+3y}$$

4.
$$\frac{z-2}{5z+3} - \frac{6z-5}{5z+3}$$

5.
$$\frac{9x-1}{3x-5} - \frac{3x+9}{3x-5}$$

6.
$$\frac{-15y}{1-5y} - \frac{3}{5y-1}$$

7. $\frac{2z}{3z-1} + \frac{z}{1-3z}$

8. $\frac{8x}{6x-5} + \frac{10-4x}{5-6x}$

9. $\frac{3y-2}{y^2-25} - \frac{4y-7}{y^2-25}$

10. $\frac{2z-5}{z^2-9} - \frac{3z-8}{z^2-9}$

11. $\frac{x}{x-y} - \frac{y}{x-y}$

12. $\frac{x^2-8x}{x-5} - \frac{15}{5-x}$

$$13. \frac{x^2}{x^2 - y^2} - \frac{y^2}{y^2 - x^2} + \frac{2xy}{y^2 - x^2}$$

$$14. \frac{x^2}{x^2 - y^2} + \frac{y^2}{x^2 - y^2} + \frac{2xy}{y^2 - x^2}$$

$$15. \frac{x + a}{x(a + b) + y(a + b)} - \frac{x - b}{x(a + b) + y(a + b)}$$

$$16. \frac{2x^2 + 1}{2x^2 - 5x - 12} - \frac{4 - x}{2x^2 - 5x - 12}$$

Add or Subtract. Simplify if possible, and assume non-zero denominators

17. $\frac{5}{6x^2} + \frac{4}{3x}$

18. $\frac{5}{3x^2y^3} - \frac{1}{6xy^4}$

19. $\frac{4z}{z^2 - 36} - \frac{2}{z - 6}$

20. $\frac{3x}{x^2 - 49} - \frac{3}{2x - 14}$

21. $\frac{y}{y^2 - 9} + \frac{3}{3 - y}$

22. $\frac{4z}{z - 1} - 4$

23. $\frac{x + 1}{x^2 - x - 6} - \frac{2}{x - 3}$

24. $\frac{4y}{y^2 - 5y} - \frac{3}{2y - 10}$

$$25. \frac{1}{z-5} - \frac{z}{z^2 - z - 20}$$

$$26. \frac{4}{x^2 - 4} + \frac{1}{2 - x} - \frac{1}{x + 2}$$

$$27. \frac{2}{y+5} + \frac{5y}{y^2 - 25} + \frac{4}{5-y}$$

$$28. \frac{z+2}{z^2 + z - 2} + \frac{3}{z^2 - 1}$$

$$29. \frac{3x + 3}{x^2 + 5x + 4} - \frac{x - 3}{x^2 + x - 12}$$

$$30. \frac{x - 1}{2x^2 + 3x + 1} - \frac{x + 1}{2x^2 - x - 1}$$

$$31. \frac{x}{2x^2 + x - 1} + \frac{3}{3x^2 + 2x - 1}$$

$$32. \frac{5}{2x^3} - \frac{3x - 9}{x^2 - 6x + 9} + \frac{12x}{4x^2 - 12x}$$

33.
$$\frac{y - 5}{(x^2 + 5x) + (xy + 5y)} + \frac{1}{x+y} - \frac{2}{x+5}$$

34.
$$\frac{2z + 11}{z^2 + z - 6} - \frac{2}{z + 3} + \frac{3}{2 - z}$$

35. The length of a rectangle is $\frac{2}{(x-4)}$ and the width is $\frac{3}{x}$. Find the perimeter of the rectangle.

36. The sum of a rational expression and $\frac{1}{(x+3)}$ is $\frac{3x}{2x^2 + 5x - 3}$. Determine the rational expression.

Answer Key – Section 3.3

1. 3
2. 6
3. 7
4. $\frac{(-5z+3)}{5z+3}$
5. 2
6. 3
7. $\frac{z}{(3z-1)}$
8. 2
9. $-\frac{1}{(y+5)}$
10. $-\frac{1}{(z+3)}$
11. 1
12. $(x-3)$
13. $\frac{(x-y)}{(x+y)}$
14. $\frac{(x-y)}{(x+y)}$
15. $\frac{1}{(x+y)}$
16. $\frac{(x-1)}{(x-4)}$
17. $\frac{8x+5}{6x^2}$
18. $\frac{10y-x}{6x^2y^4}$

19. $\frac{2}{(z+6)}$
20. $\frac{3}{2(x+7)}$
21. $\frac{(-2y-9)}{(y-3)(y+3)}$
22. $\frac{4}{(z-1)}$
23. $\frac{(-x-3)}{(x-3)(x+2)}$
24. $\frac{5}{2(y-5)}$
25. $\frac{4}{(z-5)(z+4)}$
26. $\frac{-2}{(x+2)}$
27. $\frac{3(y-10)}{(y-5)(y+5)}$
28. $\frac{(z+4)}{(z-1)(z+1)}$
29. $\frac{2}{(x+4)}$
30. $\frac{-4x}{(2x+1)(x-1)(x+1)}$
31. $\frac{3x^2+5x-3}{(2x-1)(3x-1)(x+1)}$
32. $\frac{5}{2x^3}$
33. $-\frac{1}{(x+5)}$
34. $-\frac{3}{(z+3)}$
35. $\frac{2(5x-12)}{x(x-4)}$
36. $\frac{(x+1)}{(x+3)(2x-1)}$

Extra Work Space