After Break.

Section 3.2 – Annuities and Installment Loans

Annuities

- An annuity is a **savings plan** where an investor makes **regular, fixed deposits** into a compound interest account and the interest rate does not change for the length of the investment
- These types of savings are more usual, since they do not require a large upfront investment

Example: Find the future amount of an annuity where \$1000 payment (investment) is made annually for 4 years at 5% compound interest

Solution: The payment is made at the end of each year.

End of first year: \$1000 payment

End of second year: $\$1000 \cdot 0.05 = \$50$ in interest

$\mathbf{\$1000}\ \boldsymbol{Principal} + \mathbf{\$1000}\ \boldsymbol{payment} + \mathbf{\$50}\ \boldsymbol{interest}$

Total: \$2050.00

End of third year: $\$2050 \cdot 0.05 = \$102.50$ in interest

$\mathbf{\$2050}\ \boldsymbol{Principal} + \mathbf{\$1000}\ \boldsymbol{payment} + \mathbf{\$102.50}\ \boldsymbol{interest}$

Total: \$3152.50

End of fourth year: $\$3152.50 \cdot 0.05 = \$157.63$ in interest

$\mathbf{\$3152.50}\ \boldsymbol{Principal} + \mathbf{\$1000}\ \boldsymbol{payment} + \mathbf{\$157.63}\ \boldsymbol{interest}$

Total: \$4310.13

The future value of the annuity at the end of 4 years is: $\mathbf{\$4310.13}$

- Calculating this way is very time consuming, so a formula called the Annuity Formula can be used
- It is developed from a Geometric Series (Grade 12) so will we simply give it to you here

Future Amount of an Annuity is calculated this way:

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{n\cdot t} - 1\right]}{\frac{r}{n}}$$

F: *future amount of the annuity*
r: *annual interest rate*
t: *term of the annuity in years*
R: *is the regular periodic payment*
n: *is the number of payments per year*

$$F = \frac{1000\left[\left(1+\frac{0.05}{1}\right)^{1\cdot 4} - 1\right]}{\frac{0.05}{1}}$$

$$= \frac{1000[0.21550625]}{0.05}$$

$$= 4310.125$$

$$= 4310.13$$

Example: Find the future amount of an annuity where $1000 payment (investment) is made annually for 4 years at 5% compound interest

Solution: $R = \$1000$ $r = 0.05$ $n = 1$ $t = 4$

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$$

$$F = \frac{1000\left[\left(1+\frac{0.05}{1}\right)^{1(4)}-1\right]}{\frac{0.05}{1}}$$

$$F = \frac{1000\left[0.21550625\right]}{0.05}$$

$$= \$4310.125$$

$$= \$4310.13$$

Practice Problems #1-5 stop here

Example: Suppose you want to save up to purchase a new car in 5 years. What amount must you save semi-monthly if the car will cost $50 000 and interest is calculated semi-monthly at 6%.

Solution: $F = 50000$ $r = 0.06$ $n = 24$ $t = 5$

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$$

$$50000 = \frac{R\left[\left(1+\frac{0.06}{24}\right)^{24(5)}-1\right]}{\frac{0.06}{24}}$$

$$50000 = \frac{R\left[0.349353\right]}{0.0025}$$

$$50000 = R\,(137.74)$$

$$R = \$357.80$$

Example: What is the better investment: Saving $200 monthly at 4% compounded monthly for 8 years, or saving $100 semi-monthlyat 4% compounded semi-monthly for 8 years?

Solution: Compounded Monthly $R = 200$ $r = 0.04$ $n = 12$ $t = 8$

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}} = \frac{200\left[\left(1+\frac{0.04}{12}\right)^{12(8)}-1\right]}{\frac{0.04}{12}} = 22583.71$$

Compounded Semi-Monthly $R = 100$ $r = 0.04$ $n = 24$ $t = 8$

$$F = \frac{R\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}} = \frac{100\left[\left(1+\frac{0.04}{24}\right)^{24(8)}-1\right]}{\frac{0.04}{24}} = 22605.66$$

- <u>If the interest is calculated **more times in the year**, the final value is greater.</u>
- <u>Compounding **semi-monthly** returns $21.95 more over 8 years.</u>

Practice Problems #6-10

Test Wednesday next week!

Instalment Loans (Financing)

- Instalment loans are the **opposite of annuities**
- When we buy things **we can't necessarily afford**, we can purchase over a period of time
- Whenever you see the words **zero percent interest**, and can pay over a time period, **take advantage!**
- **Otherwise** you will have to **pay some interest on the purchase price**

Example: Raffaela bought a new car with retail price of $35 000. Her down payment was $3000 and she had to pay $838.14 per month for 48 months. Calculate her total cost for the new car, including interest.

Solution Installment price $= \frac{\$838.14}{\text{month}} \cdot 48 \text{ months} = \$40\,230.72$

Total cost = Down payment + Installment price

$= \$3000 + \$40\,230.72$

$= \$43\,230.72$

Monthly Payments by Simple Interest

- Not the best type of loan to get, because you pay interest on the initial loan and it does not consider the decreasing size of the loan as payments are made

Example: New appliances for a house cost $15 000. The full cost was financed over three years at 9% simple interest per year.

a) Find the finance charge
b) Find the instalment price
c) Find the monthly payment

Solution: a) $I = Prt = 15000(0.09)3 = \$4050$

b) Installment price = Principle + Interest

$= \$15000 + \$4050$

$= \$19050$

c) Monthly Payment $= \frac{\text{Installment Price}}{\#\text{ months}}$

$= \frac{\$19050}{36}$

$= \$529.17$

Annual Percentage Rate: Annual (Yearly) Percentage Rate is the interest rate expressed for a whole year instead of monthly.

$$r_{annual} = (1 + r_{monthly})^{12} - 1$$

$$P(1+r)^t = P\left(1+\frac{r}{n}\right)^{nt}$$
$$1+r = \left(1+\frac{r}{n}\right)^n$$
$$r = \left(1+\frac{r}{n}\right)^n - 1$$

Credit Cards

- Credit cards charge interest on the unpaid balance each month. The rate is much higher than bank loan interest rates, so if you are struggling to pay off your credit card, get a bank loan at a lower rate to pay off the credit card balance

Example: On December 11st, Brad had an unpaid balance of $5280.60 on his credit card. In December he made purchases of $1235.42 for Christmas presents and made a payment of $300 on the balance. The monthly interest on the unpaid balance was 1.8%. Find the finance charge, and the new balance on January 1st.

Solution:

- December 1st balance: $5280.60
- Purchases are: $1235.42
- Payment is: - $300
- New Balance: $5280.60 + $1235.42 - $300 = $6216.02
- Interest is: $6216.02 (0.018) = $111.89 for the month of December

The new balance for January: $6216.02 + $111.89 = $6327.91

Note: If the percentage rate seems low, remember this is just for a month. The yearly percentage rate for the credit card is:

$$r_{annual} = (1 + r_{monthly})^{12} - 1$$
$$= (1 + 0.018)^{12} - 1$$
$$= 0.2387$$
$$= 23.87\%$$

Practice Problems #11-16

Section 3.2 – Practice Problems

Find the amount of each annuity

Payment	Rate	Compounded	Time	Future Value
1. $2500	5%	Annually	10 *years*	
2. $6000	6.5%	Semi-Annually	25 *years*	
3. $1200	4%	Quarterly	8 *years*	
4. $500	9%	Monthly	15 *years*	
5. $300	10%	Bi-Weekly	5 *years*	

Find the Periodic payment need to attain the future amount of each annuity

Future Value	Rate	Compounded	Time	Periodic Payment
6. $7500	3.6%	Semi-Annually	6 *years*	
7. $35 000	5.4%	Quarterly	9 *years*	
8. $1 000 000	7.5%	Monthly	25 *years*	

9. In order to plan for their retirement, a married couple decides to buy an annuity that pay 6% interest compounded semi-annually. If they invest $2500 semi-annually for 35 years, how much interest would they earn?

10. You have $5000 to invest and are offered a 5-year investment at 4.5% simple interest, or an annuity of $1000 per year for 5 years at 9% compounded annually. If your only concern is the future amount, what is the better investment?

11. Sally bought a stereo for $760. She made a down payment of $60, and paid $65 per month for a year. What was the installment price of the stereo? What was the total cost?

12. Hunter bought a 75 inch TV for $2600, including taxes. He made a down payment of 20% and paid the balance over 18 months. The financial charges were 6% of the amount financed. Determine the instalment price of the TV, and monthly payments.

13. A $12 000 loan is to be paid off in 48 monthly payments of $292.96. The borrower decides to pay off the loan after 30 payments have been made. Find the amount of interest saved.

14. Parker Publishing borrows $20 000 to be paid off with 36 monthly payments of $664.29. After good sales, they decide to pay off the loan in 24 months. Find the amount saved.

15. For the month of March, Nadine had an unpaid balance of \$2340.62 on her credit card. She purchased \$369.78 and made a payment of \$300 during the month. If the interest is 2% on the unpaid balance, what is her new balance on April 1st, and what is the annual percentage rate?

16. For the month of November, the unpaid balance on Alissa's credit card statement was \$1816.22. She purchased \$435.85 and made a payment of \$400 during the month. If the interest is 1.8% on the unpaid balance, what is her new balance on December 1st, and what is the annual percentage rate?

Extra Work Space