3.1 Velocity

A **scalar** quantity is one which has only magnitude (just a number). Examples are mass, distance, and time. A **vector** is a quantity that has magnitude and direction. Examples are velocity, acceleration, and force.

Consider an object moving in a straight line such as a rock being thrown straight up in the air, or a car being driven down the road. The position function is $s(t)$ and if the initial position ($t = 0$) of the object is taken to be zero (the origin) then the position is equal to the **displacement** (the direction and distance that an object has travelled) of the object[1]. Unless the object changes direction. In general, displacement is defined as the *change* in an objects position (Δs). Recall that the **instantaneous velocity** (speed and direction of the object) at time t is defined as the limit of the **average velocity** of the object over shorter and shorter time intervals:

$$v(t) = s'(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$$

Thus, the velocity is the derivative of the position function and in Leibniz notation is written

$$v = \frac{ds}{dt} \qquad \longleftarrow \text{derivative of position with respect to time}$$

Note

The sign convention we will use is the same as that used in physics. For vectors that are directed to the **right** or **up** the sign of the vector quantity will be *positive*. For vectors that are directed to the **left** or **down** the sign of the vector quantity will be *negative*. This **does not** apply to scalar quantities such as time, distance, or speed, they will always be positive since they have no direction associated with them.

Ex. 1

If a stone is dropped from a cliff that is 122.5 m high, then its height in metres after t seconds is $h(t) = 122.5 - 4.9t^2$. Note that this is a height *above* the ground level, so the initial position of the stone is $h = 122.5$ m *above* the ground which is referenced as the initial position. If this were done in a Physics 11 class, the position of the stone would be written $d = \frac{1}{2}gt^2$ with $g = -9.80$ m/s^2. Here, the top of the cliff is referenced as the initial position to be zero and $d = -122.5$ m.

   (a) Find its velocity after 1 s and 2 s.
   (b) When will the stone hit the ground?
   (c) With what velocity will it hit the ground?

a) $h'(t) = v(t) = -9.8t$ at $t = 1$ $\boxed{v(1) = -9.8\,m/s}$
                         $t = 2$ $\boxed{v(2) = -19.6\,m/s}$

b) hits ground when $h(t) = 0$ $\rightarrow 0 = 122.5 - 4.9t^2 \rightarrow t^2 = 25$ $\boxed{t = \pm 5}$ only take positive time

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[1] The symbol s comes from the Latin **spatium** referring to the space between two locations. If you have taken physics from me the symbol $\vec{d}$ is used for displacement (again assuming the initial position is zero) and d is used to represent distance. Recall from physics class that the arrow above the d was dropped and the context was used to determine if d represented displacement or distance. In most cases in physics class d referred to displacement. I am using s for position here because that is what is used in your textbook.

c) $v(5) = -9.8(5)$
     $= \boxed{-49\,m/s}$

Ex. 2
The position of a particle moving on a line is given by the equation
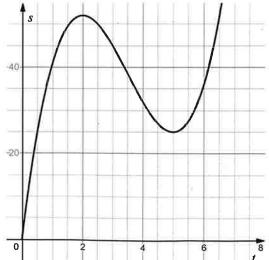
$$s(t) = 2t^3 - 21t^2 + 60t, \quad t \geq 0$$
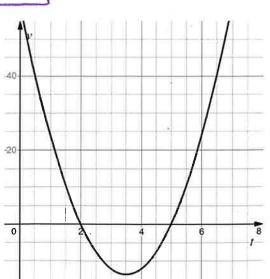
Where t is measured in second and s in metres.
   (a) What is the velocity after 3 s and after 6 s?
   (b) When is the particle at rest?
   (c) When is the particle moving in the positive direction?
   (d) Find the total *distance* travelled by the particle during the first 6 s

$v(t) = s'(t) = 6t^2 - 42t + 60$

a) $v(3) = 6(3)^2 - 42(3) + 60$

$= \boxed{-12 m/s}$

$v(6) = 6(6)^2 - 42(6) + 60$

$= \boxed{24 m/s}$

b) $v(t) = 0$ when?

$0 = 6t^2 - 42t + 60$

$0 = t^2 - 7t + 10$

$0 = (t-2)(t-5)$

$\boxed{\text{at rest at } t = 2 \quad t = 5}$

d) $|s(2) - s(0)| + |s(5) - s(2)| + |s(6) - s(5)|$

$|52 - 0| + |25 - 52| + |36 - 25|$

$52 + 27 + 11 = \boxed{90m}$

c)

Positive direction
$0 \leq t < 2$ and $t > 5$

Homework Assignments
   • Section 3.1: #1 – 9

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