3.2 Acceleration

second derivative of the position function

Acceleration is a vector and is defined as the rate of change of velocity with respect to time. The acceleration function $a(t)$ is the first derivative of the velocity function $v(t)$.

$$a(t) = v'(t) = \frac{dv}{dt}$$

Since the velocity function is the first derivative of the position function $s(t)$, then the acceleration function is the second derivative of the position function.

$$a(t) = v'(t) = s''(t)$$

Which written in Leibniz notation,

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

If all quantities are measured in SI units[2] metres, and seconds, then velocity as we saw before is measured in $\mathrm{m/s}$ and acceleration is measured in $\mathrm{m/s^2}$. The sign convention discussed in the previous section applies here as well.

Ex. 1

The position function of a particle is given by $s(t) = t^3 + 2t^2 + 2t$, where s is measured in metres and t in seconds.

(a) Find the velocity and acceleration as a function of time.
(b) Find the acceleration at 3 s.

a) $v(t) = 3t^2 + 4t + 2$

$a(t) = 6t + 4$

b) $a(3) = 6(3) + 4$

$= \boxed{22\,\mathrm{m/s^2}}$

Ex. 2

The position of a ball thrown directly upward from ground level is given by $s(t) = 24.5t - 4.9t^2$, where s is measured in metres and t in seconds.

(a) Find the initial velocity of the ball.
(b) Find the acceleration of the ball.
(c) Find the maximum height the ball reaches.

a) $v(t) = 24.5 - 9.8t$ initial velocity at $t = 0$ is $24.5\,\mathrm{m/s}$

b) $a(t) = -9.8\,\mathrm{m/s^2}$

c) Max height occurs when $v(t) = 0$

$0 = 24.5 - 9.8t$ $t = 2.5$ secs

$s(2.5) = 24.5(2.5) - 4.9(2.5)^2$

$= \boxed{30.63\,\mathrm{m}}$

Homework Assignment

- Section 3.2: #1 – 9

[2] The International System of Units is the modern form of the metric system you are familiar with.