Section 3.2 – Special Cases of Linear Equations

This booklet belongs to: __________________________ Block: ______

Horizontal Lines

- A horizontal line can be thought of as **all the points** on the graph where \( y \) has the same value
- The **slope** of a horizontal line **is 0** (The rise is 0).
- Using a slope of 0, the slope intercept equation of a line is:

\[
y = mx + b \quad \rightarrow \quad y = m(0) + b \quad \rightarrow \quad y = b
\]

**Equation of a Horizontal Line with \( y \)-intercept \((0, b)\)**

\[
y = b
\]

Vertical Lines

- A vertical line can be thought of as **all the points** on the graph where \( x \) has the same value
- The **slope** of a vertical line **is undefined** (The run is 0).
- The equation of a vertical line is \( x = a \) by definition, since the slope is undefined.

**Equation of a Vertical Line with \( x \)-intercept \((a, 0)\)**

\[
x = a
\]

Example of Horizontal Line \( y = 3 \)  
Example of Vertical Line \( x = 3 \)
Writing the Equation of a Line Through Two Points

- When two points are given, we now have the ability to write the equation of a line.

**Example:** Write the equation of a line passing through $A(5, 2)$ and $B(1, -4)$ in slope – intercept form.

**Solution:**

- First find the slope of the line. $m = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-4)}{5-1} = \frac{6}{4} = \frac{3}{2}$

- Now pick either point and substitute it into the point – slope equation.

  \[ y - y_1 = m(x - x_1) \]
  \[ y - 2 = \frac{3}{2}(x - 5) \]
  \[ y - 2 = \frac{3}{2}x - \frac{15}{2} \]
  \[ y = \frac{3}{2}x - \frac{15}{2} + 2 \]
  \[ y = \frac{3}{2}x - \frac{11}{2} \]

- You get the same answer if you use the other point

  \[ y - y_1 = m(x - x_1) \]
  \[ y - (-4) = \frac{3}{2}(x - 1) \]
  \[ y + 4 = \frac{3}{2}x - \frac{3}{2} \]
  \[ y = \frac{3}{2}x - \frac{3}{2} - 4 \]
  \[ y = \frac{3}{2}x - \frac{11}{2} \]

- So find the Slope first
- Then substitute in either one of the points for $(x_1, y_1)$
- Use algebra to get to slope – intercept form
- You can then continue the algebra to get to general form
Parallel and Perpendicular Lines

- Parallel lines have the **same slopes** but different **y – intercepts**
- Perpendicular lines have slopes that are **negative reciprocals** of each other
- Knowing this we can determine if equations are **parallel, perpendicular of neither**

**Example:** In the following system of equations, determine if the lines are parallel, perpendicular or neither.

\[
\begin{align*}
 x + 2y &= 6 \\
-2x + y &= 3
\end{align*}
\]

**Solution:**

- The short cut is remembering the slope of the **Standard Form** of a line,
  \[
  Ax + By = C, \quad \text{is:} \quad -\frac{A}{B}
  \]

\[
\begin{align*}
 x + 2y &= 6 \quad \text{has a slope of} \quad -\frac{1}{2} \\
-2x + y &= 3 \quad \text{has a slope of} \quad 2
\end{align*}
\]

The **slopes** are **negative reciprocals** of each other, so the lines are **perpendicular**.

- I don’t like relying on things to remember so it is important to be able to manipulate the equations using algebra to go from **Standard Form to Slope – Intercept Form**

**Example:** In the following system of equations, determine if the lines are parallel, perpendicular, or neither.

\[
\begin{align*}
 3x - y &= 5 \\
-6x + 2y &= 12
\end{align*}
\]

**Solution:**

- Put both equations into **Slope – intercept Form**

\[
\begin{align*}
 3x - y &= 5 \quad \rightarrow \quad -y &= -3x + 5 \quad \rightarrow \quad y &= 3x - 5 \quad \rightarrow \quad m = 3 \\
-6x + 2y &= 12 \quad \rightarrow \quad 2y &= 6x + 12 \quad \rightarrow \quad y &= 3x + 6 \quad \rightarrow \quad m = 3
\end{align*}
\]

The **slopes** are **equal**, so the lines are **parallel**.
Example: In the following system of equations, determine if the lines are parallel, perpendicular, or neither.

\[
\begin{align*}
4x + 3y &= 7 \\
2x - 4y &= 4
\end{align*}
\]

Solution:

- Using the **Standard Form** shortcut (Slope is \(-\frac{A}{B}\)):
  \[
  \begin{align*}
  4x + 3y &= 7 & \text{has a slope of } & -\frac{4}{3} \\
  2x - y &= 4 & \text{has a slope of } & 2
  \end{align*}
  \]

- Changing the system of equations to **Slope-intercept form**:
  \[
  \begin{align*}
  4x + 3y &= 7 & \rightarrow & 3y = -4x + 7 & \rightarrow & y = -\frac{4}{3}x + \frac{7}{3} & \rightarrow & m = -\frac{4}{3} \\
  2x - y &= 4 & \rightarrow & -y = -2x + 4 & \rightarrow & y = 2x - 4 & \rightarrow & m = 2
  \end{align*}
  \]

Both methods produce the same result.

The slopes aren’t the same, or negative reciprocals of one another, so the lines are neither parallel nor perpendicular.
Section 3.2 – Practice Problems

Determine the equation of the graph and explain why.

1. 
   \[ \text{Equation:} \]
   \[ \text{Why:} \]

2. 
   \[ \text{Equation:} \]
   \[ \text{Why:} \]

Determine the equation of a line through the given pair of points, in \textbf{Point – Slope Form}.

3. \((-4, 1)\) and \((6, 1)\)

4. \((1, -4)\) and \((1, 6)\)

5. \((-2, 0)\) and \((5, 0)\)

6. \((0, -2)\) and \((0, 5)\)

7. \((a, b)\) and \((c, b)\)

8. \((b, a)\) and \((b, c)\)
Write the equation of the line with the given information

9. *vertical, passes through* \((3, 6)\)  
10. *vertical, passes through* \((-2, -4)\)

11. *horizontal, passes through* \((3, 6)\)  
12. *horizontal, passes through* \((-2, -4)\)

For each pair of equations, determine whether they are parallel, perpendicular, or neither

13. \(2x + 5y = 7\)  \(\text{and}\)  \(4x + 10y = 2\)  
14. \(-4x + 3y = 7\)  \(\text{and}\)  \(-8x + 6y = 0\)

15. \(4x - 3y = 6\)  \(\text{and}\)  \(4x + 6y = -3\)  
16. \(3x - 5y = 4\)  \(\text{and}\)  \(5x - 3y = 4\)
17. $4x - 3y = 5$ \text{ and } $3x + 4y = 2$

18. $2x - 5y = -3$ \text{ and } $10x + 4y = 1$

19. $4x - y = 3$ \text{ and } $x - 4y = -2$

20. $5x - 2y = 7$ \text{ and } $2x + 5y = 7$

Write the equation of a line passing through the given set of points in \textit{slope – intercept form}

21. $(3, 5)$ and $(2, 4)$

22. $(5, -2)$ and $(-3, 1)$
23. \((-4, 1)\) and \((-2, -3)\)

24. \((-1, -2)\) and \((-6, -4)\)

25. \((6, -2)\) and \((-3, 2)\)

26. \((0, 0)\) and \((-3, 2)\)

Using the information you have learned, use reasoning to answer the following questions:

27. If a line is horizontal, what is the slope of any line perpendicular to it?

28. If the graph of a linear equation has one point that is both the \(x\) \(-\) intercept and \(y\) \(-\) intercept, where is that point?

29. What is the equation of the \(x\) \(-\) axis?

30. What is the equation of the \(y\) \(-\) axis?

31. What is the \(x\) \(-\) intercept of the line \(ax + by = c\)?

32. What is the slope of the line \(ax + by = c\)?
Answer Key

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1. \( y = 3 \)
2. \( x = -5 \)
3. \( y = 1 \)
4. \( x = 1 \)
5. \( y = 0 \)
6. \( x = 0 \)
7. \( y = b \)
8. \( x = b \)
9. \( x = 3 \)
10. \( x = -2 \)
11. \( y = 6 \)
12. \( y = -4 \)
13. Parallel
14. Parallel
15. Neither
16. Neither
17. Perpendicular
18. Perpendicular
19. Neither
20. Perpendicular
21. \( y = x + 2 \)
22. \( y = -\frac{3}{8}x - \frac{1}{8} \)
23. \( y = -2x - 7 \)
24. \( y = \frac{2}{5}x + \frac{8}{5} \)
25. \( y = -\frac{4}{9}x + \frac{2}{3} \)
26. \( y = -\frac{2}{3}x \)
27. Undefined
28. \((0, 0)\)
29. \( y = 0 \)
30. \( x = 0 \)
31. \( x = \frac{c}{a} \)
32. \( -\frac{a}{b} \)
Extra Work Space