

Section 3.2 – Multiplying and Dividing Rational Expressions

This booklet belongs to: _____ Block: _____

Multiplying Rational Expressions

- The product of **two rational expressions** is achieved exactly like the **product of two rational numbers (fractions)**
- **Numerators with Numerators** and **Denominators with Denominators**
- But just like with fractions

Simplify before you Multiply

Multiplying Rational Expressions

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational expressions, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

Example 1: Multiply $\frac{3}{10} \cdot \frac{5}{9}$

Solution 1:

$$\frac{3}{10} \cdot \frac{5}{9} = \frac{3 \cdot 5}{10 \cdot 9}$$

$$\frac{\cancel{3} \cdot \cancel{5}}{2 \cdot \cancel{5} \cdot \cancel{3} \cdot 3} = \frac{1}{6}$$

Example 2: Multiply $\frac{x-y}{2x} \cdot \frac{x^2}{(x-y)^2}$

Solution 2:

$$\frac{x-y}{2x} \cdot \frac{x^2}{(x-y)^2} = \frac{(x-y)x^2}{2x(x-y)^2}$$

$$= \frac{(x-y)x \cdot x}{2x(x-y)(x-y)}$$

$$= \frac{\cancel{(x-y)}x \cdot x}{2x(x-y)\cancel{(x-y)}} = \frac{x}{2(x-y)}$$

Example 3: Multiply $\frac{x^2 - 25}{x^2 - 4} \cdot \frac{4x - 8}{3x - 15}$

Solution 3: $\frac{x^2 - 25}{x^2 - 4} \cdot \frac{4x - 8}{3x - 15} = \frac{(\cancel{x-5})(x+5)(4)(\cancel{x-2})}{(x+2)(\cancel{x-2})(3)(\cancel{x-5})} = \frac{4(x+5)}{3(x+2)}$

Example 4: Multiply $\frac{x^2 - 3x}{x^2 - 3x - 4} \cdot \frac{x^2 - 5x + 4}{x^2 - 2x - 3}$

Solution 4:

$$\begin{aligned} \frac{x^2 - 3x}{x^2 - 3x - 4} \cdot \frac{x^2 - 5x + 4}{x^2 - 2x - 3} &= \frac{(x^2 - 3x)(x^2 - 5x + 4)}{(x^2 - 3x - 4)(x^2 - 2x - 3)} \\ &= \frac{x(\cancel{x-3})(\cancel{x-4})(x-1)}{(\cancel{x-4})(x+1)(\cancel{x-3})(x+1)} \\ &= \frac{x(x-1)}{(x+1)^2} \end{aligned}$$

Factor the
Quadratics

Cancel out
Common Factors

Example 5: Multiply $\frac{x^2 - 3x}{x^2 - x - 6} \cdot \frac{x^2 + x - 2}{x - x^2}$

Solution 5:

$$\begin{aligned} \frac{x^2 - 3x}{x^2 - x - 6} \cdot \frac{x^2 + x - 2}{x - x^2} &= \frac{(x^2 - 3x)(x^2 + x - 2)}{(x^2 - x - 6)(x - x^2)} \\ &= \frac{x(\cancel{x-3})(\cancel{x-1})(x+2)}{(\cancel{x-3})(x+2)(-x)(\cancel{-1+x})} \\ &= \frac{x}{-x} = -1 \end{aligned}$$

Factor the
Quadratics

Cancel out
Common Factors

Example 6: Multiply $(x - 7) \cdot \frac{x^2 - x}{x^2 - 8x + 7}$

Solution 6:

$$\begin{aligned} (x - 7) \cdot \frac{x^2 - x}{x^2 - 8x + 7} &= \frac{(x - 7)(x^2 - x)}{(x^2 - 8x + 7)} \\ &= \frac{(\cancel{x-7})(x)(\cancel{x-1})}{(\cancel{x-7})(\cancel{x-1})} \\ &= x \end{aligned}$$

Example 7: Multiply $\frac{6x^2 + xy - 2y^2}{4x^2 - 8xy + 3y^2} \cdot \frac{x - y}{3x + 2y} \cdot \frac{8x - 12y}{2y - 2x}$

Solution 7:

$$\begin{aligned} \frac{6x^2 + xy - 2y^2}{4x^2 - 8xy + 3y^2} \cdot \frac{x - y}{3x + 2y} \cdot \frac{8x - 12y}{2y - 2x} &= \frac{(6x^2 + xy - 2y^2)(x - y)(8x - 12y)}{(4x^2 - 8xy + 3y^2)(3x + 2y)(2y - 2x)} \\ &= \frac{(3x - 2y)(2x - y)(x - y)(4)(2x - 3y)}{(2x - 3y)(2x - y)(3x + 2y)(-2)(x - y)} \\ &= \frac{4}{-2} = -2 \end{aligned}$$

Dividing Rational Expressions

- The quotient of **two rational expressions** is achieved exactly like the **quotient of two rational numbers (fractions)**
- But just like with fractions

Flip the Second Fraction and then Multiply

Dividing Rational Expressions

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational expressions, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

Example 8: Divide $\frac{x - 2}{x + 3} \div \frac{x^2 + x - 2}{x^2 - 4}$

Solution 8:

$$\begin{aligned} \frac{x - 2}{x + 3} \div \frac{x^2 + x - 2}{x^2 - 4} &= \frac{(x - 2)(x^2 - 4)}{(x + 3)(x^2 + x - 2)} \\ &= \frac{(x - 2)(x - 2)(x + 2)}{(x + 3)(x + 2)(x - 1)} \\ &= \frac{(x - 2)^2}{(x + 3)(x - 1)} \end{aligned}$$

Section 3.2 – Practice Problems

Multiply the following, assume non-zero denominators. Simplify the final answer.

1.
$$\frac{15x^2}{18} \cdot \frac{9}{5x}$$

2.
$$\frac{8(x-2)}{y} \cdot \frac{3y}{6(x-2)^2}$$

3.
$$\frac{3}{2x-6} \cdot \frac{x-3}{6}$$

4.
$$\frac{8x-24}{3x+9} \cdot \frac{4x+12}{6x-18}$$

5.
$$\frac{x-2}{8} \cdot \frac{6}{2-x}$$

6.
$$\frac{24-3x}{4} \cdot \frac{12}{2x-16}$$

7.
$$\frac{x - 2y}{6} \cdot \frac{3}{2y - x}$$

8.
$$\frac{15}{3x - 2y} \cdot \frac{2y - 3x}{12}$$

9.
$$\frac{3(x^2 - 4)}{28(x - 2)} \cdot \frac{14x}{11(x + 2)}$$

10.
$$\frac{x^2 - x - 2}{x + 3} \cdot \frac{3x + 9}{2x + 2}$$

11.
$$\frac{x^2 - 4x - 5}{x^2 - 7x + 10} \cdot \frac{x - 4}{x + 1}$$

12.
$$\frac{y^2 + 3y + 2}{y^2 - 4y + 3} \cdot \frac{y - 1}{y + 1}$$

$$13. \frac{2z^2 - z - 1}{2z^2 + 5z + 3} \cdot \frac{2z^2 + z - 3}{4z^2 - 1}$$

$$14. \frac{4x^2 - x}{6x^2 + 10x} \cdot \frac{3x^2 + 11x + 10}{8x^2 + 2x - 1}$$

$$15. \frac{y^2 + 4y - 5}{4y^2 - 9} \cdot \frac{2y^2 - 5y - 12}{y^2 + y - 20}$$

$$16. \frac{z^2 - 9z + 18}{4z^2 - 9} \cdot \frac{2z^2 - 5z - 12}{z^2 - 10z + 24}$$

Divide the following, assume non-zero denominators. Simplify the final answer.

17. $\frac{5x^2}{21} \div \frac{15x^6}{14}$

18. $\frac{12}{2x - 3y} \div \frac{15}{3y - 2x}$

19. $\frac{x^2 - x}{x} \div (3x - 3)$

20. $\frac{5y - 10}{y + 2} \div \frac{3y - 6}{2}$

21. $\frac{2z - 2}{2z - 4} \div (z^2 - z)$

22. $\frac{y^2 + 3y}{9} \div \frac{y + 3}{3y}$

23. $\frac{6z}{z - 2} \div \frac{3z^2}{(z - 2)^2}$

24. $\frac{3x^2 - 20x - 7}{x^2 - 2x - 35} \div (3x^2 - 14x - 5)$

$$25. \frac{4a^2 - ab - 5b^2}{ax + by + ay + bx} \div (4a - 8b)$$

$$26. \frac{y^2 - 10y + 9}{y^2 - 1} \div \frac{y^2 - 5y - 36}{1 - y^2}$$

$$27. \frac{z^2 - 6z + 9}{12 - 4z} \div \frac{z^3 - 3z^2}{z^6 - 9z^4}$$

$$28. \frac{x^3 - 4x}{x - x^4} \div \frac{4x - x^3}{x^4 - x}$$

29.
$$\frac{4y - 4x}{8y^3} \div \frac{x^2 - y^2}{2x + 2y}$$

30.
$$\frac{x^2 - y^2}{3x^2 + 3xy} \div \frac{3x^2 - 2xy - y^2}{3x^2 + 6x}$$

31.
$$\frac{3x + 4y}{x^2 + 4xy + 4y^2} \div \frac{2}{x + 2y}$$

32.
$$\frac{x^2 - 4}{2y} \div \frac{2 - x}{6xy}$$

Answer Key – Section 3.2

1. $\frac{3x}{2}$
2. $\frac{4}{(x-2)}$
3. $\frac{1}{4}$
4. $\frac{16}{9}$
5. $-\frac{3}{4}$
6. $-\frac{9}{2}$
7. $-\frac{1}{2}$
8. $-\frac{5}{4}$
9. $\frac{3x}{22}$
10. $\frac{3(x-2)}{2}$
11. $\frac{(x-4)}{(x-2)}$
12. $\frac{(y+2)}{(y-3)}$
13. $\frac{(z-1)^2}{(z+1)(2z-1)}$
14. $\frac{(x+2)}{2(2x+1)}$
15. $\frac{(y-1)}{(2y-3)}$
16. $\frac{(z-3)}{(2z-3)}$

17. $\frac{2}{9x^4}$
18. $-\frac{4}{5}$
19. $\frac{1}{3}$
20. $\frac{10}{3(y+2)}$
21. $\frac{1}{z(z-2)}$
22. $\frac{y^2}{3}$
23. $\frac{2(z-2)}{z}$
24. $\frac{1}{(x+5)(x-5)}$
25. $\frac{4a-5b}{4(a-2b)(x+y)}$
26. $-\frac{(y-1)}{(y+4)}$
27. $-\frac{z^2(z-3)(z+3)}{4}$
28. 1
29. $-\frac{1}{y^3}$
30. $\frac{(x+2)}{(3x+y)}$
31. $\frac{(3x+4y)}{2(x+2y)}$
32. $-3x(x+2)$

Extra Work Space