<u>Section 2.5 – Rates of Change and Conversions</u>

<u>Rate of Change</u>

- Rates of change involve **two variables**: think km/hr
- The **Rate of Change** is the **change of one variable with respect to the other**
- The **Rate of Change is** the **Slope**
- The Greek letter Delta ($\boldsymbol{\Delta}$) is used to represent $\boldsymbol{change}$.
- We use <u>Rates of Changes</u> to help **compare** quantities with different units.
- The formula for Rate of Change is: $\boldsymbol{change\ in\ y}$ **over** $\boldsymbol{change\ in\ x}$.

Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

<u>Examples of Rates of Change:</u>

1. Kilometers per hour: km/hr or $\frac{km}{h}$
2. Miles per gallon: $miles/gal$ or $\frac{mi}{gal}$
3. Dollars per hour: $\$/hr$ or $\frac{dollars}{hour}$
4. If the city of Surrey grew by 120 000 people over a five year period.

   It has a rate of change of: $\frac{120\ 000\ people}{5\ years} = 24\ 000/yr$

5. If a person runs the $400m$ race in $56\ seconds$, they run at a rate of:

$$\frac{400m}{56\ sec} = 7.14\ meters/second.$$

- **<u>Rates of Change</u>** are just the <u>slope relationship</u> of two variables
- The variable on the $y - axis$ is the **dependent variable**
- The variable on the $x - axis$ is the **independent variable** (Usually: TIME)

Example:

Paul rents a car. The odometer read $86\ 347km$. He used the car for 3 days and when he returned it the odometer read was $86\ 721km$.

a) Determine the rate of gas consumption for the car.
b) Determine the average rate of travel per day.

Solution:

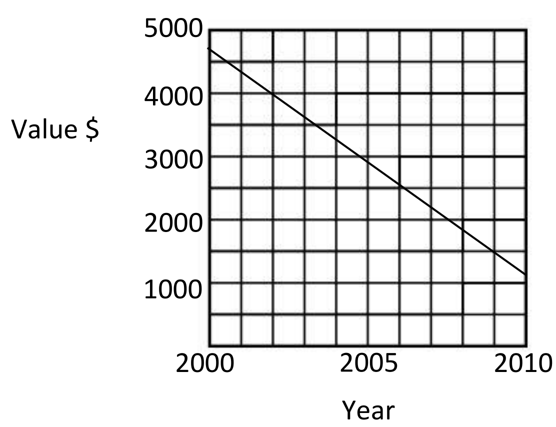
a) $\frac{\Delta y}{\Delta x} = \frac{(86\ 721 - 86\ 347)\ km}{(63 - 0) litres} = \ 5.94\ km/litre$

b) $\frac{\Delta y}{\Delta x} = \frac{(86\ 721 - 86\ 347) km}{(3 - 0) days} = \ 124.7\ km/day$

- Rates of Change can be visualized using graphs. As mentioned the **denominator** quantity is generally placed of the $\boldsymbol{x - axis}$, the **numerator** value is placed on the $\boldsymbol{y - axis}$.

Example 1: Between 2000 and 2010, the cost of a 42" LCD TV dropped from \$4600 to \$1200. Graph this result and determine the average drop in price per year.

Solution 1: Graph this information with $\boldsymbol{time}$ $(\boldsymbol{yrs})$ as the **independent axis** $(x - axis)$.



$$Rate\ of\ Change = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4600 - 1200}{2000 - 2010} = \frac{\$3400}{-10\ yrs} = \frac{\$340}{-1yr} = \$ - \mathbf{340/yr}$$

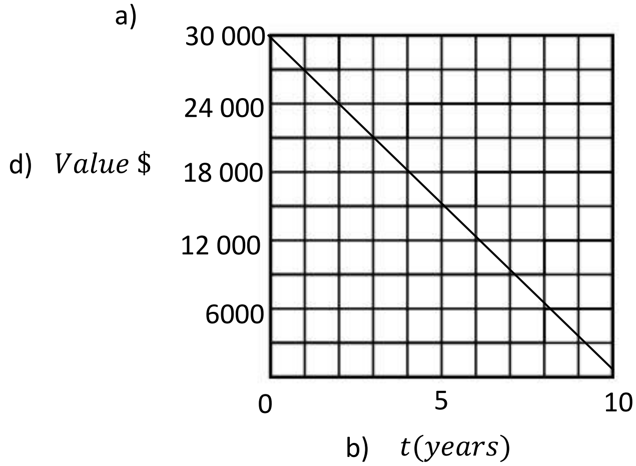
The TV PRICE DROPS by $\mathbf{\$340/yr}$

Example 2: Most cars depreciate as they age. A car costing $\$30\ 000$ will have a value of $\$2500$ at the end of $10\ years$. Determine the **DEPRECIATION RATE.**

a) Draw the graph of this information
b) Have $\boldsymbol{time\ in\ years\ as\ the\ independant\ variable}\ (\boldsymbol{x})$
c) **Have** $\boldsymbol{price\ in}\ \$\ \boldsymbol{as\ the\ dependant\ variable}\ (\boldsymbol{y})$
d) What is the rate of change of the car's value with respect to time?

Solution 2:

a)



$$Rate\ of\ Change\ = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{30\ 000 - 2500}{0 - 10} = \frac{\$27\ 500}{-10\ yrs} = \frac{\$2750}{-1yr}$$

$$= \$ - \mathbf{2750/yr}$$

The CAR DEPRECIATES by $\mathbf{\$2750/yr}$

Example 3:

Georgia sells computers. She is paid a basic monthly salary of $1500, plus $400 for every **five** computers she sells.

a) Write a formula for Rate of Change of Georgia's $\$/Computer\ Sold$
b) Determine Georgia's wage in a month when she sells 60 computers

Solution 3:

a) $Rate\ of\ Change\ = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{400 - 0}{5 - 0} = \frac{\$400}{5\ Comp} = \frac{\$80}{1\ Comp} = \$\mathbf{80/comp}$

$If\ it\ is\ \frac{\$80}{1\ Comp}$, then times that by 60 computers. $\frac{\$80}{1\ Comp} \cdot 60\ Comp = \$4800$

These cancel!

$$\boldsymbol{So\ she\ makes}: \$\mathbf{4800} + \$\mathbf{1500} = \$\mathbf{6300}\ \boldsymbol{that\ month}$$

Conversions and Measurement Systems

- When we are converting units, there will always be a known ratio that we use
- This known ratio will be between to different units

Example: $1cm = 10mm$ or $1cm : 10mm$ $\leftrightarrow$ $10mm : 1cm$

- If we know these ratios we can convert anything we are given.
- Remember always **MULTIPLY**
    - You just have to follow the following structure every time!

$$What\ you\ Have * Ratio = Answer$$

Metric System

- The Metric System is used by almost the entire world (all but three countries)
- It is easy for the purpose of conversion because it is a BASE 10 system

Example:

$1cm = 10mm$

$1m = 100cm$

$1km = 1000m$

All differ by multiples of 10

BASE 10 SYSTEM

- The Base 10 system makes the conversion quite straight forward

Here is a list of the known Metric Conversion we will use:

Equation	Ratio	Fraction (Read Top **per** Bottom)
$1cm = 10mm$	$1cm : 10mm$ $10mm : 1cm$	$\frac{1cm}{10mm} \leftrightarrow \frac{10mm}{1cm}$
$1m = 100cm$	$1m : 100cm$ $100cm : 1m$	$\frac{1m}{100cm} \leftrightarrow \frac{100cm}{1m}$
$1km = 1000m$	$1km : 1000m$ $1000m : 1km$	$\frac{1km}{1000m} \leftrightarrow \frac{1000m}{1km}$

Example 1:

How many centimeters are in 123 meters?

Solution 1:

$$123m * \frac{100cm}{1m}$$

- I use the Ratio of $cm : m$
- I set it up so the **meters** are on B (since my original is on top)
- That way they cancel out

$$123\cancel{m} * \frac{100cm}{1\cancel{m}} = \frac{123 * 100cm}{1} = \mathbf{123\ 000cm}$$

Just left with $Centimeters$

$Meters$ cancel with $meters$

Example 2:

How many km are there in 15 242 $centimeters$?

Solution 2:

Step 1:

- First I get $\boldsymbol{meters}$ using $\boldsymbol{m}$: $\boldsymbol{cm}$ **ratio**
- This time $\boldsymbol{cm}$ is on the **bottom** because I want it to **cancel out**

$$15\ 242\cancel{cm} * \frac{1m}{100\cancel{cm}} = \frac{15242m}{100} = 152.42m$$

Step 2:

$$152.42\cancel{m} * \frac{1km}{1000\cancel{m}} = \frac{152.42km}{1000} = \mathbf{0.15242km}$$

- Now I get $\boldsymbol{kilometers}$ using $\boldsymbol{km}$: $\boldsymbol{m}$ **ratio**
- This time $\boldsymbol{m}$ is on the **bottom** because I want it to **cancel out**

We can do it **all in one step**, **set up** the **ratios**, **continuous multiplication**, so the **units cancel**!

$$15242\cancel{cm} * \frac{1\cancel{m}}{100\cancel{cm}} * \frac{1km}{1000\cancel{m}} = \frac{15242km}{100 * 1000} = \frac{15242km}{100000} = 0.15242km$$

Centimeters Cancel

Meters Cancel

Imperial System (Only 3 and a Half Countries use this)

- Liberia
- Myanmar (Burma)
- USA
- Canada/UK (use it sometimes)

The **conversion ratios** for the **Imperial System** are not Base 10, so they are not as easy to visualize

Here they are:

Equation	Ratio	Fraction (Read Top **per** Bottom)
$1\ mile = 1760\ yards$	$1mi : 1760yds$ $1760yds : 1\ mi$	$\frac{1mi}{1760yds} \leftrightarrow \frac{1760yds}{1mi}$
$1\ mile = 5280\ ft$	$1mi : 5280ft$ $5280ft : 1\ mi$	$\frac{1mi}{5280ft} \leftrightarrow \frac{5280ft}{1mi}$
$1\ yards = 3\ feet$	$1yd : 3ft$ $3ft : 1yd$	$\frac{1yd}{3ft} \leftrightarrow \frac{3ft}{1yd}$
$1\ foot = 12\ inches$	$1ft : 12in$ $12in : 1ft$	$\frac{1ft}{12in} \leftrightarrow \frac{12in}{1ft}$

- Everything still gets set-up the same way
- Make sure the ratios are set-up so that the **units still cancel** out **top and bottom**

Example 3:

How many $feet$ are in 64 $inches$?

Solution 3:

$$64\cancel{in} * \frac{1ft}{12\cancel{in}} = \frac{64ft}{12} = \mathbf{5.3ft}$$

Inches cancel

Example 4:

How many inches are there in 3 $miles$?

Solution 4:

Multi Step Set-Up

$$3\cancel{mi} * \frac{1760yds}{1\cancel{mi}} = 5280yds$$

$Cancel\ miles$

$$5280\cancel{yds} * \frac{3ft}{1\cancel{yds}} = 15840ft$$

$Cancel\ yds$

$$15840\cancel{ft} * \frac{12in}{1\cancel{ft}} = \mathbf{190\ 080}\boldsymbol{in}$$

$Cancel\ feet$

One Step Set-Up

$$3\cancel{mile} * \frac{1760\cancel{yd}}{1\cancel{mi}} * \frac{3\cancel{ft}}{1\cancel{yd}} * \frac{12in}{1\cancel{ft}} = \mathbf{190\ 080}\boldsymbol{in}$$

Example 5:

How many $feet$ in 4.5 $miles$?

Solution 5:

Multi-Step

$$4.5\cancel{mi} * \frac{1760yds}{1\cancel{mi}} = 7920yds$$

$Cancel\ miles$

$$7920\cancel{yds} * \frac{3ft}{1\cancel{yd}} = \mathbf{23760}\boldsymbol{ft}$$

$Cancel\ yds$

One Step

$$4.5\cancel{mi} * \frac{1760\cancel{yds}}{1\cancel{mi}} * \frac{3ft}{1\cancel{yd}} = \mathbf{23760}\boldsymbol{ft}$$

Metric to Imperial ↔ Imperial to Metric

- Again it is the **exact same process**
- In this case since we are **dealing with approximate ratios** it is good form to switch within each **individual system and** you make the ratio **switch to the new system at the smallest units** (You'll see an example)

Here are the conversions from system to system

Equation	Ratio	Fraction (Read Top **per** Bottom)
$1\ mi \cong 1.609km$	$1mi : 1.609km$ $1.609km : 1\ mi$	$\frac{1mi}{1.609km} \leftrightarrow \frac{1.609km}{1mi}$
$1\ ft \cong 0.305\ m$	$1ft : 0.305\ m$ $0.305\ m : 1ft$	$\frac{1ft}{0.305\ m} \leftrightarrow \frac{0.305\ m}{1ft}$
$1\ in \cong 2.54cm$	$1\ in : 2.54cm$ $2.54cm : 1\ in$	$\frac{1in}{2.54cm} \leftrightarrow \frac{2.54cm}{1in}$

<u>Example 6:</u>

How many $kilometers\ are\ in\ 730ft$?

<u>Solution 6:</u>

- Since there is **NO DIRECT CONVERSION** from $\boldsymbol{km}$ **to** $\boldsymbol{feet}$, and the estimation from $km\ to\ miles$ is a larger distance**. Convert to meters first** (Least amount of discrepancy)
- **Switch** from $\boldsymbol{feet\ to\ meters}$
- Then we can **switch** from $\boldsymbol{meters\ to\ km}$ (a **DIRECT CONVERSION**)

Multi-Step

$$\boxed{Cancel\ feet} \qquad 730\cancel{ft} * \frac{0.305m}{1\cancel{ft}} = 222.65m$$

$$\boxed{Cancel\ meters} \qquad 222.65\cancel{m} * \frac{1km}{1000\cancel{m}} = \boldsymbol{0.22km}$$

One Step

$$730\cancel{ft} * \frac{0.305\cancel{m}}{1\cancel{ft}} * \frac{1km}{1000\cancel{m}} = \frac{730 * 0.305km}{1000} = \frac{222.65km}{1000} = \boldsymbol{0.22km}$$

Example 7:

How many $centimeters$ are there in $42yds$?

Solution 7:

Multi-Step

We have a small estimated direct conversion from centimeters to inches, go from yards to inches first

$$\boxed{Cancel\ yards} \quad 42\cancel{yds} * \frac{3ft}{1\cancel{yd}} = 126ft$$

$$\boxed{Cancel\ feet} \quad 126\cancel{ft} * \frac{12in}{1\cancel{ft}} = 1512in$$

$$\boxed{Cancel\ inches} \quad 1512\cancel{in} * \frac{2.54cm}{1\cancel{in}} = 3840.48cm$$

One-Step

We have a direct conversion from centimeters to inches, so let's go from yards to inches first

$$42\cancel{yds} * \frac{3\cancel{ft}}{1\cancel{yd}} * \frac{12\cancel{in}}{1\cancel{ft}} * \frac{2.54cm}{1\cancel{in}} = \mathbf{3840.48cm}$$

Example 8:

How many $feet$ are there in $4km$

Solution 8:

Multi-Step

We have a direct conversion from meters to feet, so let's go from kilometers to meters first

$$\boxed{Cancel\ kilometers} \quad 4k\cancel{m} * \frac{1000m}{1km} = 4000m$$

$$\boxed{Cancel\ meters} \quad 4000\cancel{m} * \frac{1ft}{0.305\cancel{m}} = \frac{4000}{0.305}ft$$

$$\boxed{Divide\ to\ get\ the\ Answer} \quad \frac{4000}{0.305}ft = \mathbf{13\ 114.75ft}$$

One-Step

We have a direct conversion from meters to feet, so let's go from kilometers to meters first

$$4000\cancel{km} * \frac{1000\cancel{m}}{1\cancel{km}} * \frac{1ft}{0.305\cancel{m}} = \frac{4000}{0.305}ft = \mathbf{13\ 114.75ft}$$

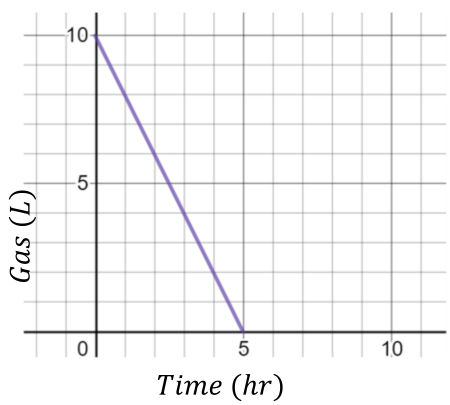
All Conversions get set-up the same way. Make sure the <u>Units Cancel</u> and then just <u>Multiply Across</u> and <u>Divide the Final Fraction</u>.

Section 2.5 – Practice Problems

1. Which slopes show an increase (circle them), a decrease (underline them), or no change (cross out)

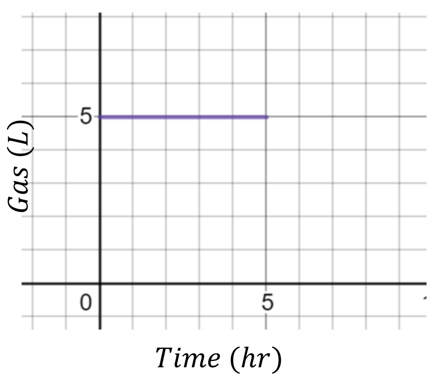
$$\frac{6}{5} \qquad -\frac{1}{5} \qquad \frac{0}{7} \qquad \frac{9}{13} \qquad -\frac{5}{4} \qquad \frac{0}{5} \qquad -\frac{2}{9} \qquad \frac{3}{3}$$

2. Graphs A, B, and C show the amount of fuel used in a car's tank over time. Describe what the rate of change represents, what could it mean about the vehicle?



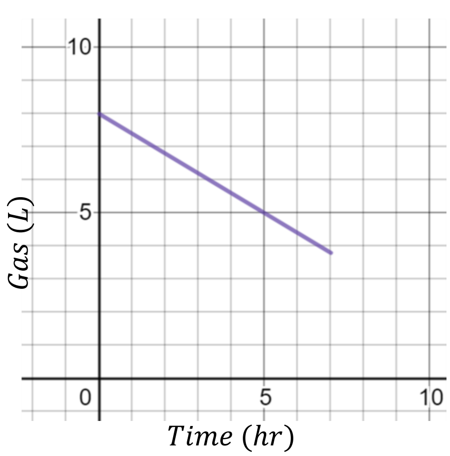
Represent:

Could Mean What about the Vehicle:



Represent:

Could Mean What about the Vehicle:

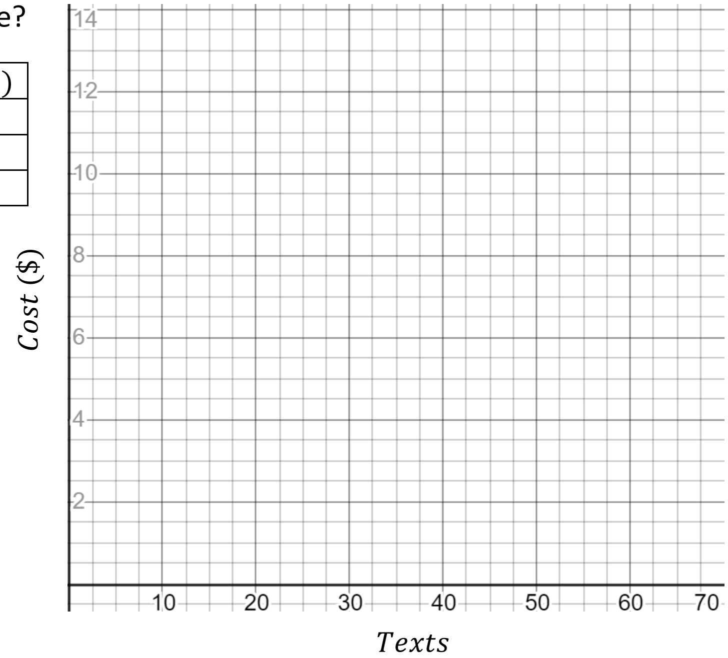


Represent:

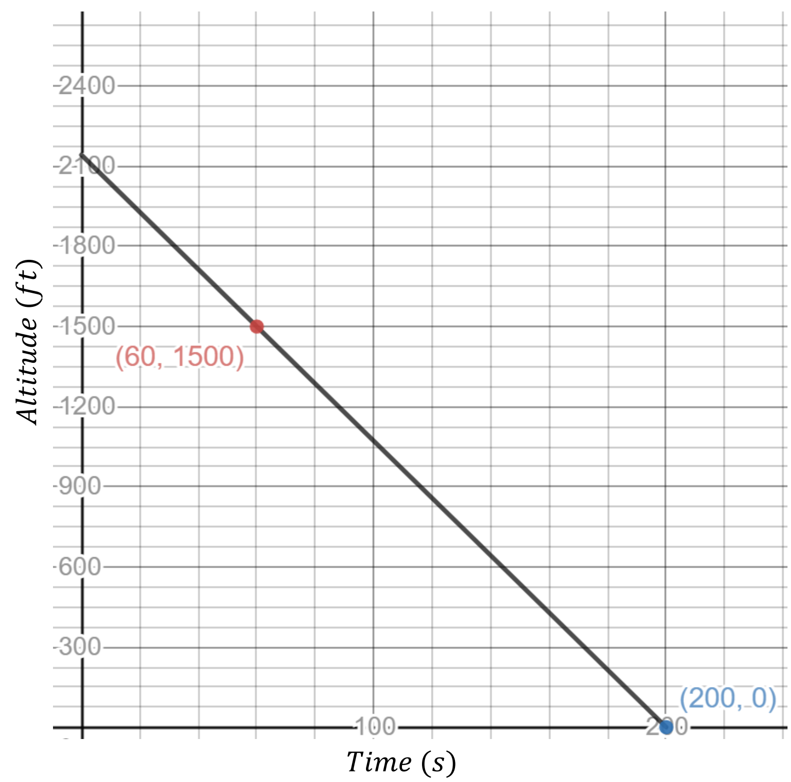
Could Mean What about the Vehicle:

3. Telus is taking advantage of me. They have me set-up on a plan where I pay per text message sent (See the grid). Graph the data (Think Dependant vs Independent variables), what is the rate of change of the line?

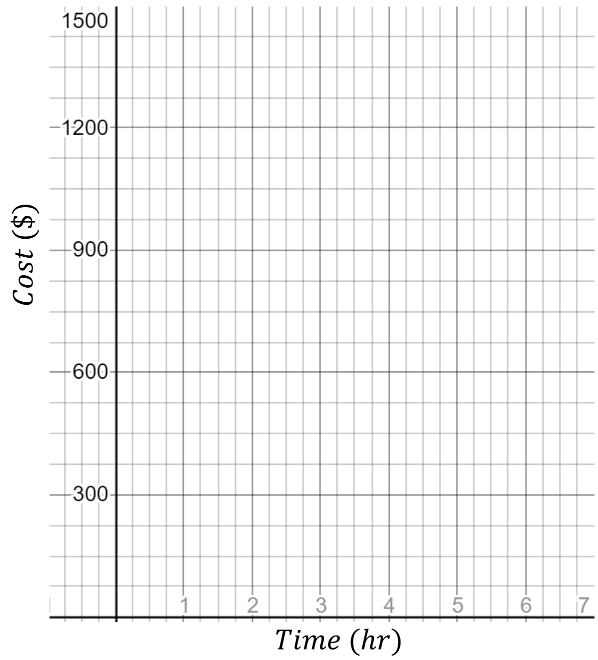
$Texts$	$Cost$ ($)
0	0
25	5
65	13



4. At what rate of change does the plane described in the graph descend at. Answer to the nearest tenth.

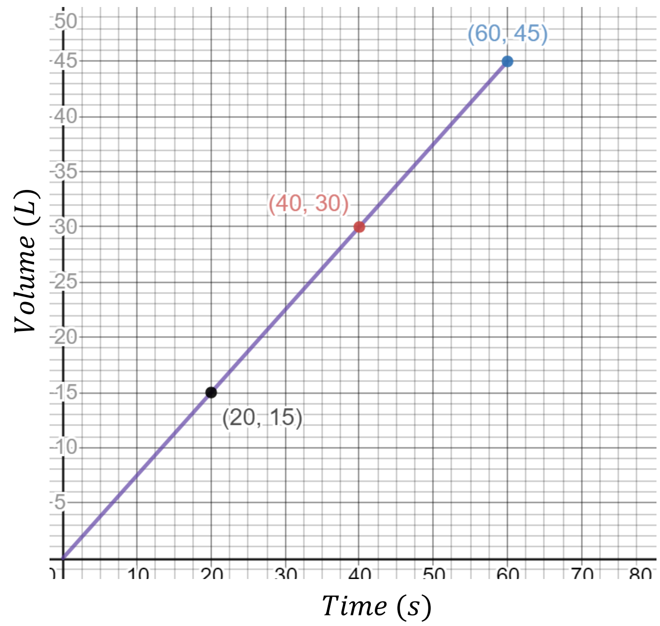


5. Mr. Phillips and his fiancé are wedding planning. They are looking to hire a DJ who charges $\$750\ for\ 3\ hours$ or $\$1200\ for\ 6\ hours$. Graph the info provided and draw a line connecting the two points.
   a) What is the slope of the line segment you have drawn? What does it represent?
   b) Extend the line to the $y-axis$, what is the DJ's flat rate?
   c) If they need the DJ for $5\ hours$, how much can they expect to pay?



6. Usain Bolt set a World Record for $100m$. He ran $100m\ in\ 9.58s$.
   a) How fast does he run in m/sec
   b) How fat does he run, if he can keep up the pace, in km/hr

7. Della is filling a pool for her kids. The graph shows the volume of water in the pool as she fills it.
   a) What is the rate of change of water in the pool (nearest hundredth)
   b) What is this rate of change in mL/min



8. The new roller coaster at the PNE has a top speed of $84 miles/hr$ What is the speed in km/hr.

9. Gregor works for the city of Sidney. He drives a hot air lancer that blasts hot air at $3000 ft/sec$. How fast does the hot air move in $meters/sec$? (Round to the nearest tenth)

10. Mr. Philips was an up and coming baseball player, he could pitch at a top speed of $85 miles/hr$. How fast could he pitch in $feet/sec$. If the distance from the mound to the plate is $50ft$ how long does the batter have to react and swing?

Section 2.5 – Answer Key

1.	$See\ Website\ Copy$
2.	i) L/hr; Not very fuel efficient ii) L/hr; No fuel consumed iii) L/hr; More fuel efficient than i)
3.	$\$0.20/Text$
4.	$Decreases\ 10.7ft/second$
5.	a) $\$150/hr$ b) $Flat\ Rate\ of\ \$300$ c) $\$1050$
6.	a) $10.4m/sec$ b) $37.4km/hr$
7.	$45\ 000mL/min$
8.	$135.3km/hr$
9.	$915m/sec$
10.	Pitches $125ft/sec$ Batter has $0.4\ seconds$ to swing