Section 2.5 – Radical Equations and Restrictions

This booklet belongs to:______________________________Block: ______

Restrictions on the Domain (Allowable values for x)

- When we think about numbers that exist (Real Numbers), $\sqrt{x+2}$ has some restrictions, because…
- We **can't have negatives** under the square root symbol
- So the restriction we are looking at is: $x+2\geq 0 \;\rightarrow\; x\geq -2$

Example: Determine the restrictions on $\sqrt{2x-3}=x-3$

Solution: All that matters is that the radicand, $2x-3\geq 0$

$2x-3\geq 0 \;\rightarrow\; 2x\geq 3 \;\rightarrow\; x\geq \frac{3}{2}$ **So the restriction is:** $\boldsymbol{x\geq \frac{3}{2}}$

Example: Determine the restrictions on $\sqrt{3x+4}-\sqrt{2x-4}=2$

Solution: Since we have two radicands, we need to **check both**

$3x+4\geq 0$	$2x-4\geq 0$
$3x\geq -4$	$2x\geq 4$
$x\geq -\dfrac{4}{3}$	$x\geq 2$

Since the restrictions **starts in the negatives** but has **another value at 2**, we need to take the larger number as the start point.

So the restriction is: $\boldsymbol{x\geq 2}$

Example: Determine the restrictions on $\sqrt{1-x}+\sqrt{x+3}=4$

Solution: Since we have two radicands, we need to **check both**

$1-x\geq 0$	$x+3\geq 0$
$-x\geq -1$	$x\geq -3$
$x\leq 1$ ← Flip inequality when multiplying or dividing by a negative	

Since the restrictions is **greater than** $-\mathbf{3}$ but also **less than 1**, we need to take the intersection point

So the restriction is: $\boldsymbol{-3\leq x\leq 1}$

Solving Radical Equations

- The first step is getting rid of the radicals, this concept is used: $If\ a = b\ then\ a^2 = b^2$
- The only issue with squaring both sides is can end up in **extraneous solutions**, solutions that don't satisfy the original equation
- So when we get our solutions, we have to test their viability in the original equation

Example: Solve $\sqrt{x+1} = x - 1$

Solution:

$\sqrt{x+1} = x - 1$

$\left(\sqrt{x+1}\right)^2 = (x-1)^2$ — Square both sides

$x + 1 = (x-1)(x-1)$ — Expand

$x + 1 = x^2 - 2x + 1$ — Simplify the expansion

$x = x^2 - 2x$ — Subtract 1 from both sides

$x^2 - 3x = 0$ — Subtract x from both sides

$x(x-3) = 0$ — Factor out the x

$x = 0, 3$ — Solve for x

Check:

For $x = 0$: $\sqrt{0+1} = 0 - 1 \quad \rightarrow \quad 0 = -1. (False)$

For $x = 3$: $\sqrt{3+1} = 3 - 1 \quad \rightarrow \quad 2 = 2. (True)$

So, $x = 0$ **is extraneous**, $x = 3$ **is a solution**

Example: Solve $\sqrt{2x+5} - \sqrt{x-1} = 2$

Solution: Convert to: $\sqrt{2x+5} = 2 + \sqrt{x-1}$

$\sqrt{2x+5} = 2 + \sqrt{x-1}$

$\left(\sqrt{2x+5}\right)^2 = \left(2 + \sqrt{x-1}\right)^2$ — Square both sides

$2x + 5 = \left(2 + \sqrt{x-1}\right)\left(2 + \sqrt{x-1}\right)$ — Expand

$2x + 5 = 4 + 4\sqrt{x-1} + x - 1$ — Simplify the expansion

$x + 2 = 4\sqrt{x-1}$ — Subtract 3 and x from both sides

$(x+2)^2 = \left(4\sqrt{x-1}\right)^2$ — Square both sides again

$x^2 + 4x + 4 = 16(x-1)$ — Expand and Simplify

$x^2 - 12x + 20 = 0$ — Factor and Solve for x

$(x-2)(x-10) = 10 \quad \rightarrow \quad x = 2, 10$

Check:

For $x = 2$: $\sqrt{2(2)+5} - \sqrt{2-1} = 2 \quad \rightarrow 3 - 1 = 2$

$2 = 2\ (True)$

For $x = 10$: $\sqrt{2(10)+5} - \sqrt{10-1} = 2 \quad \rightarrow 5 - 3 = 2$

$2 = 2\ (True)$

So, $x = 2\ and\ x = 10$ **are solutions**

Section 2.5 – Practice Problems

Square the expression

1. $\sqrt{x+2}$

2. $\sqrt{x}+2$

3. $\sqrt{3x-5}$

4. $\sqrt{3x}-5$

5. $\sqrt{1-4x}$

6. $1-4\sqrt{x}$

7. $x-3$

8. $\sqrt{x}-\sqrt{3}$

Determine the restriction on the radical equation

9. $\sqrt{x+5}=4$

10. $\sqrt{9-x}=5$

11. $\sqrt{2x+3}=6$

12. $\sqrt{10x-8}=3\sqrt{x}$

13. $\sqrt{5x-5}=\sqrt{4x-1}$

14. $\sqrt{3x+3}=\sqrt{5x-1}$

Solve the radical equation

15. $\sqrt{2t-3}=5$

16. $\sqrt{3t+4}=-2$

17. $\sqrt{1-3x}=-2$

18. $2\sqrt{x-1}=x$

19. $\sqrt{2x+3}-\sqrt{x+2}=2$

20. $-\sqrt{x+3}=\sqrt{3x+5}$

21. $\sqrt{2x+1}=x-7$

22. $\sqrt{3x+10}+5=2x$

23. $x+3=\left(\sqrt{x+1}\right)\left(\sqrt{x+6}\right)$

24. $\sqrt{y-8}+\sqrt{y}=2$

25. The maximum distance, d, in kilometers that a person can see from a height, h, in kilometers above the ground is $d = 111.7\sqrt{h}$. Find the height in meters that would allow a person to see 75 kilometers.

26. The formula $v = \sqrt{2gh}$ relates velocity, V, in meters per second of an object after h meters accelerated by gravity, g, in meters per second squared. If g is approximately $9.8m/s^2$, how far has an object fallen if its velocity is $30m/s$?

Answer Key – Section 2.5

1. $x+2$	14. $x \geq \frac{1}{5}$
2. $x+4\sqrt{x}+4$	15. $t=14$
3. $3x-5$	16. $No\ Solution$
4. $3x-10\sqrt{3x}+25$	17. $No\ Solution$
5. $1-4x$	18. $x=2$
6. $1-8\sqrt{x}+16x$	19. $x=23$
7. x^2-6x+9	20. $No\ Solution$
8. $x-2\sqrt{3x}+3$	21. $x=12$
9. $x \geq -5$	22. $x=5$
10. $x \leq 9$	23. $x=3$
11. $x \geq -\frac{3}{2}$	24. $No\ Solution$
12. $x \geq \frac{4}{5}$	25. $h=451m$
13. $x \geq 1$	26. $h=45.9m$

Extra Work Space