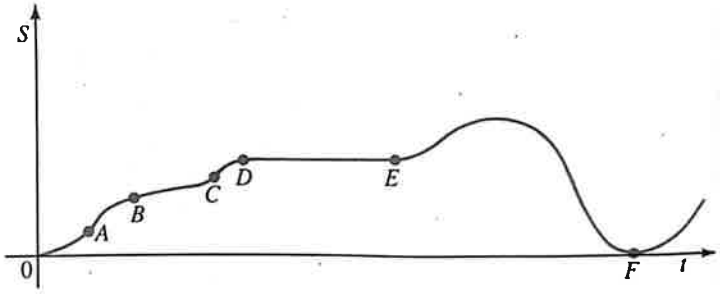
Section 3.1 – Practice Problems

1. The graph shows the position function of a car.



a) What was the initial velocity of the car?

0 m/s

b) Was the car going faster at B or C?

C (slope of target line is steeper)

c) Was the car slowing down or speeding up at A, B, and C?

Speeding up at A and C slowing down at B

d) What happened between D and E?

The car stopped

e) What happened at F?

The car returned to where it started (position function)

2. The position functions give s (in meters) as a function of t (in seconds). Find the velocity as a function of time and the velocities after 2s and 4s.

a) $s = 5 + 12t$

$s'(t) = 12$

at 2 secs 12 m/s

at 4 secs 12 m/s

b) $s = 8t^2 - 24t + 5$

$s'(t) = 16t - 24$

at 2 secs 8 m/s

at 4 secs 40 m/s

c) $s = t^3 - 6t^2$

$s'(t) = 3t^2 - 12t$

at 2 secs $\boxed{-12 \text{ m/s}}$

at 4 secs $\boxed{0 \text{ m/s}}$

d) $s = \frac{5t}{1+t}$

$s'(t) = \frac{(1+t)(5) - 1(5t)}{(1+t)^2}$

$\frac{5 + 5t - 5t}{(1+t)^2} = \frac{5}{(1+t)^2}$

at 2 secs $\boxed{\frac{5}{9} \text{ m/s}}$

at 4 secs $\boxed{\frac{1}{5} \text{ m/s}}$

3. If a stone is thrown downward with a speed of 15m/s from a cliff that is 80m high, its height in meters after t seconds is $h = 80 - 15t - 4.9t^2$. Find the velocity after 1s and 2s.

$v(t) = -15 - 9.8t$

$v(1) = \boxed{-24.8 \text{ m/s}}$

$v(2) = \boxed{-34.6 \text{ m/s}}$

4. If a ball is thrown directly upward with an initial velocity of 24.5m/s, then its height after t seconds, in meters, is $h = 24.5t - 4.9t^2$.

a) Find the velocity after 1s, 2s, 3s, and 4s.

$v(t) = 24.5 - 9.8t$

$v(1) = \boxed{14.7 \text{ m/s}}$

$v(2) = \boxed{4.9 \text{ m/s}}$

$v(3) = \boxed{-4.9 \text{ m/s}}$

$v(4) = \boxed{-14.7 \text{ m/s}}$

b) When does he ball reach its maximum height?

$v(t) = 0$ is max height

$0 = 24.5 - 9.8t \rightarrow \frac{-24.5}{-9.8} = t$

$t = 2.5$ secs

c) What is the maximum height?

$h(2.5) = 24.5(2.5) - 4.9(2.5)^2$

$= 30.6\text{m}$

d) When does it hit the ground?

This occurs at $(h(t)) = 0$

$0 = 24.5t - 4.9t^2$

$0 = t(24.5 - 4.9t)$

$24.5 - 4.9t = 0$

$\frac{24.5}{4.9} = t$ $t = 5$ sec

e) With what velocity does it hit the ground?

$v(5) = 24.5 - 9.8(5)$

$= -24.5\text{ m/s}$

5. The distance travelled by a car is given by $s = 160t^2 + 20t$, where t is measured in hours and s in kilometers. When did the velocity reach 100km/h?

$v(t) = s'(t) = 320t + 20$

$100 = 320t + 20$

$80 = 320t$

$\frac{80}{320} = t$ $t = \frac{1}{4}\text{ hr} = 15\text{ min}$

6. The position function of a particle is $s = t^3 - 3t^2 - 5t, t \geq 0$, where t is measured in seconds and s in meters. When does the particle reach a velocity of 4m/s?

$s'(t) = 3t^2 - 6t - 5$

$4 = 3t^2 - 6t - 5$

$0 = 3t^2 - 6t - 9$

$0 = (t^2 - 2t - 3)$

$0 = (t-3)(t+1)$ Reject

$\boxed{t = 3}$ or $t = -1$

7. The position of a particle is given by $s = t^2 - 4t + 4, t \geq 0$ where s is measured in meters and t is measured in seconds.

a) Find the velocity after 1s and 3s.

$s'(t) = 2t - 4$

$s'(1) = -2\text{m/s}$ $s'(3) = 2\text{m/s}$

b) When is the particle at rest?

$0 = 2t - 4$

$t = \boxed{2\text{sec}}$

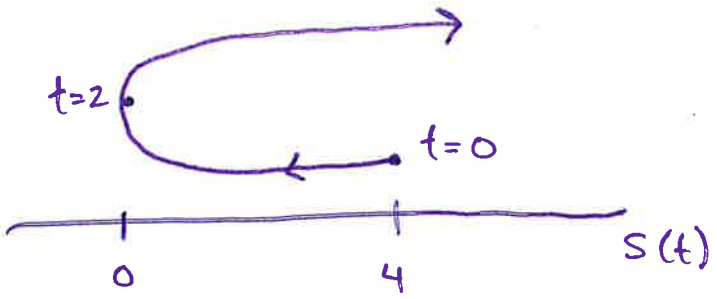
c) When is the particle moving in the positive direction?

$2t - 4 > 0$

$2t > 4$ all time after 2 secs.

$t > 2$

d) Draw a diagram to illustrate the motion of the particle.



8. The motion of a particle is described by the position function $s = t^3 - 15t^2 + 63t, t \geq 0$ where t is measured in seconds and s is measured in meters.

a) When is the particle at rest?

$v(t) = 3t^2 - 30t + 63$

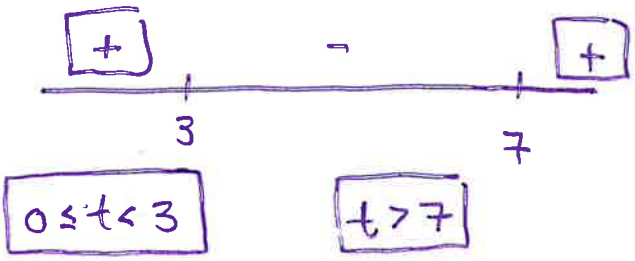
at rest at 3 secs and 7 secs

$0 = 3t^2 - 30t + 63$

$0 = t^2 - 10t + 21$

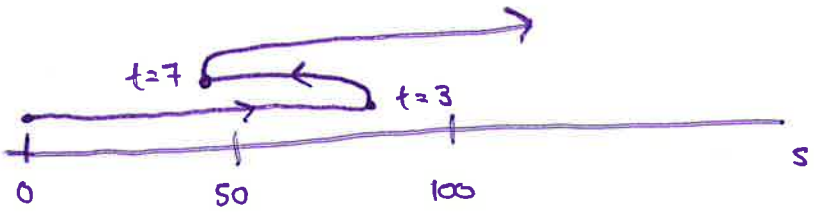
$0 = (t-3)(t-7)$

b) When is the particle moving in the positive direction?



$0 \leq t < 3$ $t > 7$

c) Draw a diagram to illustrate the motion of the particle.



when $t = 3$

$s(3) = 81$

$t = 7$

$s(7) = 49$

$t = 0$

$s(0) = 0$

d) Find the total distance travelled in the first 10s.

$|s(3) - s(0)| + |s(7) - s(3)| + |s(10) - s(7)|$

$|81 - 0| + |49 - 81| + |130 - 49|$

$81 + 32 + 81$

194m

9. If a ball is thrown upward with a velocity of 10m/s from top of the CN Tower, 450m above the ground. Then the distance, in meters, of the ball above the ground level after t seconds is $s = 450 + 10t - 5t^2$

a) When does the ball reach its maximum height?

$s'(t) = 0$ is max height $\quad s'(t) = 10 - 10t$

$0 = 10 - 10t$

$\frac{-10}{-10} = t \quad t = 1$ at $\boxed{1 \text{ sec}}$

b) Use the quadratic formula to find how long it takes for the ball to reach the ground.

$0 = 450 + 10t - 5t^2$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow \frac{-10 \pm \sqrt{10^2 - 4(-5)(450)}}{2(-5)}$

Positive time only $= \frac{-10 \pm \sqrt{9100}}{-10} = \frac{-10 \pm 10\sqrt{91}}{-10}$

$1 + \sqrt{91} \approx \boxed{10.5 \text{ sec}}$ $\quad = 1 \pm \sqrt{91}$

c) Find the approximate velocity with which the ball strikes the ground.

$V(10.5) = 10 - 10(10.5)$

approx $\boxed{-95 \text{ m/s}}$