Section 3.1 – Practice Problems

1. Suppose $y = f(x)$ has the point (a, b). Write (a, b) with the transformations described.

a) A polynomial is continuous, because it has no gaps, breaks, of holes.

b) A polynomial of **even degree** has at most how many zeros? At least how many zeros? How many turning points?

At least: 0
At most: n
n-1 turning points

c) A polynomial of **odd degree** has at most how many zeros? At least how many zeros? How many turning points?

At least: 1
At most: n
Turning Points: n-1

d) If $x = a$ is a zero of a polynomial then:

$x = a$ is also called a solution

A factor of the polynomial is: (x-a)

$(a, 0)$ is what kind of intercept?

x-intercept

e) A polynomial written in Standard Form is written with powers on the variables in what kind of order?

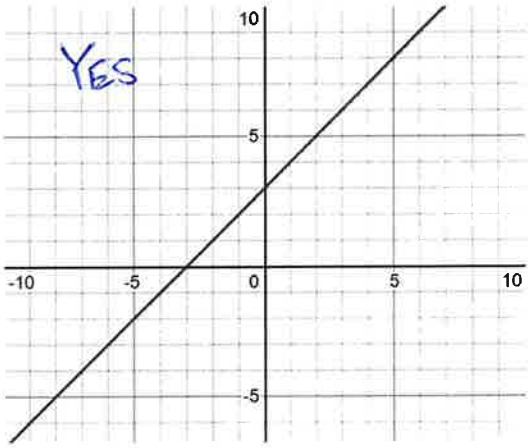
Descending Order

2. Are the following equations polynomials? If so, state the degree, leading coefficient, and special name (if available), if it is not a polynomial, state the reason.

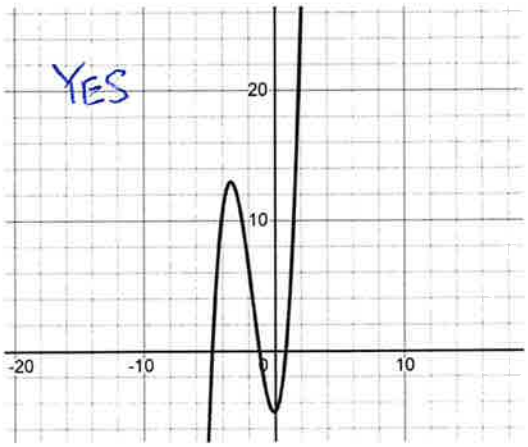
Equation	Polynomial Y/N	Degree	Leading Coefficient	Special Name
a) $-2x^3 + x^2 - 5$	YES	3	-2	cubic
b) $\sqrt{2}x^4 - \sqrt{3x} + 2$	NO	$\sqrt{3x}$ is not allowed for a polynomial		
c) $-\frac{1}{3}x^2 + \sqrt{-2}x + 1$	NO	$\sqrt{-2}$ is not a real number		
d) $3x + 2$	YES	1	3	Linear
e) 5	YES	0	5	Constant

3. Which of the following graphs are Polynomial? If they are not, explain why.

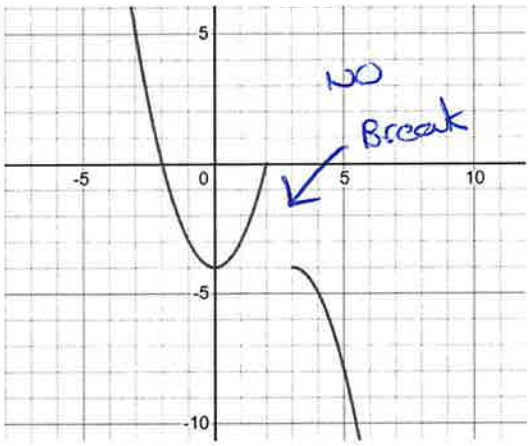
a)



b)



c)



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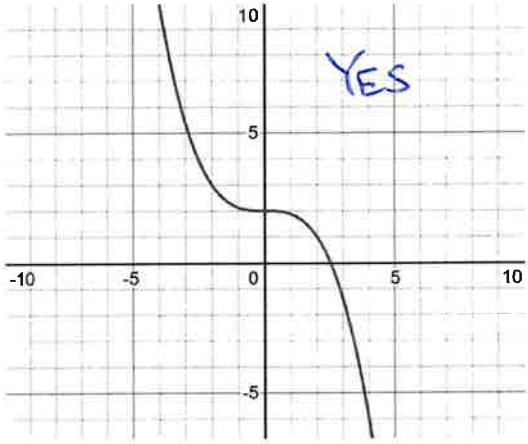
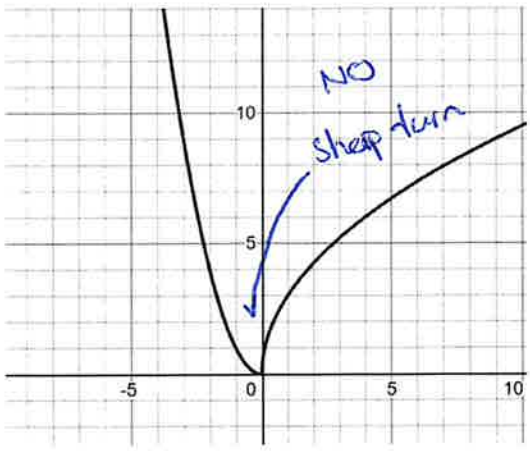
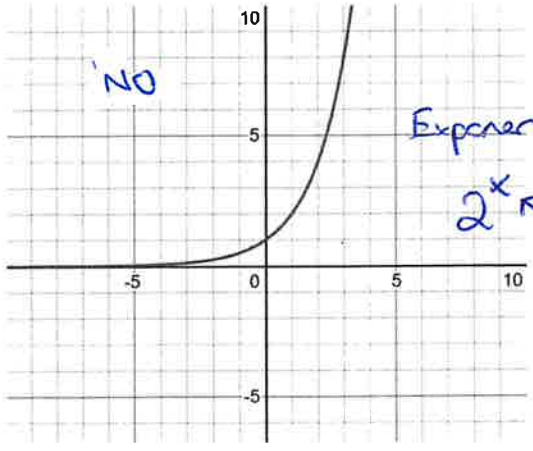


e)

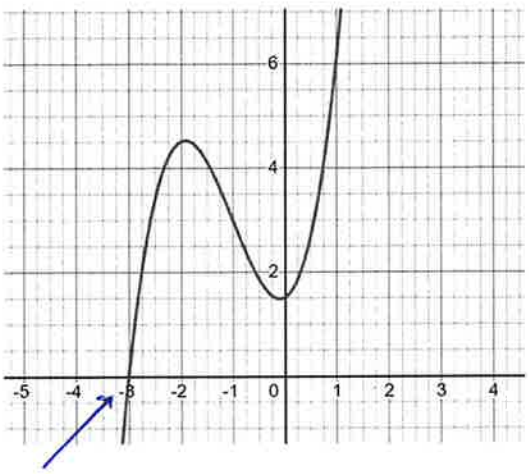


f)



4. What are the real zeros of the following polynomials?

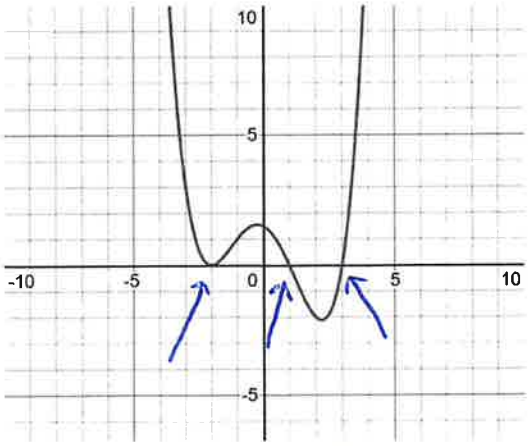
a)



$x = -3$

$(x+3)$

b)



$x = -2$ $(x+2)$

$x = 1$ $(x-1)$

$x = 3$ $(x-3)$

5. What is the max/min number of zeros of the following Polynomials.

a) $3x^5 + 3x^4 - 2x^2 + 1 = 0$

odd degree so min root is 1

max is 5

$8x^6 + 3x^4 - 2x^2 + 1 = 0$

Even degree so min root is 0

max root is 6

b) Given a general polynomial:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

What is the max/min number of roots when n is odd? max: n min: 1

What is the max/min number of roots when n is even? max: n min: 0

6. State whether the following are polynomial functions. If so, what is the degree, if not why?

a) $f(x) = -x^4 + 4x^4 + 2$

Yes, degree 4

b) $f(x) = \sqrt{x}$

NO; $x^{\frac{1}{2}}$ not a polynomial

c) $f(x) = \frac{1}{x}$ → x^{-1}

NO; not a polynomial

d) $f(x) = 0$

Yes; degree 0

e) $f(x) = (x-2)^3$

Yes; degree 3

f) $f(x) = (x+1)^{-2}$

NO; -2 not a polynomial

g) $f(x) = x^3 - \sqrt{2}x + \frac{1}{3}$

Yes; degree 3

h) $f(x) = 2^{-3}x^2$ → $\frac{x^2}{2^3}$ → $\frac{x^2}{8}$

Yes; degree 2

i) $f(x) = \sqrt{2}x^2$

Yes; degree 2

j) $f(x) = \frac{1}{x+1}$ → $(x+1)^{-1}$

NO; Exp -1 not allowed

7. Determine the end behaviour of the polynomials below.

a) $f(x) = 3x$

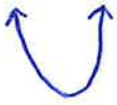
Linear; positive slope

b) $f(x) = -3x$

Linear; negative slope

c) $f(x) = 2x + 3x^2$ → $3x^2 + 2x$

Quadratics



d) $f(x) = 2x - 3x^2$ → $-3x^2 + 2x$

Quadratic

e) $f(x) = -2x + x^2 + 3x^3$

$3x^3 + x^2 - 2x$

Cubic

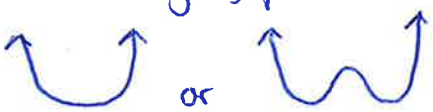
f) $f(x) = 2x - x^2 - 3x^3$

$-3x^3 - x^2 + 2x$

Cubic

g) $f(x) = 3x^4 - x^2 + 1$

Fourth Degree; positive



h) $f(x) = -3x^4 + x^2 - 1$

Fourth Degree; negative

i) $f(x) = x^4 + 2x^3 + x^5 - 2$

$x^5 + x^4 + 2x^3 - 2$

Fifth Degree

j) $f(x) = x^4 - 2x^3 - x^5 + 2$

$-x^5 + x^4 - 2x^3 + 2$

Fifth Degree

8. Find a function in the form $y = cx^n$ that has the same end behaviour as the given function.

Answers Vary

a) $f(x) = -3x^3 - 2x^2 + 1$

Rises left, lowers right

can do linear

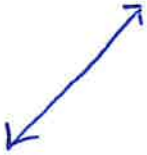
$-3x^n$

b) $g(x) = 2x^3 + x^2 - 1$

Rises right, lowers left

can be linear, positive slope

$2x^n$



c) $h(x) = 2.3x^4 - 4x^2 + 6x$

Rises both left/right

$2.3x^n$

n is even

d) $k(x) = -2.4x^5 + 3x^4 - 2x - 1$

Rises Left, lowers right

$-2.4x^n$

n is odd

9. Find all the real zeros, and the multiplicity of each zero.

a) $f(x) = x^2 - 4$

$f(x) = (x+2)(x-2)$

$x = \pm 2$

b) $f(x) = (x-4)^2$

$(x-4)(x-4)$

$x = 4$ multiplicity of 2

c) $g(x) = x^3 - 4x^2 + 4x$

$x(x^2 - 4x + 4)$

$x(x-2)(x-2)$

$x = 0$

$x = 2$ (multiplicity of 2)

d) $g(x) = 2x(x^2 - 2x - 1)$

Quadratic Equation

$2x(x - 2x - 1)$

$x = 0$

$x = 1 + \sqrt{2}$

$x = 1 - \sqrt{2}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)}$

$\frac{2 \pm \sqrt{8}}{2}$

$\frac{2 \pm 2\sqrt{2}}{2} = 1 \pm \sqrt{2}$

e) $h(x) = x^4 - x^3 - 20x^2$

$x^2(x^2 - x - 20)$

$x^2(x-5)(x+4)$

$x = 0$ (multiplicity of 2)

$x = 5$

$x = -4$

f) $h(x) = \frac{1}{3}x^4 - \frac{1}{3}$

$= \frac{1}{3}(x^4 - 1) \rightarrow \frac{1}{3}(x^2 - 1)(x^2 + 1)$

$\frac{1}{3}(x+1)(x-1)(x^2+1)$

not a real root

$x = \pm 1$

g) $k(x) = x^4 + 3x^2 + 2$

let $x^2 = z$

$z^2 + 3z + 2$

$(z+2)(z+1)$

$(x^2+2)(x^2+1)$

no real root

$x = \varnothing$ ← empty set, no solutions

h) $k(x) = x^3 - 4x^2 - 25x + 100$

Factor by grouping

$(x^3 - 4x^2)(-25x + 100)$

$x^2(x-4) - 25(x-4)$

$(x^2 - 25)(x-4)$

$(x+5)(x-5)(x-4)$

$x = \pm 5$

$x = 4$

i) $l(x) = -x^3 - 3x^2 + 4x + 12$

$(-x^3 - 3x^2)(+4x+12)$

$-x^2(x+3) + 4(x+3)$

$(-x^2+4)(x+3) \rightarrow -1(x^2-4)(x+3)$

$-1(x+2)(x-2)(x+3)$

$x = \pm 2 \quad x = -3$

j) $l(x) = x^3 - 5x^2 - x + 5$

$(x^3 - 5x^2)(-x+5)$

$x^2(x-5) - 1(x-5)$

$(x^2-1)(x-5)$

$(x+1)(x-1)(x-5)$

$x = \pm 1 \quad x = 5$

k) $m(x) = x^4 - 2x^2 + 1$

Let $z = x^2$

$z^2 - 2z + 1$

$(z-1)(z-1)$ sub back in x^2

$(x^2-1)(x^2-1)$ $x = \pm 1$

$(x-1)(x+1)(x-1)(x+1)$ both multiplicity of 2

l) $m(x) = -x^4 + 3x^2 - 2$

Factor -1

$-1(x^4 - 3x^2 + 2)$ Let $z = x^2$

$-1(z^2 - 3z + 2)$

$-1(z-1)(z-2) \rightarrow -1(x^2-1)(x^2-2)$

$-1(x+1)(x-1)(x^2-2)$

$x = -1 \quad x = \sqrt{2}$

$x = 1 \quad x = -\sqrt{2}$

m) $n(x) = -x^4 + 4x^3 - 4x^2$

$-x^2(x^2 - 4x + 4)$

$-x^2(x-2)(x-2)$

$x = 0$ multiplicity 2

$x = 2$ multiplicity 2

n) $n(x) = -x^2(x^2-1) + 4(x^2-1)$

$(-x^2+4)(x^2-1)$

$-1(x^2-4)(x^2-1)$

$-1(x+2)(x-2)(x+1)(x-1)$

$x = \pm 2$

$x = \pm 1$

See Website for Detailed Answer Key of the Remainder of the Questions