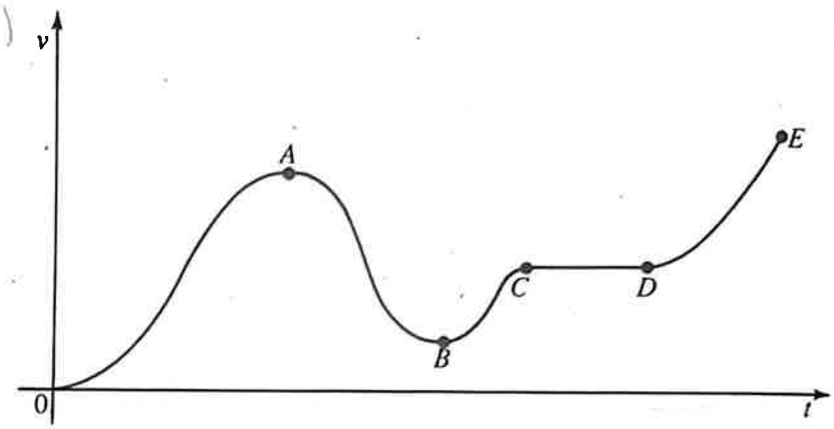
Section 3.2 – Practice Problems

1. The graph of a velocity function is shown. State whether the acceleration is positive, zero, or negative from:

a) 0 to A	b) A to B	c) B to C	d) C to D	e) D to E
Positive	Negative	Positive	Zero	Positive

consider the slope of the tangent line

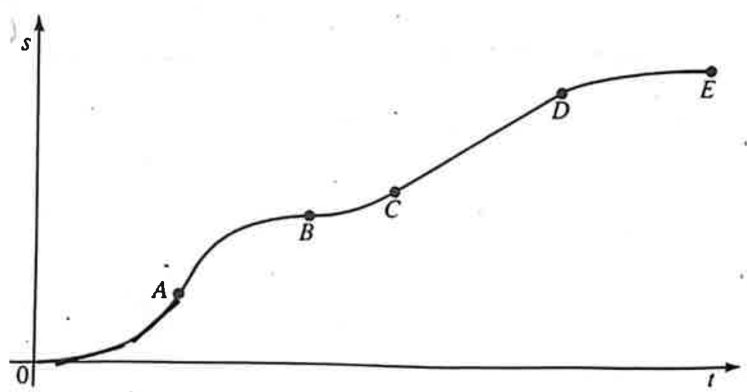


2. The graph of a position function is shown.

a) For the part of the graph from 0 to A, use slopes of tangents to decide whether the velocity is increasing or decreasing. Is the acceleration positive or negative?

velocity increasing

acceleration positive



b) State whether the acceleration is positive, zero, or negative from:

a) A to B	b) B to C	c) C to D	d) D to E
Negative	Positive	Zero	Negative

3. The position functions give the displacement s as a function of the time t. Find the velocity and acceleration as functions of t.

a) $s = 12 + 30t$

$v(t) = 30$

$a(t) = 0$

b) $s = 16t^2 + 5t - 10$

$v(t) = 32t + 5$

$a(t) = 32$

c) $s = t^3 + 5t^2 + t + 1$

$v(t) = 3t^2 + 10t + 1$

$a(t) = 6t + 10$

d) $s = \sqrt{t^2 + t}$

$v(t) = \frac{1}{2\sqrt{t^2+t}} \cdot (2t+1) = \boxed{\frac{2t+1}{2\sqrt{t^2+t}}}$

$a(t) = \frac{1}{2}\left[\frac{\sqrt{t^2+t}\,(2) - (2t+1)\left(\frac{1\,(2t+1)}{2\sqrt{t^2+t}}\right)}{t^2+t}\right]$

$a(t) = \frac{1}{2}\left[\frac{(t^2+t)(4) - (2t+1)^2}{2\sqrt{t^2+t}\,(t^2+t)}\right] = \boxed{\frac{-1}{4(t^2+t)^{3/2}}}$

4. The position functions give s (in meters) as a function of t (in seconds). Find the acceleration at 4s.

a) $s = 100 - 15t - 4.9t^2$

$v(t) = -15 - 9.8t$

$a(t) = -9.8$

$a(4) = \boxed{-9.8\,m/s^2}$

b) $s = t^3 - t^2$

$v(t) = 3t^2 - 2t$

$a(t) = 6t - 2$

$a(4) = 24 - 2$

$= \boxed{22\,m/s^2}$

c) $s = t^3 - 2t^2 + 3t - 5$

$v(t) = 3t^2 - 4t + 3$

$a(t) = 6t - 4$

$a(4) = 24 - 4$

$= \boxed{20 m/s^2}$

d) $s = \frac{5t}{1+t}$

$v(t) = \frac{(1+t)(5) - 5t(1)}{(1+t)^2}$

$v(t) = \frac{5 + 5t - 5t}{(1+t)^2} = \boxed{\frac{5}{(1+t)^2}}$

$a(t) = \frac{(1+t)^2(0) - 5(2(1+t)(1))}{(1+t)^4}$

$= \frac{-10(1+t)}{(1+t)^4} = \frac{-10}{(1+t)^3}$ at $t=4$

$\frac{-10}{125} = \boxed{-0.08 m/s^2}$

5. A position function is given by $s = s_0 + v_0 t + \frac{1}{2} g t^2$, where s_0, v_0 and g are constants. Find:

a) The initial position

at $t=0$

initial position is $\boxed{S_0}$

b) The initial velocity

$v(t) = V_0 + gt$

initial velocity is $\boxed{V_0}$

c) The acceleration

$a(t) = g$ initial acceleration is $\boxed{g}$

6. The position function of a particle is $s = t^3 - 12t, t \geq 0$, where s is measured in meters and t is measured in seconds. Find the acceleration at the instant when the velocity is 0.

$v(t) = 3t^2 - 12$ → when $v(t) = 0$ $\quad 0 = 3t^2 - 12$

$t^2 = 4 \quad t = 2$ secs

$a(t) = 6t$

when $v(t) = 0$ is at $t = 2$

$a(2) = 6(2) = \boxed{12\,m/s^2}$

7. A particle moves according to the equation of motion $s = t^3 - 9t^2 + 18t$, where s is measured in meters and t is measured in seconds.

a) When is the acceleration 0.

$v(t) = 3t^2 - 18t + 18$

$a(t) = 6t - 18$

$0 = 6t - 18 \quad \boxed{t = 3 \text{ secs}}$

b) Find the displacement and velocity at that time.

$s(3) = 3^3 - 9(3)^2 + 18(3)$

$27 - 81 + 54$

$= \boxed{0\,m}$

$v(3) = 3(3)^2 - 18(3) + 18$

$= 27 - 54 + 18$

$= \boxed{-9\,m/s}$

8. The position function of a particle is $s = t^4 - 12t^3 + 30t^2 + 5t, t \geq 0$. When is the acceleration positive and when is it negative?

$v(t) = 4t^3 - 36t^2 + 60t + 5$

$a(t) = 12t^2 - 72t + 60$

$0 = 12t^2 - 72t + 60$

$0 = 12(t^2 - 6t + 5)$

$0 = (t-5)(t-1)$

acceleration is positive $\boxed{0 \leq t < 1}$ and $\boxed{t > 5}$

+ − +

1 5

acceleration is negative

$\boxed{1 < t < 5}$

9. A car is travelling at 72km/h and the brakes are fully applied, producing a constant deceleration of 12m/s^2

a) Verify that the velocity function $v(t) = -12t + 20$, where t is measured in seconds, gives this deceleration and initial velocity.

$a(t) = -12\ m/s^2$ ✓

$\frac{72\ km}{1\ hr} \cdot \frac{1000\ m}{1\ km} \cdot \frac{1\ hr}{60\ min} \cdot \frac{1\ min}{60\ sec} \Rightarrow 20\ m/sec$

$v(0) = 20\ m/s$

b) How long does it take for the car to come to a complete stop?

Time to stop gives velocity 0

$0 = -12t + 20$

$-20 = -12t$

$\frac{-20}{-12} = t$

$\boxed{t = \frac{5}{3}\ secs}$