

Section 2.6 – Practice Problems

1. Suppose $y = f(x)$ has the point (a, b) . Write (a, b) with the transformations described.

a) $y = f(x - 1) + 1$ $(a, b) \rightarrow (a+1, b+1)$	b) $y = f(1 - x) \rightarrow y = f[-1(x-1)]$ $(a, b) \rightarrow (-a, b) \rightarrow (-a+1, b)$
c) $y = -f(-x)$ $(a, b) \rightarrow (-a, -b)$	d) $y = f(x) + 1$ $(a, b) \rightarrow (a, b+1)$
e) $y = f(-x)$ $(a, b) \rightarrow (-a, b)$	f) $y = -f(x)$ $(a, b) \rightarrow (a, -b)$
g) $y = f(x + 1)$ $(a, b) \rightarrow (a-1, b)$	h) $y = f^{-1}(x)$ $(a, b) \rightarrow (b, a)$
i) $y = -f^{-1}(x)$ $(a, b) \rightarrow (b, a) \rightarrow (b, -a)$	j) $y = f^{-1}(x) + 1$ $(a, b) \rightarrow (b, a) \rightarrow (b, a+1)$
k) $y = f^{-1}(x - 1)$ $(a, b) \rightarrow (b, a) \rightarrow (b+1, a)$	l) $y = f^{-1}(-x) + 1$ $(a, b) \rightarrow (b, a) \rightarrow (-b, a+1)$
m) $y = f^{-1}(x) + 1$ $(a, b) \rightarrow (b, a) \rightarrow (b, a+1)$	n) $y = -f^{-1}(-x) + 1$ $(a, b) \rightarrow (b, a) \rightarrow (-b, -a)$ $\rightarrow (-b, -a+1)$

2. If points $(4, -2)$ and (a, b) are on the graph of $y = f(x)$, what points must be on the following graphs?

a) $y = f(x - 1) - 3$

$$(4, -2) \rightarrow (5, -5)$$

$$(a, b) \rightarrow (a+1, b-3)$$

b) $y = -f(-x) + 1$

$$(4, -2) \rightarrow (-4, 2) \rightarrow (-4, 3)$$

$$(a, b) \rightarrow (-a, -b) \rightarrow (-a, -b+1)$$

c) $y = -f(x + 2) - 1$

$$(4, -2) \rightarrow (2, 2) \rightarrow (2, 1)$$

$$(a, b) \rightarrow (a-2, -b) \rightarrow (a-2, -b-1)$$

d) $y = |f(2x)|$

$$(4, -2) \rightarrow (4, 2) \rightarrow (2, 2)$$

$$(a, b) \rightarrow (a, |b|) \rightarrow (\frac{1}{2}a, |b|)$$

e) $y = \frac{1}{2}f(x - 1) + 4$

$$(4, -2) \rightarrow (5, -1) \rightarrow (5, 3)$$

$$(a, b) \rightarrow (a+1, \frac{1}{2}b) \rightarrow (a+1, \frac{1}{2}b+4)$$

f) $y = -|f(x - 2)|$

$$(4, -2) \rightarrow (6, 2) \rightarrow (6, -2)$$

$$(a, b) \rightarrow (a+2, -|b|)$$

g) $y = f\left(-\frac{1}{2}x\right) + 1$

$$(4, -2) \rightarrow (-8, -2) \rightarrow (-8, -1)$$

$$(a, b) \rightarrow (-2a, b+1)$$

h) $y = -f(1 - x) \rightarrow -f[-1(x - 1)]$

$$(4, -2) \rightarrow (-4, 2) \rightarrow (-3, 2)$$

$$(a, b) \rightarrow (-a, -b) \rightarrow (-a+1, -b)$$

i) $y = f^{-1}(x) + 2$

$(4, -2) \rightarrow (-2, 4) \rightarrow (-2, 6)$

$(a, b) \rightarrow (b, a) \rightarrow (b, a+2)$

j) $y = f^{-1}(x+1)$

$(4, -2) \rightarrow (-2, 4) \rightarrow (-3, 4)$

$(a, b) \rightarrow (b, a) \rightarrow (b-1, a)$

3. If $f(x) = x^2 - 1$, determine the equation after each of the following transformations.

a) $y = f(x+2)$

$y = (x+2)^2 - 1$

b) $y = f\left(\frac{1}{2}x\right) + 1$

$y = \left(\frac{1}{2}x\right)^2 - 1 + 1$

$y = \frac{1}{4}x^2$

c) $y = -f(x-1) + 2$

$-[(x-1)^2 - 1] + 2$

$-(x-1)^2 + 1 + 2 \rightarrow y = -(x-1)^2 + 3$

d) $y = 2f(1-x) + 3$

$y = 2[(1-x)^2 - 1] + 3$

$y = 2(1-x)^2 - 2 + 3$

$y = 2(1-x)^2 + 1$

e) Expand vertically by a factor of 3

$f(x) \rightarrow 3f(x)$

$y = x^2 - 1 \rightarrow 3(x^2 - 1)$

$3x^2 - 3$

f) Expand horizontally by a factor of 3

$f(x) \rightarrow f\left(\frac{1}{3}x\right)$

$f\left(\frac{1}{3}x\right) = \left(\frac{1}{3}x\right)^2 - 1$

$\frac{1}{9}x^2 - 1$

4. If $4x^2 + y^2 = 36$, determine the equation after each of the following transformations (these are not intuitive, is it in the form $y = f(x)$?)

would be $y = \frac{1}{3}f(x)$ so $\left(\frac{y}{\frac{1}{3}}\right)$

a) Expand horizontally by a factor of 2

$4\left(\frac{1}{2}x\right)^2 + y^2 = 36$

$x^2 + y^2 = 36$

b) Compress vertically by a factor of $\frac{1}{3}$

$4x^2 + \left(\frac{y}{\frac{1}{3}}\right)^2 = 36 \rightarrow 4x^2 + 9y^2 = 36$

would be $y = \frac{4}{3}f(x)$ so $\frac{3}{4}y$

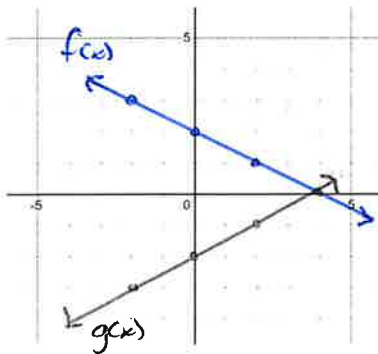
c) Compress horizontally by a factor of $\frac{1}{2}$ and expand vertically by a factor of $\frac{4}{3}$

$$4(2x)^2 + \left(\frac{3}{4}y\right)^2 = 36 \rightarrow 16 \cdot 16x^2 + \frac{9}{16}y^2 = 36 \rightarrow \boxed{256x^2 + 9y^2 = 576}$$

5. Write an expression for $f(x)$ obtained by reflecting the graph $g(x) = \frac{1}{2}x - 2$, about the:

Drawings may help.

a) x -axis



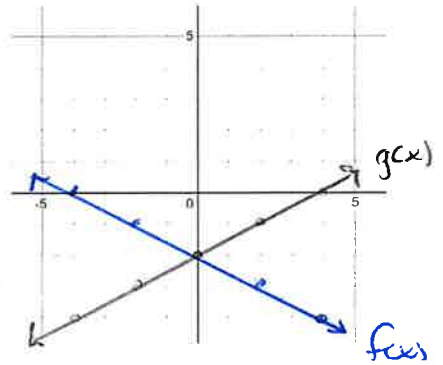
change in y -values

$$f(x) = -g(x)$$

$$-\left(\frac{1}{2}x - 2\right)$$

$$\boxed{f(x) = -\frac{1}{2}x + 2}$$

b) y -axis



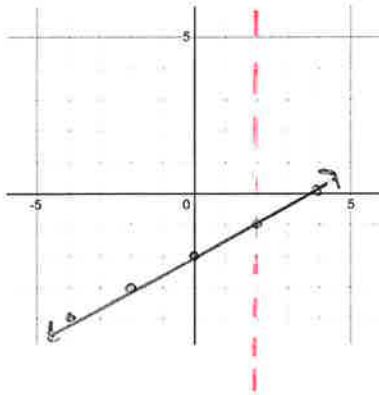
change in x -values

$$f(x) = g(-x)$$

$$g(-x) = \frac{1}{2}(-x) - 2$$

$$\boxed{f(x) = -\frac{1}{2}x - 2}$$

c) line $x = 2$



Shift $g(x)$ so line of reflection becomes y -axis

$$g(x) \rightarrow g(x+2)$$

$$g(x+2) = \frac{1}{2}(x+2) - 2 \rightarrow \frac{1}{2}x + 1 - 2$$

$$= \frac{1}{2}x - 1$$

* shifting the function shifts the line of reflection

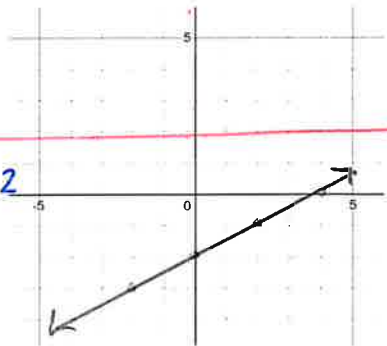
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Then reflect x -values

$$-\frac{1}{2}(x-2) - 1 = \boxed{-\frac{1}{2}x}$$

$-\frac{1}{2}x - 1$ then shift back two units

d) line $y = 2$



$$g(x) \rightarrow g(x) - 2$$

$$g(x) - 2$$

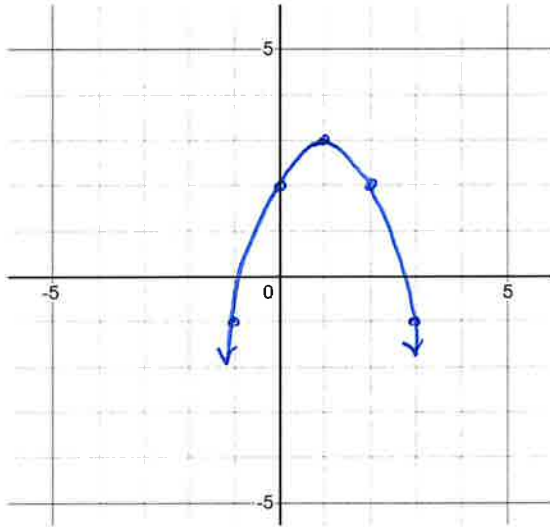
$\frac{1}{2}x - 4$ then reflect y -values

$$-\left[\frac{1}{2}x - 4\right] \rightarrow -\frac{1}{2}x + 4$$

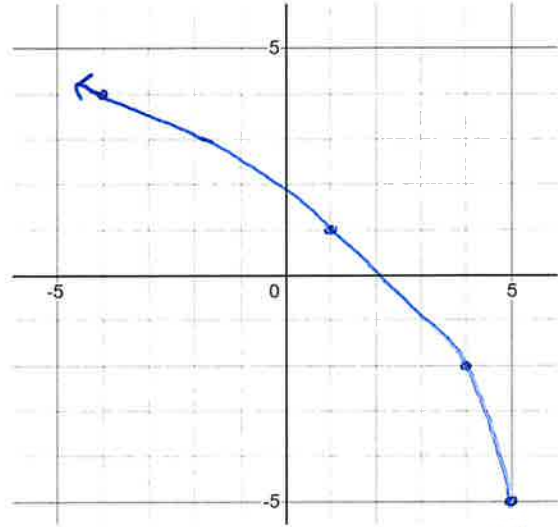
then shift up 2: $\boxed{-\frac{1}{2}x + 6}$

6. Graph the following functions without using Desmos, graph the basic form first, then graph the transformation and erase the original.

a) $f(x) = -(x - 1)^2 + 3$ Parabola
vertex (1,3)

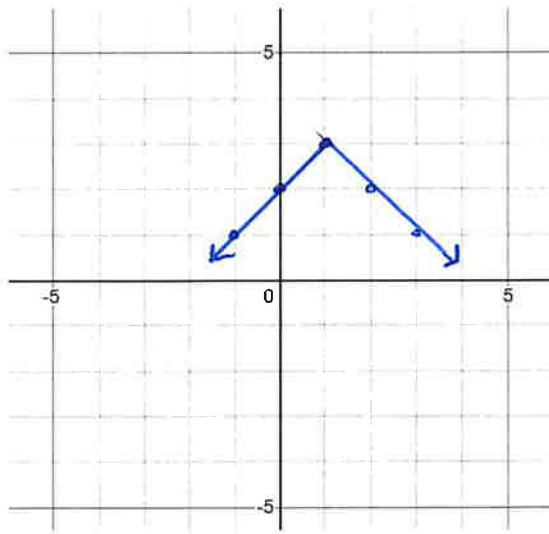


b) $f(x) = 3\sqrt{5-x} - 5$ $D: x \leq 5$
 $\rightarrow 3\sqrt{-1(x-5)} - 5$

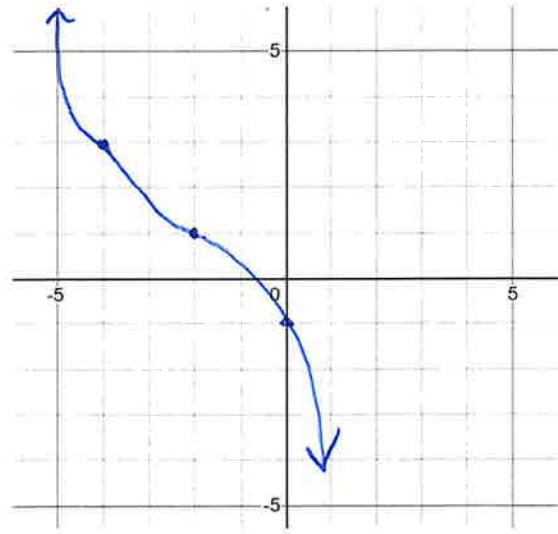


$x=5, y=-5$
 $x=4, y=-2$
 $x=1, y=1$
 $x=-4, y=4$

c) $f(x) = -|1-x| + 3$

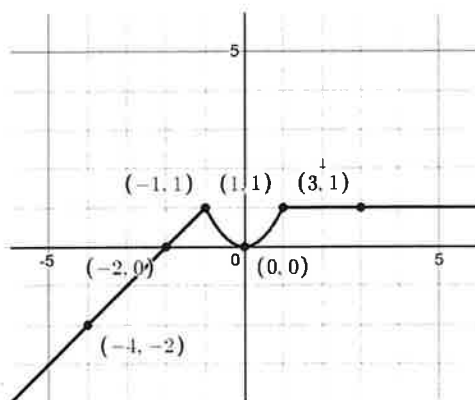


d) $f(x) = -\frac{1}{4}(x+2)^3 + 1$

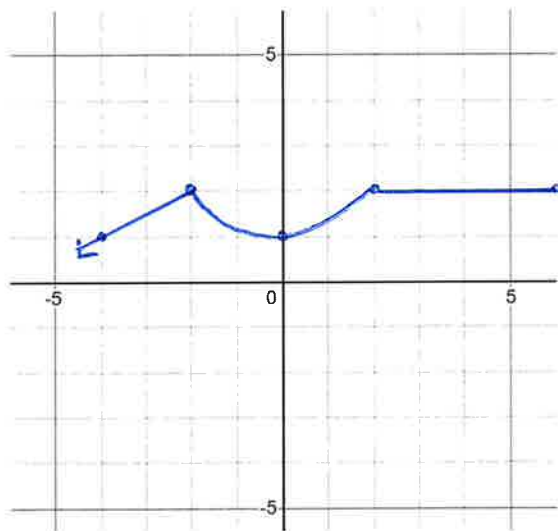


7. Given the graph of $y = f(x)$ below, sketch the graphs of the following:

Use Reference Points to make this easier.



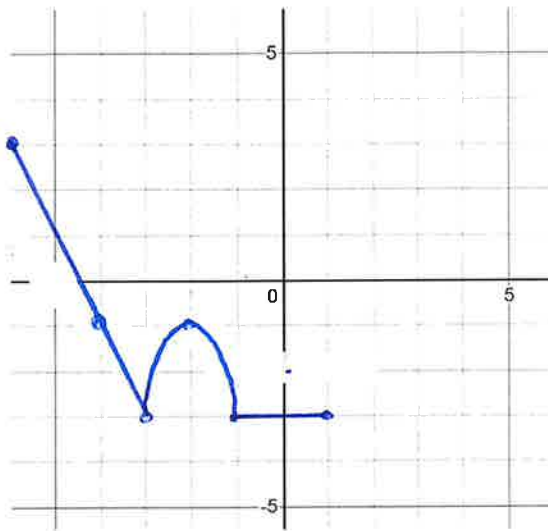
a) $y = f\left(\frac{1}{2}x\right) + 1$



- $(-4, -2) \rightarrow (-8, -1)$
- $(-2, 0) \rightarrow (-4, 1)$
- $(-1, 1) \rightarrow (-2, 2)$
- $(0, 0) \rightarrow (0, 1)$
- $(1, 1) \rightarrow (2, 2)$
- $(3, 1) \rightarrow (6, 2)$

Horizontal expansion factor of 2
Vertical Translation up 1

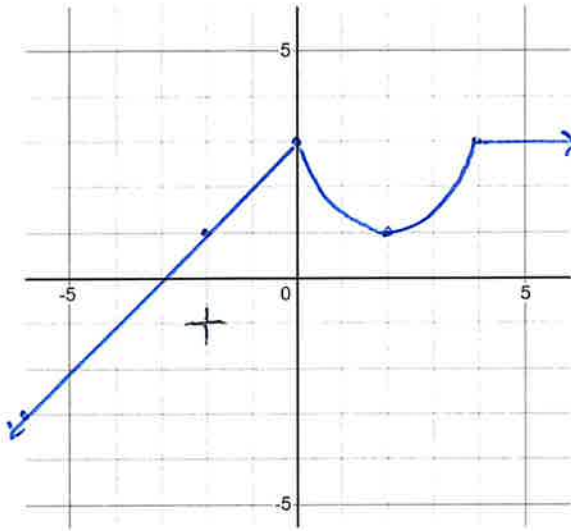
b) $y = -2f(x + 2) - 1$



- $(-4, -2) \rightarrow (-6, 3)$
- $(-2, 0) \rightarrow (-4, -1)$
- $(-1, 1) \rightarrow (-3, -3)$
- $(0, 0) \rightarrow (-2, -1)$
- $(1, 1) \rightarrow (-1, -3)$
- $(3, 1) \rightarrow (1, -3)$

Reflection of y -values, vertical stretch factor 2
Horizontal shift -2
vertical drop 1

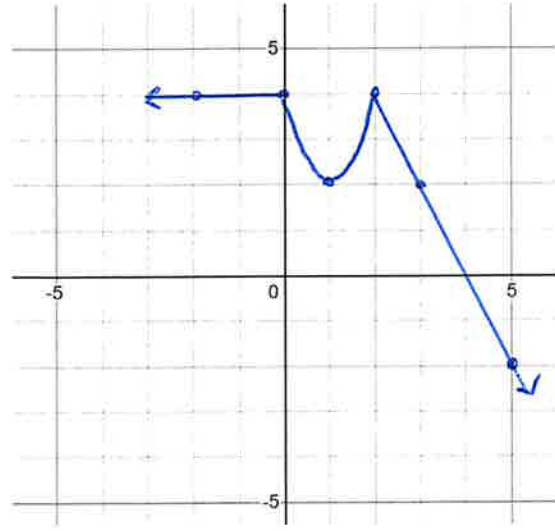
c) $y = 2f\left(\frac{1}{2}x - 1\right) + 1$ $\rightarrow 2f\left[\frac{1}{2}(x-2)\right] + 1$



- $(-4, -2) \rightarrow (-6, -3)$
- $(-2, 0) \rightarrow (-2, 1)$
- $(-1, 1) \rightarrow (0, 3)$
- $(0, 0) \rightarrow (2, 1)$
- $(1, 1) \rightarrow (4, 3)$
- $(3, 1) \rightarrow (8, 3)$

vertical stretch factor of 2, shift up 1
 horizontal stretch factor of 2, shift right 2

d) $y = 2f(1-x) + 2 \rightarrow 2f[-1(x-1)] + 2$

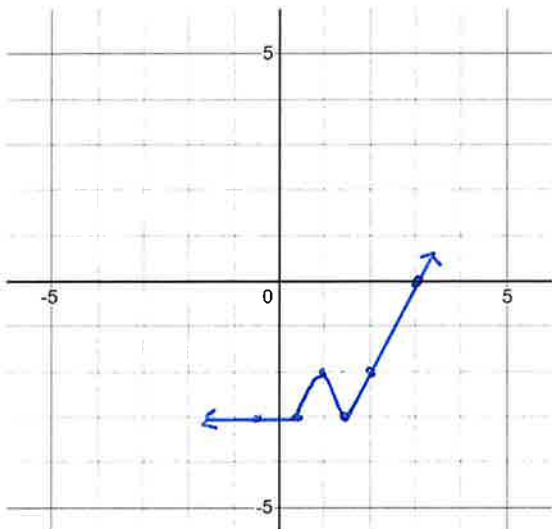


- $(-4, -2) \rightarrow (5, -2)$
- $(-2, 0) \rightarrow (3, 2)$
- $(-1, 1) \rightarrow (2, 4)$
- $(0, 0) \rightarrow (1, 2)$
- $(1, 1) \rightarrow (0, 4)$
- $(3, 1) \rightarrow (-2, 4)$

vertical stretch factor of 2, up 2
 horizontal reflection of x-values
 shift 1 right

$$-f[-2(x-1)]-2$$

$$e) y = -f(2-2x) - 2$$



$$(-4, -2) \rightarrow (3, 0)$$

$$(-2, 0) \rightarrow (2, -2)$$

$$(-1, 1) \rightarrow (3/2, -3)$$

$$(0, 0) \rightarrow (1, -2)$$

$$(1, 1) \rightarrow (1/2, -3)$$

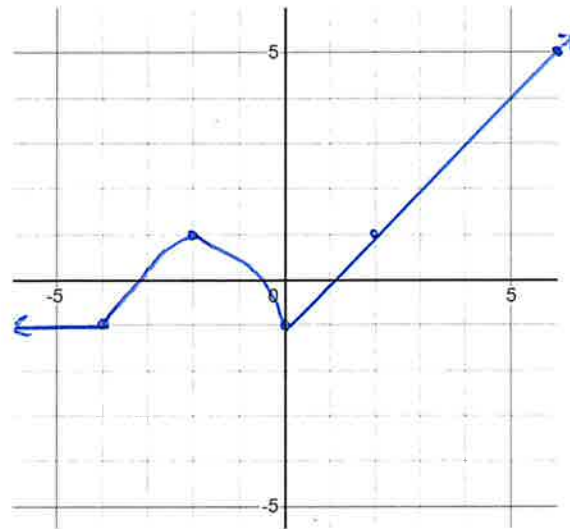
$$(3, 1) \rightarrow (-1/2, -3)$$

Reflection of y-values, down 2

Reflection of x-values, compression by $\frac{1}{2}$,
right 1

$$-2f[-\frac{1}{2}(x+2)] + 1$$

$$f) y = -2f(-\frac{1}{2}x - 1) + 1$$



$$(-4, -2) \rightarrow (6, 5)$$

$$(-2, 0) \rightarrow (2, 1)$$

$$(-1, 1) \rightarrow (0, -1)$$

$$(0, 0) \rightarrow (-2, 1)$$

$$(1, 1) \rightarrow (-4, -1)$$

$$(3, 1) \rightarrow (-8, -1)$$

Reflection of y-values, expansion factor of 2,
up 1

Reflection of x-values, expansion factor of 2,
left 2

Answer Key for Number 1

1.

a) $(a + 1, b + 1)$	b) $(1 - a, b)$	c) $(-a, -b)$
d) $a, b + 1)$	e) $(-a, b)$	f) $(a, -b)$
g) $a - 1, b)$	h) (b, a)	i) $(b, -a)$
j) $(b, a + 1)$	k) $(b + 1, a)$	l) $(-b, a + 1)$
m) $(b, 1 - a)$	n) $(-b, 1 - a)$	

See Website for Detailed Answer Key of the Remainder of the Questions