Section 3.1 – One and Two Step Equations

This booklet belongs to:______________________________**Block:** ______

So, it begins….

- When we think algebra, what comes to mind?
    - Headaches, moans & groans, anxiety…
- Don't get yourself too riled up. **Algebra** is just the **logical manipulation** of an equation.
- That's where we start. With an equation.
- In order to be considered an equation you need a statement of inequality.

Either: $=$ $<$ $>$ $\leq$ $\geq$

- Whenever you have one of these in a statement it makes it an **equation**
- One side maintains equality with the other

In other words:

BALANCE

Whatever we do from this point on in an equation, we have to use logical rules in order to maintain that balance, that equality.

Addition and Subtraction

- It's called the **ADDITION PRINCIPLE (ADDING TO MAKE 0)**

Consider this,

$3 = 3$ we have BALANCE

- So, if we **ADD** something to **one side** we have to **add it to both**:

$$3 + 2 = 3 + 2$$

- We use this concept to help **eliminate information** from one side of an equation
- This in turns adds it to the other side

Example 1: $r - 4 = 7$

On the left we have an unknown. We need to get that unknown by itself on one side of the equals sign.

How do we do that?

- Well we have -4, in order to **eliminate it**, we need it to be 0
- So, what do we **add** to -4 to make it 0, we need to **add** $+\mathbf{4}$

Solution 1:

$$r - 4 + 4 = 7 + 4$$

Add $+4$ to both sides

And now, since $-4 + 4 = 0$, we get $r + 0$ on the left, which is r

So, after **the elimination** we get: $\boldsymbol{r = 11}$ and we have **solved for the unknown**

- Now the previous example saw us **subtracting** from the unknown so we had to **add a positive** to both sides.
- When we **add** with the unknown, we have to **add a negative (subtract)** from both sides.

Example 2:

$q + 5 = 15$

$q + 5 - 5 = 15 - 5$

$q = 10$

Added a negative to both sides,
in other words:

Subtracted

Example 3:

$$r - 4 = 7$$

$$r - 4 + 4 = 7 + 4$$

$$r = 11$$

$$t + 5 = 2$$

$$t + 5 - 5 = 2 - 5$$

$$t = -3$$

Example 4:

$$q - 8 = 10$$

$$q - 8 + 8 = 10 + 8$$

$$q = 18$$

$$x + 4 = -6$$

$$x + 4 - 4 = -6 - 4$$

$$x = -10$$

$$a - 6 = -13$$

$$a - 6 + 6 = -13 + 6$$

$$a = -7$$

$$b + 8 = -2$$

$$b + 8 - 8 = -2 - 8$$

$$b = -10$$

Multiplication and Division

It's called the **MULTIPLICATION PRINCIPLE (Multiplying to get 1)**

- **Multiplication and Division are inverses** of one another
- Much like **adding a negative** is the same as **subtraction**
- **Multiplying a fraction** is the same as **dividing**

Now for multiplication and division the number we want isn't 0, it's 1

- When we are **multiplying with the variable** we have to **divide** to end up with 1

Example 5:

$$3x = 12$$

- I don't want $3x$, I want $1x$, so I'll have to **divide by 3 (or multiply by $\frac{1}{3}$)**

$$\frac{3x}{3} = 1x$$

But don't forget the whole **balance thing**. We need to **divide both sides**

$$\frac{3x}{3} = \frac{12}{3}, \qquad 1x = 4 \; or \; x = 4$$

- When we **multiply** with the **variable**, we do the **inverse**, **division**
- Then, if we **divide** with the **variable**, we do the **inverse**, **multiplication**

Consider this,

$$\frac{1}{2} \cdot 2 = 1$$

- If you **multiply a fraction** by its **denominator** the cancel one another out, because the top and bottom divide to give you 1

$$\frac{1}{2} \cdot 2 = \frac{1 \cdot 2}{2} = \frac{2}{2} = 1$$

So,

$$\frac{t}{5} = 10$$

- Since we are **dividing** with the **variable**, we have to **multiply**

$$\frac{t}{5} = 10, \qquad 5 \cdot \frac{t}{5} = 10 \cdot 5, \qquad \frac{5t}{5} = 50, \qquad t = 50$$

Multiply both sides by 5 Divide the left out

Example 6:

$5x = 10$	$-3r = 27$
$\frac{5x}{5} = \frac{10}{5}$	$\frac{-3r}{-3} = \frac{27}{-3}$
$x = 2$	$r = -9$

Example 7:

$4a = 3$ $\frac{4a}{4} = \frac{3}{4}$ $a = \frac{3}{4}$	$8n = 2$ $\frac{8n}{8} = \frac{2}{8}$ $n = \frac{1}{4}$
$\frac{q}{5} = 2$ $5 \cdot \frac{q}{5} = 2 \cdot 5$ $q = 10$	$\frac{d}{-4} = -8$ $-4 \cdot \frac{d}{-4} = -8 \cdot -4$ $d = 32$
$\frac{b}{7} = 2$ $7 \cdot \frac{b}{7} = 2 \cdot 7$ $b = 14$	$\frac{v}{4} = -12$ $4 \cdot \frac{v}{4} = -12 \cdot 4$ $v = -48$

- These are all $\boldsymbol{1 - Step}$ equations
- They take 1 step to get your answer
- Addition, Subtraction, Multiplication, and Division

Next, we will see examples that require **2 or more Steps**

Two Steps

If you are **multiplying with a constant with a variable and adding or subtracting a number to it**.

- We need **2 *Steps***

Example 8:

$$2x + 5 = 11$$

- First **get rid of the number** that is **being added or subtract**, leaving **a constant-variable product**

$$2x + 5 - 5 = 11 - 5$$

- The 5's cancel, leave us with:

$$2x = 6$$

- Then **divide the 2, on both sides,** to **isolate the variable** and solve

$$\frac{2x}{2} = \frac{6}{2} \quad \rightarrow \quad \boldsymbol{x = 3}$$

Example 9:

$$3q - 8 = 10$$

$$3q - 8 + 8 = 10 + 8$$

$$3q = 18$$

$$\frac{3q}{3} = \frac{18}{3} \quad \rightarrow \quad q = 6$$

$$5x + 4 = -6$$

$$5x + 4 - 4 = -6 - 4$$

$$5x = -10$$

$$\frac{5x}{5} = \frac{-10}{5} \quad \rightarrow \quad x = -2$$

$$3a - 6 = -13$$

$$3a - 6 + 6 = -13 + 6$$

$$3a = -7$$

$$\frac{3a}{3} = \frac{-7}{3} \quad \rightarrow \quad a = -\frac{7}{3}$$

$$11b + 8 = -2$$

$$11b + 8 - 8 = -2 - 8$$

$$11b = -10$$

$$\frac{11b}{11} = \frac{-10}{11} \quad \rightarrow \quad b = -\frac{10}{11}$$

If you have **one fraction multiplying with a variable**?

- We need **2** $\boldsymbol{Steps}$

Example 10:

$$\frac{2}{3}x = 6$$

- **First Multiply** by the **Denominator** on **both** sides

$$3 \cdot \frac{2}{3}x = 6 \cdot 3$$

- The $3's$ cancel on the left

$$2x = 18$$

- **Then Divide** by the **Numerator** on **both** sides

$$\frac{2x}{2} = \frac{18}{2}$$

- The $2's$ cancel out on the left

$$x = 9$$

Example 11:

$$\frac{4}{5}x = 4 \quad \rightarrow \quad 5 \cdot \frac{4}{5}x = 4 \cdot 5 \quad \rightarrow \quad 4x = 20 \quad \rightarrow \quad \frac{4x}{4} = \frac{20}{4} \quad \rightarrow \quad x = 5$$

- These can be done in **1 step** by **multiplying by the reciprocal**.
- It only works in when you have **1 fraction**, a **variable** and the **answer**

$$\frac{2}{3}x = 8 \quad \rightarrow \quad \frac{3}{2} \cdot \frac{2}{3}x = 8 \cdot \frac{3}{2} \quad \rightarrow \quad x = \frac{24}{2} \quad \rightarrow \quad x = 12$$

- Algebra is much than just solving for the unknown, at its center, it is the study and understanding of how to manipulate an equation.
- How to free-up a piece of information.
- Your experiences in math, and physics and science in particular, you will experience scenarios where you need to solve for an unknown piece of information. That is algebra.

<u>Example 12:</u> Solve for a in terms of b,c, and d. $ab + c = d$

<u>Solution 12:</u> What this means is that we need $\boldsymbol{a} =$

Step 1: Subtract c from both sides $ab + c - \boldsymbol{c} = d - \boldsymbol{c} \quad \rightarrow \quad \boldsymbol{ab = d - c}$

Step 2: Divided both sides by b $ab = d - c \quad \rightarrow \quad \frac{a\cancel{b}}{\cancel{b}} = \frac{d-c}{b}$

Step 3: Write your solution as $a =$ $\boldsymbol{a = \frac{d-c}{b}}$

<u>Example 13:</u> Here is a classic physics equation: $F = ma$ (Force equals mass times acceleration). Solve for $mass$ in terms of Force and acceleration.

<u>Solution 13:</u> So, we want m alone.

$$F = ma \quad \rightarrow \quad \frac{F}{a} = \frac{m\cancel{a}}{\cancel{a}} \quad \rightarrow \quad m = \frac{F}{a}$$

Divide both sides by a

Mass = Force divided by acceleration

<u>Example 14:</u> Solve for b. $a - b = c$

<u>Solution 14:</u> It's important to remember that we need b alone, not $-b$. There are a number of ways to tackle this, but here is my example.

Step 1: Subtract a from both sides $a - b - \boldsymbol{a} = c - \boldsymbol{a} \quad \rightarrow \quad \boldsymbol{-b = c - a}$

Step 2: Multiply every term by (-1) $(-1)(-b) = (-1)c - (-1)a \quad \rightarrow \quad b = -c + a$

Step 3: Write your solution as $b =$ $\boldsymbol{b = -c + a}$ **or** $\boldsymbol{b = a - c}$

Section 3.1 – Practice Questions

EMERGING LEVEL QUESTIONS

Use the Addition and Subtraction Principle. ISOLATE THE VARIABLE, show steps.

1. $w + 4 = 7$
2. $x + 16 = -4$
3. $t - 12 = -4$
4. $t + 9 = -3$
5. $k - 6 = -8$
6. $w - 3 = -8$
7. $z + 2 = 5$
8. $8 = j + 7$
9. $r + 5 = 12$
10. $12 = l - 4$
11. $15 = r + 12$
12. $x + 23 = -7$
13. $j + 7 = -4$
14. $23 + f = -17$
15. $j + 5 = -11$

Use the Multiplication and Division Principle. ISOLATE THE VARIABLE, show steps.

16. $3x = 12$	17. $2x = 24$	18. $4t = -13$
19. $-3t = -6$	20. $-4r = 12$	21. $-12m = 156$
22. $3t = 17$	23. $-x = 4$	24. $7h = 2$
25. $\frac{z}{7} = 9$	26. $\frac{k}{6} = -2$	27. $\frac{t}{8} = 4$
28. $\frac{r}{3} = -3$	29. $-\frac{j}{4} = -6$	30. $\frac{r}{6} = 35$
31. $-\frac{t}{2} = 5$	32. $\frac{a}{7} = 0$	33. $-\frac{w}{7} = -4$

Use two-step processes to solve the following

34. $3x - 5 = 10$	35. $-2x + 5 = 7$	36. $-4r - 4 = -4$
37. $6x + 12 = -12$	38. $3x - 7 = -5$	39. $2x + 3 = 4$
40. $-2r - 5 = 9$	41. $6x + 3 = -4$	42. $12x - 54 = 6$
43. $-7q - 2 = -9$	44. $3r - 5 = 7$	45. $-x + 3 = 8$

PROFICIENT LEVEL QUESTIONS

46. $\frac{2}{3}x = 8$

47. $-\frac{2}{5}x = 4$

48. $\frac{2}{r} = 4$

49. $-\frac{3}{7}x = 5$

50. $-\frac{6}{7}x = 2$

51. $\frac{2}{5}t = 13$

EXTENDING LEVEL QUESTIONS

In each case, solve for the variable a

52. $ab + c = d$

53. $-a + b = -d$

54. $\frac{b}{a} = cd$

Answer Key – Section 3.1

1. $w = 3$	2. $x = -20$	3. $t = 8$	4. $t = -12$
5. $k = -2$	6. $w = -5$	7. $z = 3$	8. $j = 1$
9. $r = 7$	10. $l = 16$	11. $r = 3$	12. $x = -30$
13. $j = -11$	14. $f = -40$	15. $j = -16$	16. $x = 4$
17. $x = 12$	18. $t = -\frac{13}{4}$	19. $t = 2$	20. $r = -3$
21. $m = -13$	22. $t = \frac{17}{3}$	23. $x = -4$	24. $h = \frac{2}{7}$
25. $z = 63$	26. $k = -12$	27. $t = 32$	28. $r = -9$
29. $j = 24$	30. $r = 210$	31. $t = -10$	32. $a = 0$
33. $w = 28$	34. $x = 5$	35. $x = -1$	36. $r = 0$
37. $x = -4$	38. $x = \frac{2}{3}$	39. $x = \frac{1}{2}$	40. $r = -7$
41. $x = -\frac{7}{6}$	42. $x = 5$	43. $q = 1$	44. $r = 4$
45. $x = -5$	46. $x = 12$	47. $x = -10$	48. $r = \frac{1}{2}$
49. $x = -\frac{35}{3}$	50. $x = -\frac{7}{3}$	51. $t = \frac{65}{2}$	
52. $a = \frac{d-c}{b}$	53. $a = d + b$	54. $a = \frac{b}{cd}$	

Extra Work Space