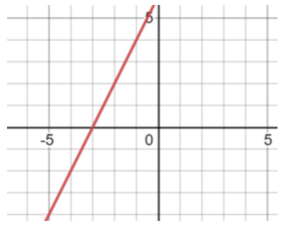
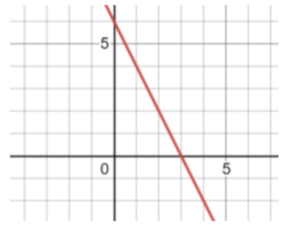
# <u>Section 3.2 – Graphing Polynomials</u>

- In order to start making a rough sketch of a Polynomial, it is a great place to start by considering the roots (zeros, solutions, $x-intercepts$), and the multiplicity at each root.

a) If $(x-a)$ has **multiplicity of 1**, the graph **crosses the $\boldsymbol{x-axis}$ at that root**

$$y=(x-a)^1$$



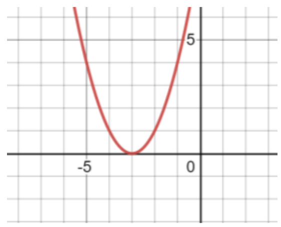



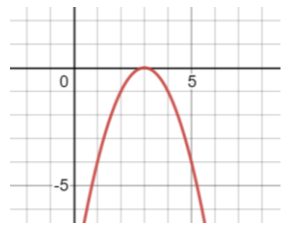

b) If $(x-a)$ has **even multiplicity**, the graph **touches** the $x-axis$ at that root and **bounces back**

$y=(x-a)^2$ …

$y=-(x-a)^4$ …

$etc.$



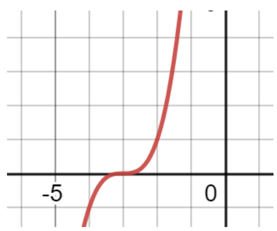



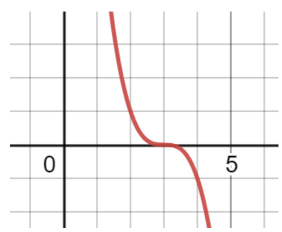

c) If $(x-a)$ has odd multiplicity, the graph **crosse**s the $x-axis$ and **looks like it lies on the root first**

$y=(x-a)^3$ …

$y=-(x-a)^5$ …

$etc.$



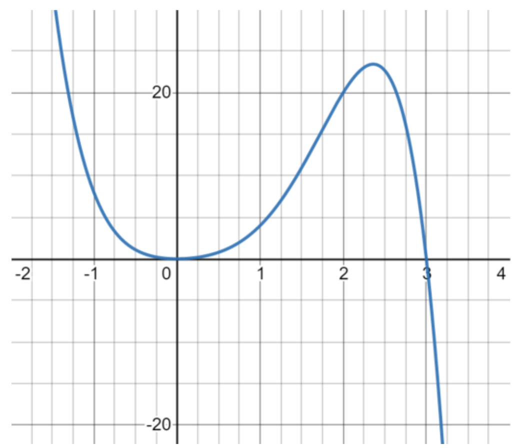

| **<u>Steps to Graph a Polynomial</u>** |
|---|
| 1) Find the $x-ints$ by solving the equation when $f(x)=0$ (by factoring) |
| 2) Find the $y-int$ by solving the equation $f(0)$ (when $x=0$) |
| 3) Determine the general shape at the $x-ints$ |
| 4) Use the leading coefficient and degree to determine end-behaviour |
| 5) Determine the number of turning points |
| 6) Use the $x-value$ between $x-ints$ to estimate the peaks and valleys of the graph |
| 7) Plot a decent number of points and roughly sketch a smooth curve |

**Example 1:** Graph $y = x^2(3-x)(x^2+1)$

**Solution 1:**

- Multiplied out $x^2(3-x)(x^2+1)$ gives us a **fifth power polynomial**, and the $(3-x) \rightarrow -1(x-3)$
- This give us a **degree 5 polynomial** with a $-\mathbf{1}$ **as a leading coefficient.**
- So, our graph starts **up left and ends down right**
- If $x = 0$ our $\boldsymbol{y-int}$ is 0 $\quad (\mathbf{0}, \mathbf{0})$
- Our $\boldsymbol{x-ints}$ are given when $y = 0$ so, $(\mathbf{0},\mathbf{0})\ \boldsymbol{and}\ (\mathbf{3},\mathbf{0})$
- $\boldsymbol{x^2+1=0}$ **has no solution**, therefore no $x-int$
- Since $\boldsymbol{x=0}$ **has multiplicity of 2** the **graph bounces back** at that root, but **goes through when** $\boldsymbol{x=3}$
- Plot additional points, **pick points on either side of the roots** to get the basic shape of the graph

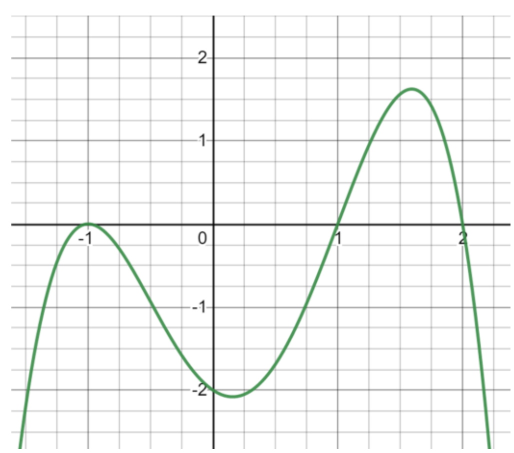
| $x$ | $y$ |
|---|---|
| $-2$ | $100$ |
| $-1$ | $8$ |
| $1$ | $4$ |
| $2$ | $20$ |
| $2.5$ | $23$ |
| $4$ | $-272$ |




**Example 2:** Graph $y = (x-2)(1-x)(x+1)^2$

**Solution 2:**

- Multiplied out $(x-2)(1-x)(x+1)^2$ gives us a **fourth power polynomial**, and the $(1-x) \rightarrow -1(x-1)$
- This give us a **degree 4 polynomial** with a $-\mathbf{1}$ **as a leading coefficient.**
- So, our graph **starts and ends down**
- If $x = 0$ our $\boldsymbol{y-int}$ is $-2$ $\quad (\mathbf{0}, -\mathbf{2})$
- $\boldsymbol{x-ints}$ are when $y = 0$ so, $(\mathbf{2},\mathbf{0}), (\mathbf{1},\mathbf{0}), \boldsymbol{and}\ (-\mathbf{1},\mathbf{0})$
- Since $(\boldsymbol{x}+\mathbf{1})$ **has multiplicity of 2** the **graph bounces back** at that root, but **goes through when** $\boldsymbol{x = 1\ and\ x = 2}$
- Plot a point at $x = 1.5$ to get our only discrepancy

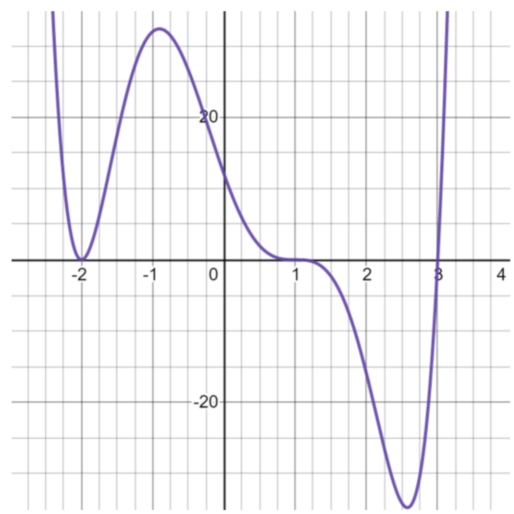


**Example 3:** Graph $y = (x-3)(x+2)^2(x-1)^3$

**Solution 3:**

- Multiplied out $(x-3)(x+2)^2(x-1)^3$ gives us a **sixth power polynomial**
- This give us a **degree 6 polynomial** with a $\boldsymbol{positive}$ **leading coefficient.**
- So, our graph **starts and ends up**
- If $x = 0$ our $\boldsymbol{y - int}$ is $12$ $\quad(\mathbf{0}, \mathbf{12})$
- Our $\boldsymbol{x - ints}$ are given when $y = 0$ so, $(-\mathbf{2}, \mathbf{0}), (\mathbf{1}, \mathbf{0}), \boldsymbol{and}\ (\mathbf{3}, \mathbf{0})$
- Since $\boldsymbol{x = -2}$ **has multiplicity of 2** the **graph bounces back** at that root, but **goes through when** $\boldsymbol{x = 3}$**, and goes through but lays on** $\boldsymbol{x = 1}$
- Plot additional points, **pick points on either side of the roots** to get the basic shape of the graph

| $x$ | $y$ |
|---|---|
| $-3$ | $2058$ |
| $-1$ | $32$ |
| $2$ | $-16$ |
| $2.5$ | $-34.172$ |




## Equation of Polynomial Functions

- Much like how we solved for the equation of a parabola in grade 11, we can use roots, intercepts, and given points to write the factored form of the polynomial and then solve for the a-value of

**Example 4:** A polynomial has $-1, -1, 0, 2$ as its roots, and $p(1) = 5$. What is the equation of the polynomial?

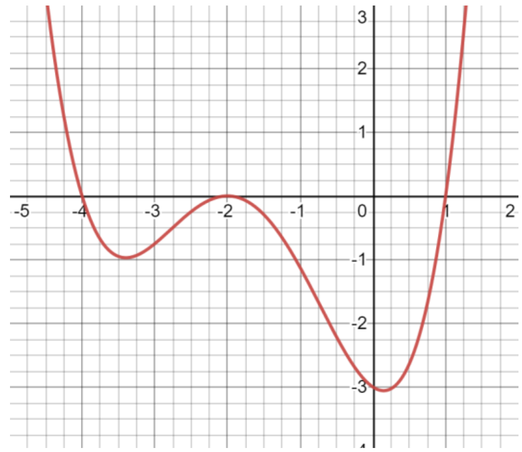
**Solution 4:**

$$p(x) = a(x+1)^2(x)(x-2), \ \ but\ with\ \ p(1) = 5$$

$p(1) = a(1+1)^2(1)(1-2) = 5 \quad \rightarrow \quad 5 = a(4)(1)(-1)$

$5 = -4a \quad \rightarrow \quad \boldsymbol{a = -\frac{5}{4}} \quad \rightarrow \quad \boldsymbol{p(x) = -\frac{5}{4}(x+1)^2(x)(x-2)}$

**Example 5:** Write the equation of the polynomial in lowest degree that is represented by the graph




**Solution 5:**

We have solutions (zeros, roots, etc.) at:

| Zeros | Multiplicity |
|---|---|
| $-4$ | $1$ |
| $-2$ | $2$ |
| $1$ | $1$ |

$$p(x) = a(x+2)^2(x+4)(x-1), \quad \textit{and pick any point on the graph} \quad p(0) = -3$$

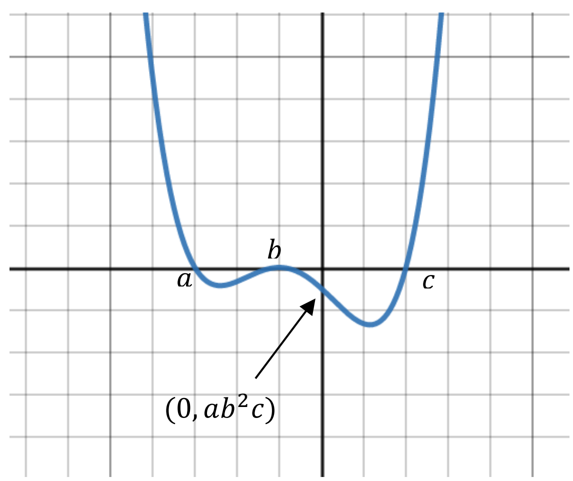
$p(0) = a(0+2)^2(0+4)(0-1) = -3 \quad \rightarrow \quad -3 = a(4)(4)(-1)$

$-3 = -16a \quad \rightarrow \quad \boldsymbol{a = \frac{-3}{-16} = \frac{3}{16}} \quad \rightarrow \quad \boldsymbol{p(x) = \frac{3}{16}(x+2)^2(x+4)(x-1)}$

**Example 6:** Sketch the graph of $f(x) = (x-a)(x-b)^2(x-c)$ where $a < b < 0 < c$

- Identify the $y - intercept, where\ f(x) > 0, and\ where\ f(x) < 0$

**Solution 6:**




- **Fourth power** Polynomial with **positive leading coefficient**
- **Starts up and ends up**
- **Goes through** $root\ a$ (most left)
- Then **bounces back** at $b$ (right of a but left of 0)
- **Goes through** $c$
- $y - int$ is when $x = 0$,
  $so\ (0-a)(0-b)^2(0-c) = ab^2c$

  $(0, ab^2c)$

- $f(x) > 0\ when\ x < a\ and\ x > c$
- $f(x) < 0\ when\ a \leq x \leq c$

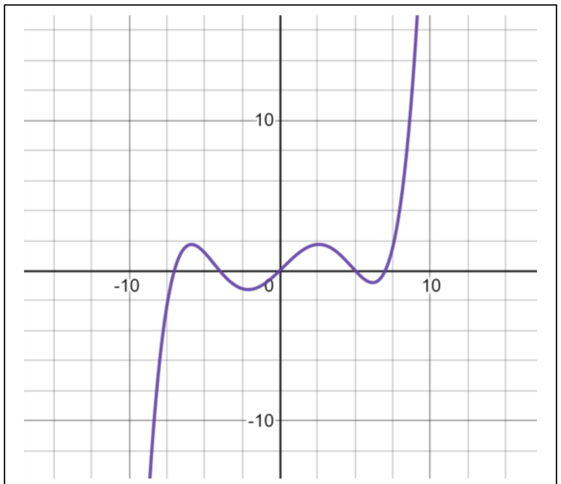
***DESMOS IS A FANTASTIC GRAPHING APP AND WEBSITE TO SEE BEHAVIOURS OF GRAPHS***

***WE WILL LOOK AT THE BASICS AS A CLASS***

## Section 3.2 – Practice Problems

1. What is the minimum degree and number of turning points of the following:

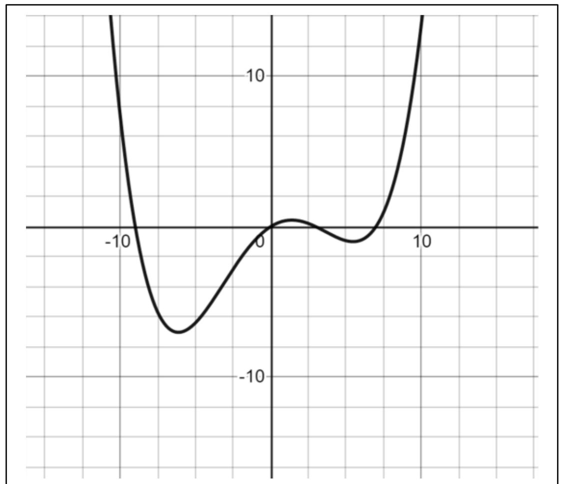
a)




Min Degree:

Turning Points:

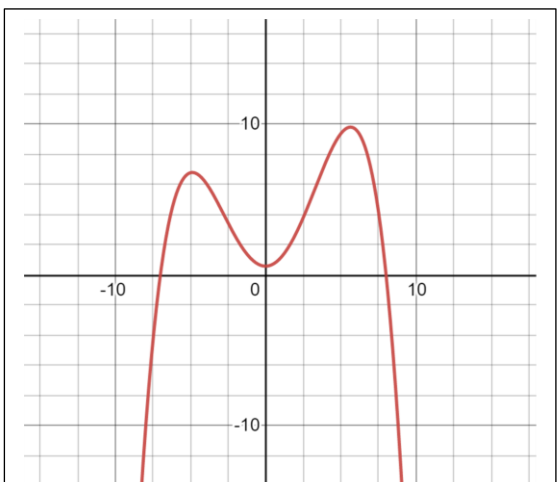
b)




Min Degree:

Turning Points:

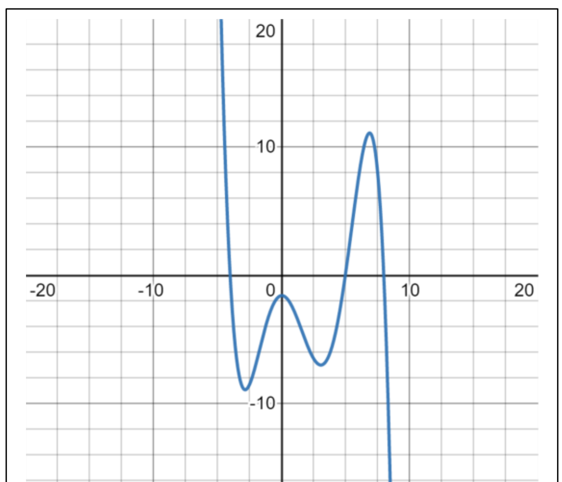
c)




Min Degree:

Turning Points:

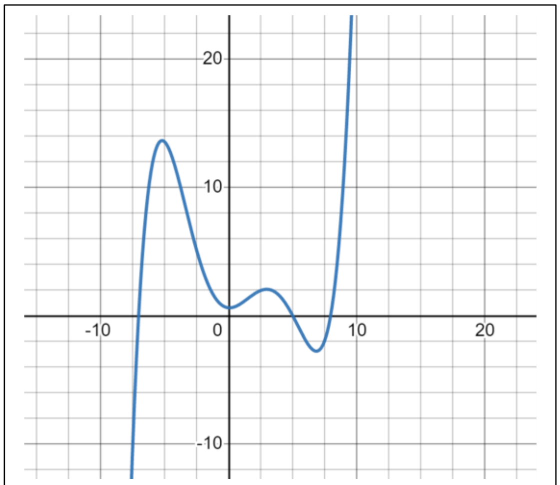
d)




Min Degree:

Turning Points:

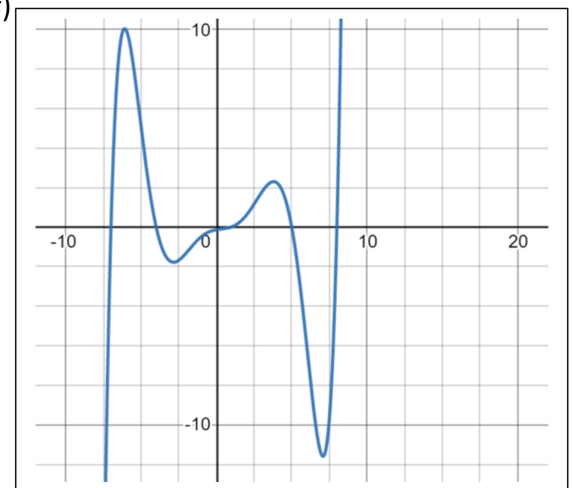
e)




Min Degree:

Turning Points:

f)




Min Degree:

Turning Points:

2. Using the table below, what is the minimum number of zeros possible for the polynomial function. Discuss at which integer or between which two integers, does the zero (root) occur.

a)

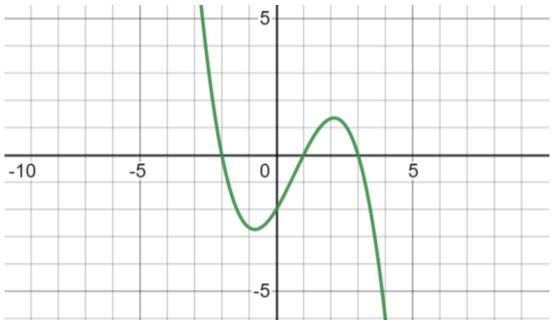
| $x$ | $-5$ | $-4$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ | $4$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $P(x)$ | $-25$ | $-8$ | $3$ | $0$ | $-6$ | $-24$ | $-33$ | $-32$ | $-15$ | $5$ |

b)

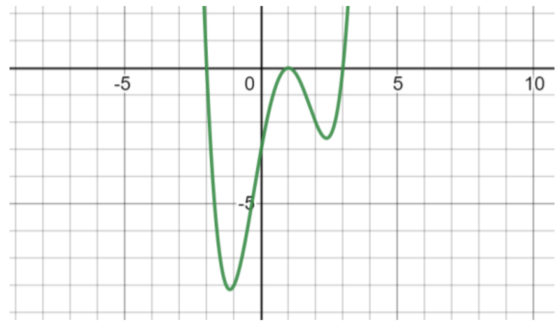
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|---|
| $P(x)$ | $-75$ | $-1$ | $3$ | $-6$ | $-15$ | $0$ | $63$ |

3. Determine the equation, in factored form, of a polynomial that fits the following graphs.

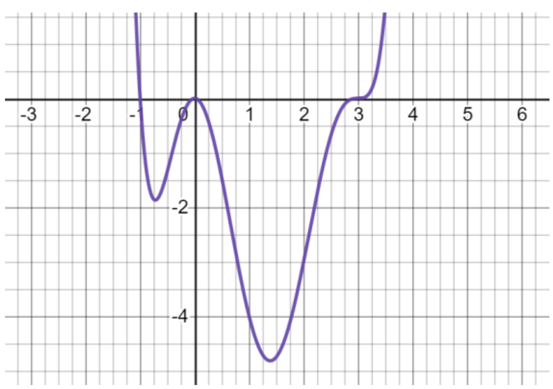
a)



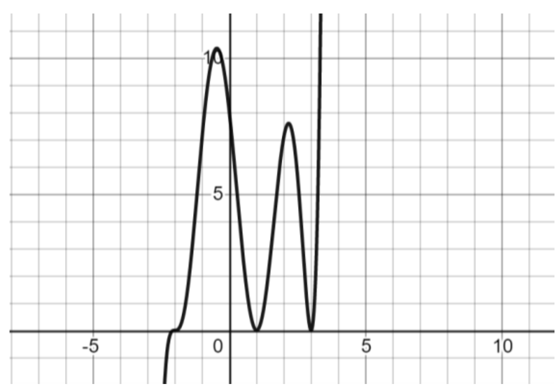
b)



c)



d)



4. Find all values of $x$ where the outputs, $f(x)$ are $\geq 0$.

a) $f(x) = -\frac{1}{3}x^3$

b) $f(x) = \frac{1}{2}x^3 + 4$

c) $f(x) = -\frac{1}{16}x^4 + 1$

d) $f(x) = x^5 - 1$

e) $f(x) = x^4 - 4x^2$

f) $f(x) = 9x - x^3$

g) $f(x) = -x^3 - 3x^2 + 10x$

h) $f(x) = x^4 - 3x^3 - 4x^2$

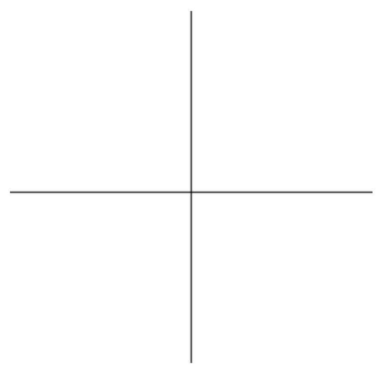
i) $f(x) = x(x+1)^2(x-2)(x-4)$

j) $f(x) = x^2(x+1)^2(x-1)$

5. Sketch the polynomial of lowest degree given the following details (Roughly estimate the peaks and valleys).

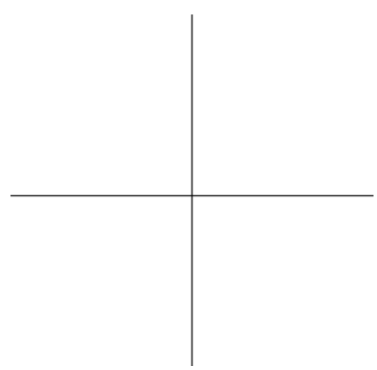
a)

| $f(x)$ | $-$ | | $-$ | | $+$ | | $-$ | | $+$ |
|---|---|---|---|---|---|---|---|---|---|
| Roots | | $-4$ | | $-2$ | | $0$ | | $2$ | |



b)

| $f(x)$ | $-$ | | $+$ | | $+$ | | $-$ | | $-$ |
|---|---|---|---|---|---|---|---|---|---|
| Roots | | $-2$ | | $0$ | | $1$ | | $3$ | |



6. Sketch a graph given the following information.

a) $f(x) = (x-a)(x-b)^2(x-c),$

$$where\ \ a < b < 0 < c$$

b) $f(x) = (x-a)^3(x-b)(x-c)$

$$where\ \ a < 0 < b < c$$

7. Determine the equation of the polynomial in factored form:

a) with zeros at $-1, -1, 2$ and a $y-int$ of $(0, -4)$

b) with zeros at $0, 0, 1, 1, 1, 2$ and $P(-1) = 12$

c) with zeros at $\frac{1}{2}, -\frac{2}{3}, -\frac{2}{3}$ and goes through $(0, -3)$

d) of degree 4 that has $-\frac{1}{2}$ as a root of multiplicity 3, and $2x^2 - x - 1$ as a factor

e) of degree 4, whose zeros include 0, 2, which has a factor of $x^2 - 2x - 5$, and whose graph contains the point $(3, 12)$

f) of the least degree that is symmetric to the $y - axis$, touches but does not go through the $x - axis$ at $x = 3$, and has $P(0) = 27$

8. If a polynomial function of even degree has a positive leading coefficient and a negative $y - intercept$, what is the minimum number of real roots it could have?

**See Website for Detailed Answer Key**

**Extra Work Space**