

Section 3.1 – Properties of Rational Expressions

This booklet belongs to: _____ Block: _____

- A rational expressions is one where there are two polynomials
 - One is the numerator the other the denominator

Example: $\frac{a}{b}$, where $b \neq 0$

- Recall the Domain of a function is all allowable values of x that satisfy an equation
- With Rational functions there will be values that are not permitted (make denominator 0)
- These values are called: Undefined Values

Undefined Values

- Since dividing by zero is not allowed, set the denominator to 0 and solve for the unknown
- That value becomes your undefined value and is not in the Domain of the Rational Function

<u>Example</u>	<u>Solution</u>
a) $\frac{2}{x}$	a) $x \neq 0$
b) $\frac{x+2}{x-3}$	b) $x-3 \neq 0$ $x \neq 3$
c) $\frac{1}{x^2-9}$	c) $x^2-9 \neq 0$ $(x+3)(x-3) \neq 0$ $x \neq 3, x \neq -3$
d) $\frac{x-3}{x^2+9}$	d) $x^2+9 \neq 0$ $x^2 \neq -9$ Not possible for any values of x So No Undefined Values

- To get an accurate telling of a rational function, it is important to remember that they can be simplified, much the way we simplified fraction in grade 9.
- The simplified portion can cancel out and we get a unique graphing feature we will explore later this year

Example: Simplify $\frac{3x - 3}{6x - 6}$

Solution:

$$\frac{3x - 3}{6x - 6} = \frac{3(x - 1)}{6(x - 1)}, \text{ Where } x - 1 \neq 0, \text{ so } x \neq 1$$

$$\frac{3(x - 1)}{6(x - 1)} = \frac{\cancel{3}(x - \cancel{1})}{2 \cdot \cancel{3}(x - \cancel{1})} = \frac{1}{2}$$

Example: Simplify $\frac{x - 2}{x^2 - 4}$

Solution:

$$\frac{x - 2}{x^2 - 4} = \frac{(x - 2)}{(x + 2)(x - 2)}, \text{ Where } x - 2 \neq 0, \text{ and } x + 2 \neq 0 \text{ so } x \neq 2, -2$$

$$\frac{\cancel{(x - 2)}}{\cancel{(x - 2)}(x + 2)} = \frac{1}{(x + 2)}$$

Example: Simplify $\frac{x^2y + xy^2}{xy + y^2}$

Solution:

$$\frac{x^2y + xy^2}{xy + y^2} = \frac{xy(x + y)}{y(x + y)}, \text{ Where } x \neq -y, \text{ and } y \neq 0$$

$$\frac{\cancel{xy}(x + \cancel{y})}{\cancel{y}(x + \cancel{y})} = x$$

Example: Simplify $\frac{x^2 - x - 6}{x^2 - 4}$

Solution:

$$\frac{x^2 - x - 6}{x^2 - 4} = \frac{(x - 3)(x + 2)}{(x - 2)(x + 2)}, \text{ Where } x \neq -2, 2$$

$$\frac{(x - 3)\cancel{(x + 2)}}{(x - 2)\cancel{(x + 2)}} = \frac{(x - 3)}{(x - 2)}$$

Example: Simplify $\frac{x^2 - 9}{x^2 - 7x + 12}$

Solution:

$$\frac{x^2 - 9}{x^2 - 7x + 12} = \frac{(x - 3)(x + 3)}{(x - 3)(x - 4)}, \text{ Where } x \neq 3, 4$$

$$\frac{\cancel{(x - 3)}(x + 3)}{\cancel{(x - 3)}(x - 4)} = \frac{(x + 3)}{(x - 4)}$$

Common Errors

The **Fundamental Principle of Rational Expressions** applies to **Common Factors**, not **common Terms**

$\frac{\cancel{x} + 1}{\cancel{x}}$	←	In this expression, x is not a factor of the numerator and denominator, therefore the simplification shown is incorrect.
$\frac{x^2 + x + 3}{x^2 + 3}$	←	x^2 and 3 are terms, not factors, therefore they cannot be simplified. This expression is already in lowest terms.
$\frac{\cancel{x} - 2}{\cancel{x} + 1}$	←	x is a term, not a factor
$\frac{2\cancel{x}^2 - 9}{2\cancel{x} - 3}$	←	$2x^2$, $2x$ and 9 , 3 are terms not factors

Section 3.1 – Practice Problems

Simplify the expressions, assume non-zero denominators

1. $\frac{x-1}{x-1}$

2. $\frac{x-1}{1-x}$

3. $\frac{x+1}{1+x}$

4. $\frac{x+2}{-2-x}$

5. $\frac{(x-1)(x+1)}{(1-x)(-x-1)}$

6. $\frac{3-x}{x+3}$

7.
$$\frac{(x - 2)(x - 1)(x + 2)}{(2 - x)(1 - x)(2 - x)}$$

8.
$$\frac{x^2 - 4}{4 - x^2}$$

9.
$$\frac{x^2 - 2x + 1}{-x^2 + 2x - 1}$$

10.
$$\frac{a - b + c}{b - c - a}$$

11.
$$\frac{8x}{14y}$$

12.
$$\frac{15xy}{24x}$$

13. $\frac{6(x + 2)}{10(x + 2)}$

14. $\frac{(x + 1)(x - 1)}{(x - 1)^2}$

15. $\frac{x^2 - xy}{x^2}$

16. $\frac{2x^2 - 8x}{4 - x}$

17. $\frac{x^3}{x^3 + x^2y}$

18. $\frac{4x^2 + 16x}{x^2 - 16}$

19. $\frac{x^2 + 2x}{x^2 + 3x + 2}$

20. $\frac{x^2 + 9x + 18}{x^2 + 6x}$

21. $\frac{2x^2 + 5x - 3}{x^2 - 9}$

22. $\frac{3x^2 - 10x - 8}{x^2 - 16}$

23. $\frac{2x^2 + 17x + 35}{3x^2 + 19x + 20}$

24. $\frac{5x^2 - 32x + 12}{4x^2 - 27x + 18}$

25.
$$\frac{7x^2 + 61x - 18}{7x^2 + 19x - 6}$$

26.
$$\frac{8x^2 - 51x + 18}{8x^2 + 29x - 12}$$

27.
$$\frac{3x - 21}{28 - 4x}$$

28.
$$\frac{x^2 + 7x - 18}{12 - 4x - x^2}$$

29.
$$\frac{x^2 + 2xy - 3y^2}{2x^2 - xy - y^2}$$

30.
$$\frac{x^2 - 3xy + 2y^2}{x^2 - 4y^2}$$

Answer Key – Section 3.1

1. 1
2. -1
3. 1
4. -1
5. 1
6. <i>Does Not Simplify</i>
7. $-\frac{(x+2)}{(x-2)}$
8. -1
9. -1
10. -1
11. $\frac{4x}{7y}$
12. $\frac{5y}{8}$
13. $\frac{3}{5}$
14. $\frac{(x+1)}{(x-1)}$
15. $\frac{(x-y)}{x}$

16. $-2x$
17. $\frac{x}{(x+y)}$
18. $\frac{4x}{x-4}$
19. $\frac{x}{x+1}$
20. $\frac{x+3}{x}$
21. $\frac{(2x-1)}{(x-3)}$
22. $\frac{(3x+2)}{(x+4)}$
23. $\frac{2x+7}{3x+4}$
24. $\frac{5x-2}{4x-3}$
25. $\frac{x+9}{x+3}$
26. $\frac{x-6}{x+4}$
27. $-\frac{3}{4}$
28. $-\frac{(x+9)}{(x+6)}$
29. $\frac{x+3y}{2x+y}$
30. $\frac{x-y}{x+2y}$

Extra Work Space