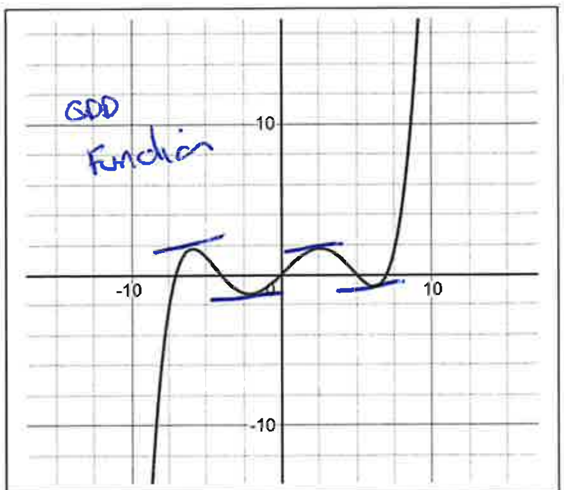
Section 3.2 – Practice Problems

1. What is the minimum degree and number of turning points of the following:

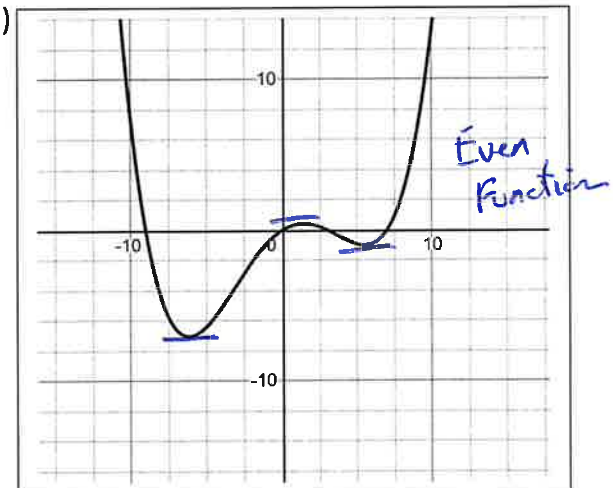
a)



Min Degree: 5

Turning Points: 4

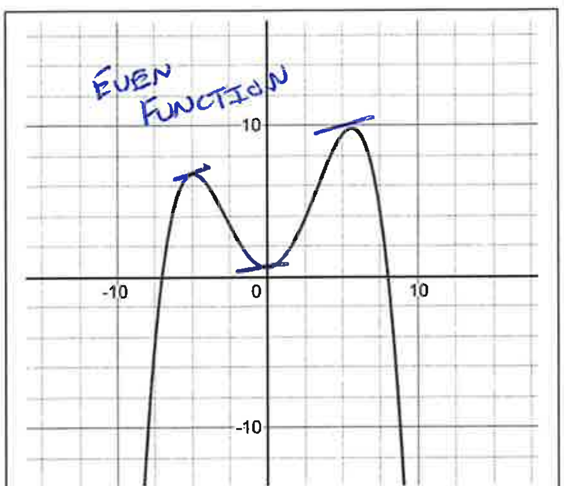
b)



Min Degree: 4

Turning Points: 3

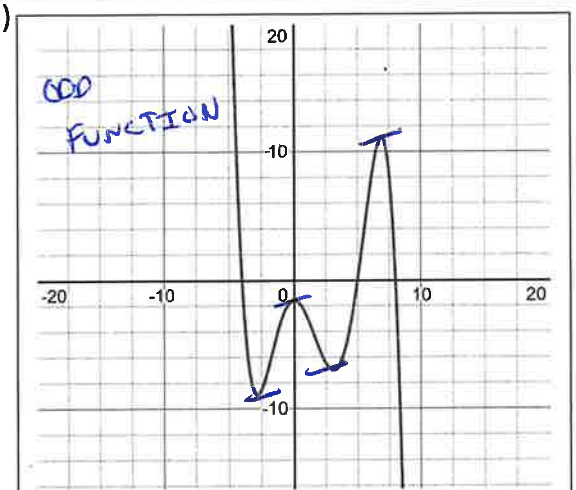
c)



Min Degree: 4

Turning Points: 3

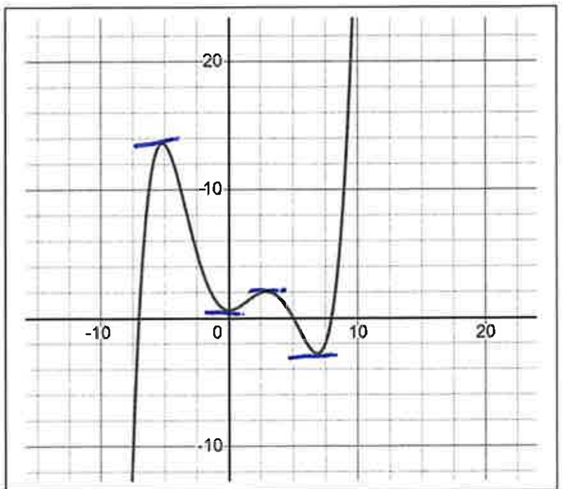
d)



Min Degree: 5

Turning Points: 4

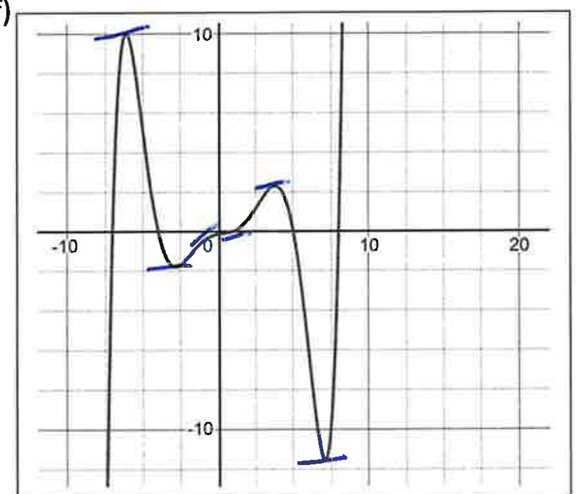
e)



Min Degree: 5

Turning Points: 4

f)



Min Degree: 7

Turning Points: 6

2. Using the table below, what is the minimum number of zeros possible for the polynomial function. Discuss at which integer or between which two integers, does the zero (root) occur.

a)

x	-5	-4	-3	-2	-1	0	1	2	3	4
$P(x)$	-25	-8	3	0	-6	-24	-33	-32	-15	5

y goes neg → pos

y goes pos → neg

y goes neg → pos

min zeros: 3 occurs between −4, −3 ; at −2 ; between 3 and 4

b)

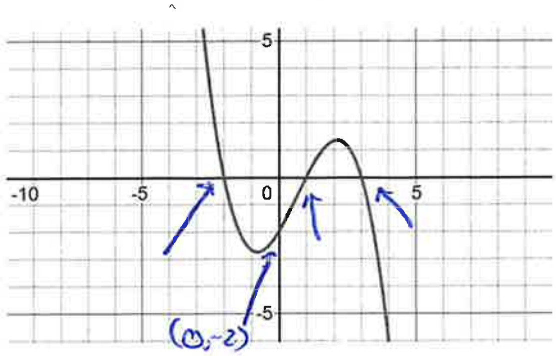
x	-3	-2	-1	0	1	2	3
$P(x)$	-75	-1	3	-6	-15	0	63

min zeros: 3

occurs: between −2 and −1
between −1 and 0
at 2

3. Determine the equation, in factored form, of a polynomial that fits the following graphs.

a)



$y = a(x+2)(x-1)(x-3)$

use a point (x,y) to solve for a

$-2 = a(0+2)(0-1)(0-3)$

$-2 = a(2)(-1)(-3)$

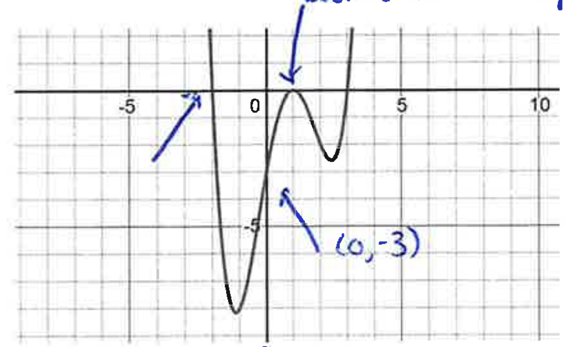
$-2 = a(6)$

$a = -\frac{2}{6} = -\frac{1}{3}$

$\boxed{y = -\frac{1}{3}(x+2)(x-1)(x-3)}$

b)

bounces off so multiplicity 2



$y = a(x+2)(x-1)^2(x-3)$

$-3 = a(0+2)(0-1)^2(0-3)$

$-3 = a(2)(-1)^2(-3)$

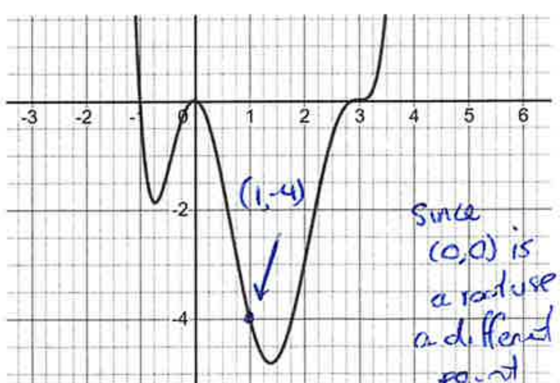
$-3 = a(-6)$

$\frac{-3}{-6} = a$

$a = \frac{1}{2}$

$\boxed{y = \frac{1}{2}(x+2)(x-1)^2(x-3)}$

c)



$y = a(x+1)x^2(x-3)^3$

$-4 = a(1+1)(1)^2(1-3)^3$

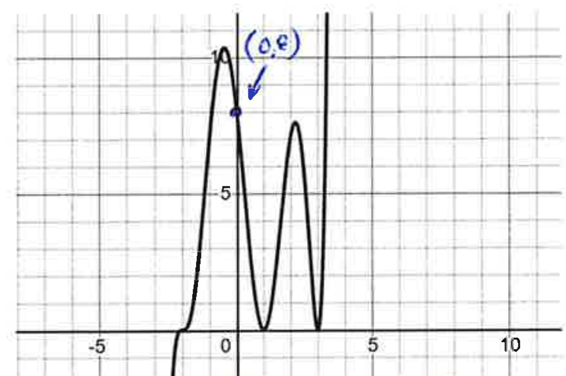
$-4 = a(2)(1)(-8)$

$-4 = -16a$

$a = \frac{4}{16} = \frac{1}{4}$

$\boxed{y = \frac{1}{4}x^2(x-3)^3(x+1)}$

d)



$y = a(x+2)^3(x-1)^2(x-3)^2$

$y = a(0+2)^3(0-1)^2(0-3)^2$

$8 = a(8)(1)(9)$

$8 = 72a$

$a = \frac{8}{72} = \frac{1}{9}$

$\boxed{y = \frac{1}{9}(x+2)^3(x-1)^2(x-3)^2}$

4. Find all values of x where the outputs, $f(x)$ are ≥ 0.

a) $f(x) = \underbrace{-\frac{1}{3}}_{\text{need to be } \geq 0} x^3$

$-\frac{1}{3}x^3 \geq 0 \rightarrow x^3 \leq 0$

multiplied by neg. so flip

$\boxed{x \leq 0}$

b) $f(x) = \frac{1}{2}x^3 + 4$

$\frac{1}{2}x^3 + 4 \geq 0$

$\frac{1}{2}x^3 \geq -4$

$x^3 \geq -8$

$x \geq \sqrt[3]{-8}$

$\boxed{x \geq -2}$

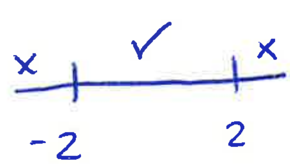
c) $f(x) = -\frac{1}{16}x^4 + 1$

$-\frac{1}{16}x^4 + 1 \geq 0$

$-\frac{1}{16}x^4 \geq -1$

$x^4 \leq 16$ ← Area that satisfies

$|x| \leq 2$



$\boxed{-2 \leq x \leq 2}$

d) $f(x) = x^5 - 1$

$x^5 - 1 \geq 0$

$x^5 \geq 1$

$\boxed{x \geq 1}$

e) $f(x) = x^4 - 4x^2$

$x^4 - 4x^2 \geq 0$

$x^2(x^2-4) \geq 0$

$x^2(x+2)(x-2) \geq 0$

test conditions

✓ | − | − | ✓

−2 0 2

and $x = 0$ ✓

$$\boxed{\begin{array}{l} x \leq -2 \\ x \geq 2 \\ x = 0 \end{array}}$$

f) $f(x) = 9x - x^3$

$-x^3 + 9x \geq 0$

$-x(x^2-9) \geq 0$

$-x(x+3)(x-3) \geq 0$

✓ | × | ✓ | ×

−3 0 3

$$\boxed{x \leq -3 \qquad 0 \leq x \leq 3}$$

g) $f(x) = -x^3 - 3x^2 + 10x$

$-x(x^2+3x-10) \geq 0$

$-x(x+5)(x-2) \geq 0$

✓ | × | ✓ | ×

−5 0 2

$$\boxed{\begin{array}{l} x \leq -5 \\ 0 \leq x \leq 2 \end{array}}$$

h) $f(x) = x^4 - 3x^3 - 4x^2$

$x^2(x^2-3x-4)$

$x^2(x-4)(x+1)$

✓ | × | × | ✓

−1 0 4

$$\boxed{\begin{array}{l} x \leq -1 \\ x \geq 4 \\ x = 0 \end{array}}$$

i) $f(x) = x(x+1)^2(x-2)(x-4)$

$x(x+1)^2(x-2)(x-4) \geq 0$

✗		✗		✓		✗		✓
	-1		0		2		4	

$x = -1$

$0 \leq x \leq 2$

$x \geq 4$

j) $f(x) = x^2(x+1)^2(x-1)$

$x^2(x+1)^2(x-1)$

✗		✗		✗		✓
	-1		0		1	

$x = -1$

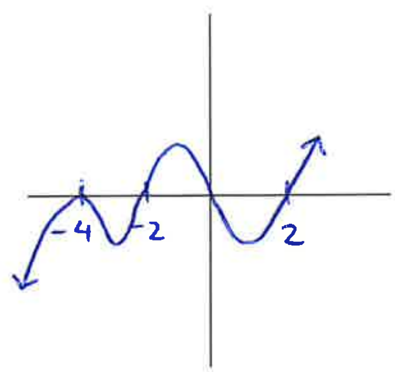
$x = 0$

$x \geq 1$

5. Sketch the polynomial of lowest degree given the following details (Roughly estimate the peaks and valleys).

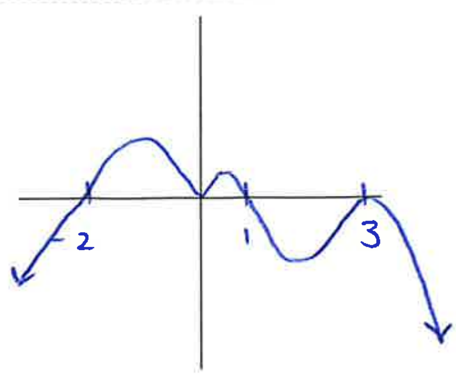
a)

$f(x)$	$-$		$-$		$+$		$-$		$+$
Roots		-4		-2		0		2	



b)

$f(x)$	$-$		$+$		$+$		$-$		$-$
Roots		-2		0		1		3	



6. Sketch a graph given the following information.

✓ bounces off

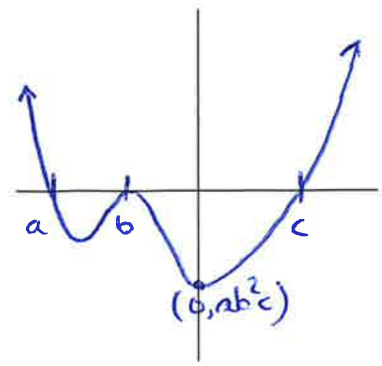
a) $f(x) = (x-a)(x-b)^2(x-c)$,

$$where \quad a < b < 0 < c$$

Degree 4

starts up, ends up

3 direction changes



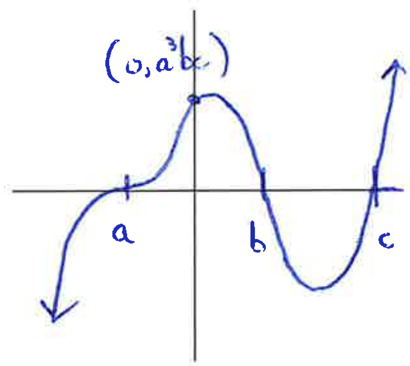
b) $f(x) = (x-a)^3(x-b)(x-c)$

$$where \quad a < 0 < b < c$$

Degree 5

4 direction changes

starts down, ends up



7. Determine the equation of the polynomial in factored form:

a) with zeros at $-1, -1, 2$ and a $y-int$ of $(0, -4)$

$y = a(x+1)^2(x-2)$

$-4 = a(0+1)^2(0-2)$

$-4 = a(1)^2(-2)$

$-4 = -2a$

$a = 2$

$\boxed{P(x) = 2(x+1)^2(x-2)}$

b) with zeros at $0, 0, 1, 1, 1, 2$ and $P(-1) = 12$

$y = ax^2(x-1)^3(x-2)$

$12 = a(-1)^2(-1-1)^3(-1-2)$

$12 = a(1)(-2)^3(-3)$

$12 = a(1)(-8)(-3)$

$12 = 24a$

$a = \frac{1}{2}$

$\boxed{P(x) = \frac{1}{2}x^2(x-1)^3(x-2)}$

c) with zeros at $\frac{1}{2}, -\frac{2}{3}, -\frac{2}{3}$ and goes through $(0, -3)$

$y = a(x - \frac{1}{2})(x + \frac{2}{3})^2$

$-3 = a(x - \frac{1}{2})(x + \frac{2}{3})^2$

$-3 = a(0 - \frac{1}{2})(0 + \frac{2}{3})^2$

$-3 = a(-\frac{1}{2})(\frac{4}{9})$

$-3 = -\frac{2}{9}a$

$a = \frac{27}{2}$

or

$\boxed{P(x) = \frac{27}{2}(x - \frac{1}{2})(x + \frac{2}{3})^2}$

$-3 = a(2(0) - 1)(3(0) + 2)^2$

$-3 = a(-1)(4)$

$a = 3/4$

$\boxed{P(x) = \frac{3}{4}(2x - 1)(3x + 2)^2}$

d) of degree 4 that has $-\frac{1}{2}$ as a root of multiplicity 3, and $2x^2 - x - 1$ as a factor

$(x + \frac{1}{2}) \rightarrow (2x + 1)^3$

$2x^2 - x - 1 \rightarrow (2x + 1)(x - 1)$

so we get $(2x + 1)^3(x - 1)$

$P(x) = a(2x + 1)^3(x - 1)$

not enough info to go further.

e) of degree 4, whose zeros include 0, 2, which has a factor of $x^2 - 2x - 5$, and whose graph contains the point $(3, 12)$

$x^2 - 2x - 5$ does not factor easily.

$P(x) = ax(x - 2)(x^2 - 2x - 5)$

$12 = a(3)(3 - 2)(3^2 - 2(3) - 5)$

$12 = a(3)(9 - 6 - 5)$

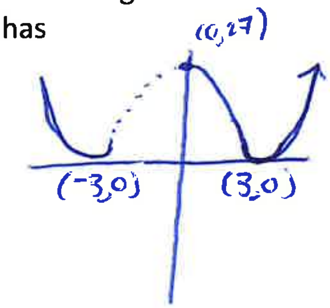
$12 = a(3)(-2)$

$12 = -6a \quad a = -2$

$\boxed{P(x) = -2x(x - 2)(x^2 - 2x - 5)}$

f) of the least degree that is symmetric to the $y - axis$, touvhes but does not go through the $x - axis$, and has $P(0) = 27$

at $x = 3$



$P(x) = a(x + 3)^2(x - 3)^2$

$P(x) = a(0 + 3)^2(0 - 3)^2$

$P(x) = a(3)^2(-3)^2$

$27 = a3^4$

$27 = 81a$

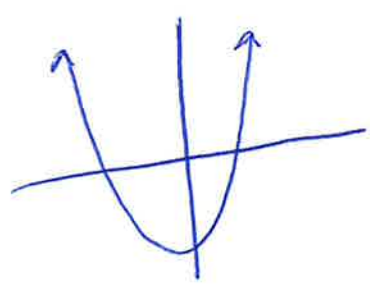
$a = \frac{27}{81} = \frac{1}{3}$

$\boxed{P(x) = \frac{1}{3}(x + 3)^2(x - 3)^2}$

8. If a polynomial function of even degree has a positive leading coefficient and a negative $y - intercept$, what is the minimum number of real roots it could have?

Visualize:
- Even degree means starts ends same direction
- Positive Leading Coefficent means up
- neg y-int

$\boxed{\text{minimum \# of real roots is } 2}$



See Website for Detailed Answer Key