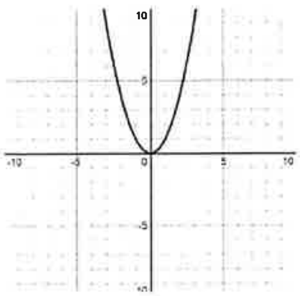
Section 2.5 – Practice Problems

1. The following are graphs of functions. Will they have inverse functions? Yes/No and Why?

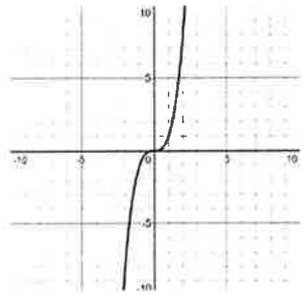
a) 

Yes or No

Function

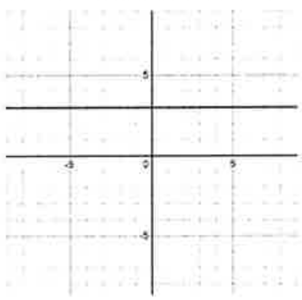
Inverse Function NO!

Why? NOT 1-1 Function

b) 

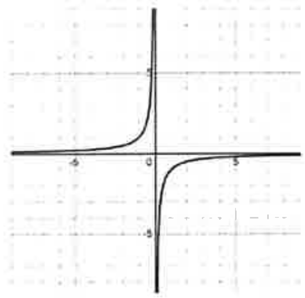
Yes or No

Why? 1-1 Graph

c) 

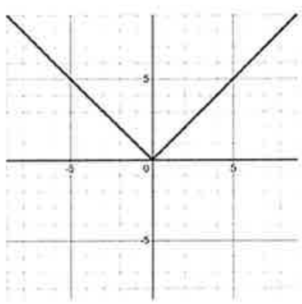
Yes or No

Why? NOT 1-1 Function

d) 

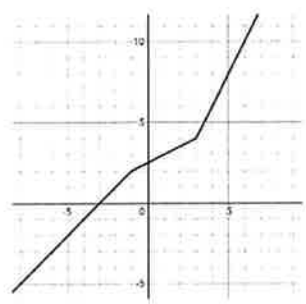
Yes or No

Why? It has asymptotes, but still 1-1

e) 

Yes or No

Why? NOT 1-1 Function

f) 

Yes or No

Why? It has sharp points but it is still 1-1

2. Determine whether the functions are inverses of each other by calculating $(f \circ g)(x)$ and $(g \circ f)(x)$

a) $f(x) = \frac{3}{5}x, g(x) = \frac{5}{3}x$

$(f \circ g)(x) \to \frac{3}{5}\left(\frac{5}{3}x\right) = \boxed{x}$

$(g \circ f)(x) \to \frac{5}{3}\left(\frac{3}{5}x\right) = \boxed{x}$

YES THEY ARE

INVERSES

b) $f(x) = x - 3,\ g(x) = x + 3$

$(f \circ g)(x) \to x+3-3 = \boxed{x}$

$(g \circ f)(x) \to x-3+3 = \boxed{x}$

YES.

c) $f(x) = 3 - 4x, g(x) = \frac{3-x}{4}$

$(f \circ g)(x) = 3 - 4\left(\frac{3-x}{4}\right)$

$= 3 - (3-x)$

$= 3-3+x \Rightarrow \boxed{x}$

$(g \circ f)(x) \to \frac{3-(3-4x)}{4}$

$= \frac{3-3+4x}{4} = \frac{4x}{4} = \boxed{x}$

YES.

d) $f(x) = x^3 - 2,\ g(x) = \sqrt[3]{x+2}$

$(f \circ g)(x) \to \sqrt[3]{x+2}^{\,3} - 2$

$= x+2-2 \to \boxed{x}$

$(g \circ f)(x) \to \sqrt[3]{x^3-2+2}$

$= \sqrt[3]{x^3} \Rightarrow \boxed{x}$

YES.

e) $f(x) = \sqrt{x-1},\ g(x) = x^2 + 1$

$(f \circ g)(x) = \sqrt{x^2+1-1} = \sqrt{x^2}$

$= \pm x$ but we use $|x|$

more accurate

$(g \circ f)(x) = \sqrt{x-1}^{\,2} + 1$

$= x - 1 + 1$

$= x$

$x \neq |x|$ so NO! NOT INVERSES

f) $f(x) = \sqrt[4]{x}, x \geq 0,\ g(x) = x^4$

$(f \circ g)(x) \rightarrow \sqrt[4]{x^4} = |x|$

$(g \circ f)(x) \rightarrow \sqrt[4]{x}^{\,4}$

$= x$

Since Domain Restriction is not on $g(x)$

$|x| \neq x$

No, not inverses

g) $f(x) = \dfrac{5x+3}{1-2x},\ g(x) = \dfrac{x-3}{2x+5}$

$(f \circ g)(x) \rightarrow \dfrac{5\left(\frac{x-3}{2x+5}\right) + 3\left(\frac{2x+5}{2x+5}\right)}{\left(\frac{2x+5}{2x+5}\right)1 - 2\left(\frac{x-3}{2x+5}\right)}$ common denominator

$= \dfrac{\frac{5x-15+6x+15}{2x+5}}{\frac{2x+5-2x+6}{2x+5}} = \dfrac{\frac{11x}{2x+5}}{\frac{11}{2x+5}} = \dfrac{11x}{11} = \boxed{x}$

$(g \circ f)(x) = \dfrac{\frac{5x+3}{1-2x} - 3\left(\frac{1-2x}{1-2x}\right)}{2\left(\frac{5x+3}{1-2x}\right) + 5\left(\frac{1-2x}{1-2x}\right)}$

$\boxed{\text{YES}}$

h) $f(x) = \sqrt[3]{x+1},\ g(x) = x^3 - 1$

$(f \circ g)(x) = \sqrt[3]{x^3-1+1} = \sqrt[3]{x^3} = \boxed{x}$

$(g \circ f)(x) = \sqrt[3]{x+1}^{\,3} - 1 = x+1-1 = \boxed{x}$

YES.

$\dfrac{\frac{5x+3-3+6x}{1-2x}}{\frac{10x+6+5-10x}{1-2x}} = \dfrac{\frac{11x}{1-2x}}{\frac{11}{1-2x}} = \dfrac{11x}{11} = \boxed{x}$

DESMOS CAN HELP VISUALIZE

3. Determine the restrictions on each of the following functions in order for its inverse to be a function

a) $f(x) = x^2$

since it is symmetric about y-axis

$x \geq 0$ or $x \leq 0$

b) $f(x) = x^2 + 2$

vertical shift so still symmetric about the y-axis

$x \geq 0$ or $x \leq 0$

c) $f(x) = (x-2)^2$

vertex shifted to $x = 2$ so symmetric there

$x \geq 2$ or $x \leq 2$

d) $f(x) = |x+1| - 2$

absolute value symmetric about the shift, similar to parabola

$x \geq -1$ or $x \leq -1$

4. Find the inverse of the following functions. State if the inverse is a function, a one-to-one function, or neither.

a) $f(x) = 2x - 3$

Inverse will be a function but... this is 1-1

$f(x) = 2x - 3$

$y = 2x - 3$

$x = 2y - 3$

$x + 3 = 2y$

$\frac{x+3}{2} = y$

$f^{-1}(x) = \frac{x+3}{2}$

b) $f(x) = \sqrt{2x-1}$

Domain: $x \geq \frac{1}{2}$

Range: $y \geq 0$

$y = \sqrt{2x-1}$

$x = \sqrt{2y-1}$

$x^2 = 2y - 1$

$x^2 + 1 = 2y$

$\frac{x^2+1}{2} = y$

- Recall Domain of $f(x)$ is Range of $f^{-1}(x)$
- Range of $f(x)$ is Domain of $f^{-1}(x)$

so this Domain is restricted to $x \geq 0$ so 1-1

$f^{-1}(x) = \frac{x^2+1}{2}$; $x \geq 0$

c) $f(x) = x^2 + 1$

NOT 1-1 so inverse will not be a Function

$y = x^2 + 1$

$x = y^2 + 1$

$x - 1 = y^2$

$y = \pm\sqrt{x-1}$

this has Domain Restricted to $x \geq 1$

Range: $y \geq 1$

d) $f(x) = \dfrac{1}{3x-2}$

Domain: $x \neq 2/3$

Range: $y \neq 0$

$y = \dfrac{1}{3x-2}$

$x = \dfrac{1}{3y-2} \rightarrow (3y-2)x = 1$

$3y - 2 = \dfrac{1}{x} \rightarrow 3y = \dfrac{1}{x} + 2$ common denominator

$3y = \dfrac{1+2x}{x} \rightarrow y = \dfrac{1+2x}{3x}$

1-1 $f^{-1}(x) = \dfrac{1+2x}{3x}$ D: $x \neq 0$ R: $y \neq 2/3$

(check using Desmos)

e) $f(x) = \dfrac{x}{1-x}$ Domain: $x \neq 1$

$f(x) = \dfrac{x}{1-x} \rightarrow y = \dfrac{x}{1-x}$

$x = \dfrac{y}{1-y} \rightarrow (1-y)x = y$

$x - xy = y \rightarrow x = xy + y$

$x = y(x+1)$

$\dfrac{x}{x+1} = y$

$f^{-1}(x) = \dfrac{x}{x+1}$; $f^{-1}(x) \neq 1$

1-1 Function

f) $f(x) = \dfrac{2x-1}{3x+2}$ D: $x \neq -2/3$

$y = \dfrac{2x-1}{3x+2} \rightarrow x = \dfrac{2y-1}{3y+2}$

$x(3y+2) = 2y - 1$

$3xy + 2x = 2y - 1$

$3xy - 2y = -2x - 1$

$y(3x-2) = -2x - 1 \rightarrow y = \dfrac{-2x-1}{3x-2}$

$f^{-1}(x) = \dfrac{-2x-1}{3x-2}$ $f^{-1}(x) \neq -2/3$

1-1 Function

5. Let $f(x) = 2x - 1, g(x) = \frac{1}{2}x + 3$, find $f^{-1}(x)$ and $g^{-1}(x)$, then determine

$f^{-1}(x) = ?$

$y = 2x - 1$

$x = 2y - 1 \rightarrow x + 1 = 2y$

$y = \frac{x+1}{2}$

$\boxed{f^{-1}(x) = \frac{x+1}{2}}$

$g^{-1}(x) = ?$

$y = \frac{1}{2}x + 3$

$x = \frac{1}{2}y + 3 \rightarrow x - 3 = \frac{1}{2}y$

$2(x-3) = y$

$2x - 6 = y \rightarrow \boxed{g^{-1}(x) = 2x - 6}$

a) $(f^{-1} \circ g)(x)$

$f^{-1}(g(x)) \rightarrow \frac{\frac{1}{2}x + 3 + 1}{2}$

$= \frac{\frac{1}{2}x + 4}{2} \rightarrow \frac{\frac{1}{2}x}{2} + 2$

$\boxed{\frac{1}{4}x + 2}$

b) $(g^{-1} \circ f^{-1})(x)$

$g^{-1}(f^{-1}(x)) \rightarrow 2\left(\frac{x+1}{2}\right) - 6$

$= \frac{\cancel{2}(x+1)}{\cancel{2}} - 6$

$= \boxed{x - 5}$

c) $(g \circ f^{-1})(x)$

$(g \circ f^{-1})(x) \rightarrow g(f^{-1}(x))$

$\frac{1}{2}\left(\frac{x+1}{2}\right) + 3$

$\frac{x+1}{4} + 3 \rightarrow \frac{x+1}{4} + \frac{12}{4}$

$\frac{x+1+12}{4} = \boxed{\frac{x+13}{4}}$

d) $(f \circ g^{-1})(x)$

$f(g^{-1}(x)) \rightarrow 2(2x-6) - 1$

$4x - 12 - 1$

$\boxed{4x - 13}$

e) $(f^{-1} \circ g^{-1})(x)$

$f^{-1}(g^{-1}(x))$

$\frac{2x-6+1}{2} \rightarrow \frac{2x-5}{2}$

$\frac{2x}{2} - \frac{5}{2} = \boxed{x - \frac{5}{2}}$

f) $(f \circ g)^{-1}(x)$

$f(g(x))$ 1st then find the inverse

$2\left(\frac{1}{2}x+3\right)-1 \rightarrow x+6-1$

$(f \circ g)(x) = x+5$

$y = x+5$

$x = y+5$

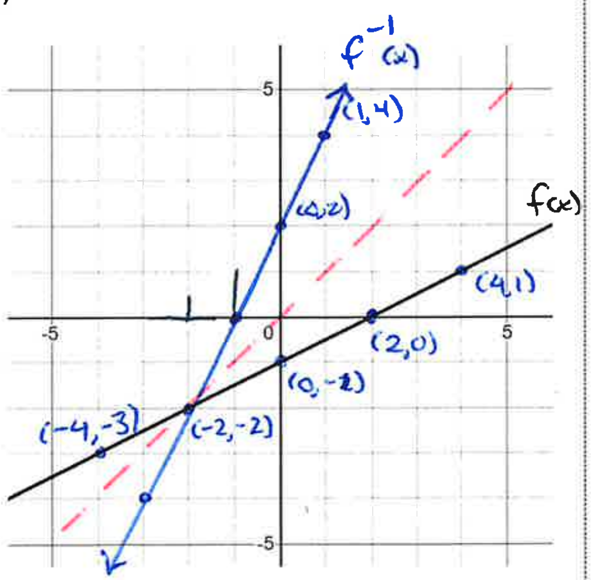
$x-5 = y$

$\boxed{(f \circ g)^{-1}(x) = x-5}$

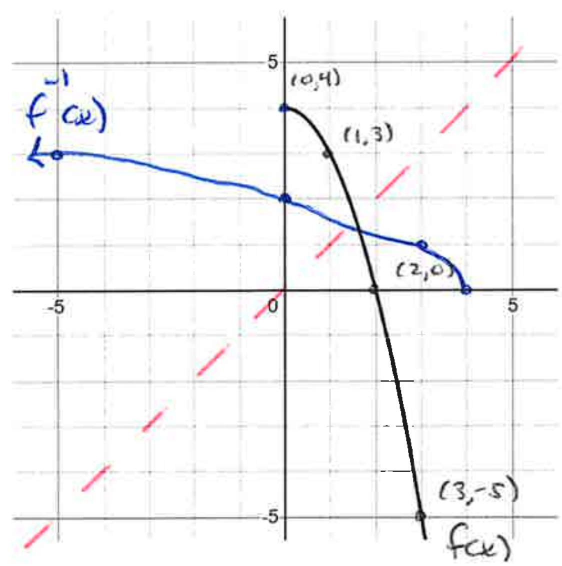
6. Given the graph of f, on the same grid draw the graph of the inverse of f.

Reflect over $y=x$ or just switch (x,y) coorindinates

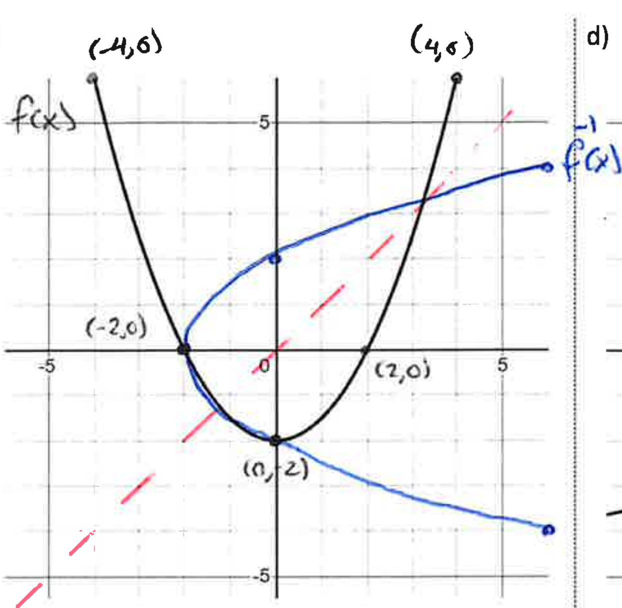
a)



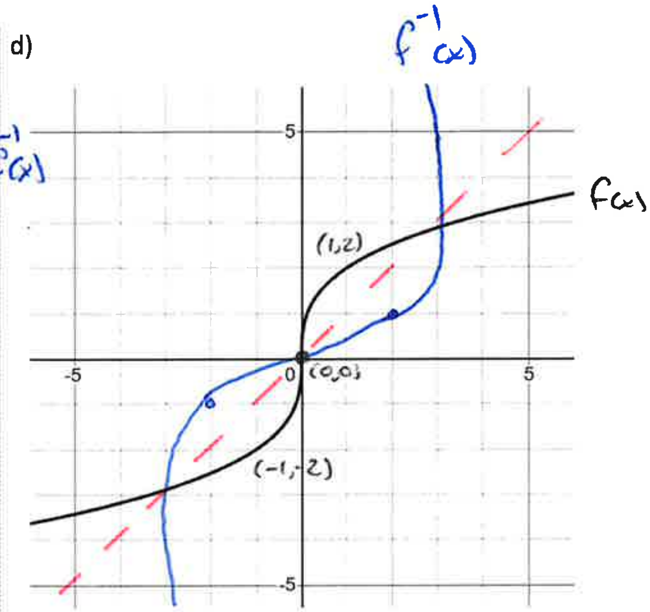
b)



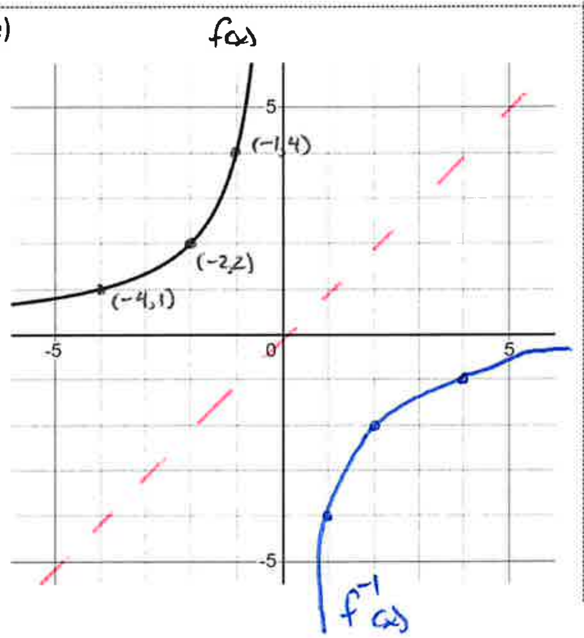
c)



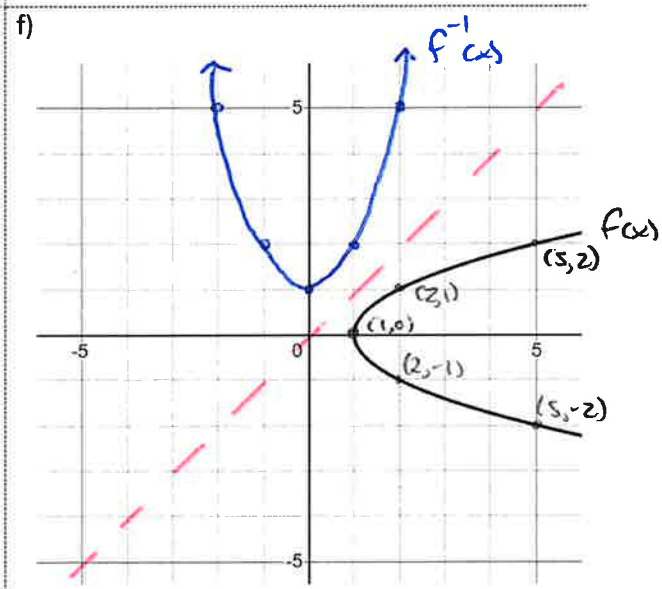
d)



e)



f)



7. If $(-1, 2)$ *or* (a, b) is a point of the graph of $y = f(x)$, what must be a point on the graph for the following?

a) $y = f^{-1}(x)$

$(-1,2) \rightarrow (2,-1)$

$(a,b) \rightarrow (b,a)$

b) $y = f^{-1}(x) - 1$

$(-1,2) \rightarrow (2,-1) \rightarrow (2,-2)$

$(a,b) \rightarrow (b,a) \rightarrow (b,a-1)$

c) $y = f^{-1}(x+2)$

$(-1,2) \rightarrow (2,-1) \rightarrow (0,-1)$

$(a,b) \rightarrow (b,a) \rightarrow (b-2,a)$

d) $y = -f^{-1}(-x)$

$(-1,2) \rightarrow (2,-1) \rightarrow (-2,1)$

$(a,b) \rightarrow (b,a) \rightarrow (-b,-a)$

e) $y = 1 - f^{-1}(-x)$ $\rightarrow -f^{-1}(-x)+1$

$(-1,2) \rightarrow (2,-1) \rightarrow (-2,1) \rightarrow (-2,2)$

$(a,b) \rightarrow (b,a) \rightarrow (-b,-a) \rightarrow (-b,-a+1)$

f) $y = f^{-1}(x+1)$

$(-1,2) \rightarrow (2,-1) \rightarrow (1,-1)$

$(a,b) \rightarrow (b,a) \rightarrow (b-1,a)$

8. Use Desmos to graph the following functions and their inverses. State if the inverse is a function, a one-to-one function, or neither.

a) $f(x) = 2x - 1$

USE DESMOS TO VISUALIZE

$y = 2x-1$

$x = 2y-1$

$\frac{x+1}{2} = y$

$f(x) = 2x-1$

$f^{-1}(x) = \frac{x+1}{2}$

$\boxed{1-1}$

b) $f(x) = x^2 + 1$

$y = x^2+1$

$x = y^2+1$

$x-1 = y^2$

$y = \pm\sqrt{x-1}$

$f(x) = x^2+1$

$f^{-1}(x) = \pm\sqrt{x-1}$

$\boxed{\text{NOT A FUNCTION}}$

c) $f(x) = x^3 - 1$

$y = x^3-1$

$x = y^3-1$

$x+1 = y^3$

$y = \sqrt[3]{x+1}$

$f(x) = x^3-1$

$f^{-1}(x) = \sqrt[3]{x+1}$

$\boxed{1-1}$

d) $f(x) = \sqrt{x^2 - 4}$ R: $y \geq 0$

$y = \sqrt{x^2-4}$

$x = \sqrt{y^2-4}$

$x^2 = y^2-4$

$x^2+4 = y^2$

$y = \pm\sqrt{x^2+4}$ D: $x \geq 0$

$f(x) = \sqrt{x^2-4}$

$f^{-1}(x) = \pm\sqrt{x^2+4}$

$\boxed{\text{NOT A FUNCTION}}$

9. The function $f(x) = a(-x^3 - x + 2)$ has an inverse function such that $f^{-1}(6) = -2$. Find a.

If $f^{-1}(6) = -2$ Then $f(-2) = 6$ x y

$y = a(-x^3-x+2)$

$6 = a(-(-2)^3-(-2)+2)$

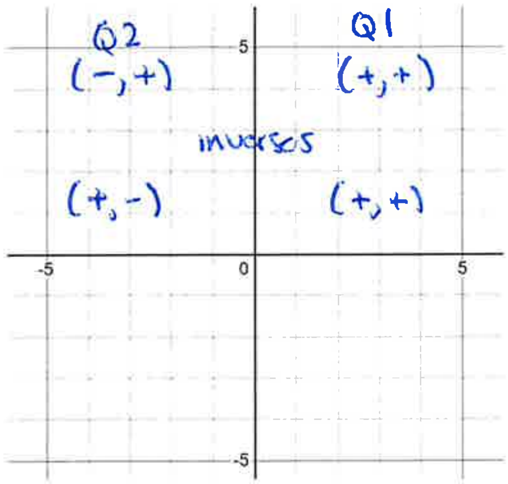
$6 = a(8+2+2)$

$6 = a(12)$

$a = \frac{6}{12}$

$\boxed{a = \frac{1}{2}}$

10. If the graph of f contains points in Quadrant I and II, the graph of f^{-1} must contain points in which Quadrant(s)? (Use the grid provided to help visualize)



so Q1 and Q4

11. The formulas for Fahrenheit and Celsius temperatures are:

$$F = \frac{9}{5}C + 32 \quad and \quad C = \frac{5}{9}(F - 32)$$

Show that these two functions are inverses of each other.

$(F \circ C)(x) \rightarrow F = \frac{9}{5}\left(\frac{5}{9}(F-32)\right) + 32 \rightarrow F = F - 32 + 32$

$F = \boxed{F}$

$(C \circ F)(x) \rightarrow C = \frac{5}{9}\left(\frac{9}{5}C + 32 - 32\right)$

$\rightarrow C = C$

BOTH INVERSES.

12. Show that for the one-to-one function $f(x) = 2x + 1$ and $g(x) = \frac{1}{4}x - 3$, that:

$$(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$$

$(f \circ g)^{-1}(x) \rightarrow 2\left(\frac{1}{4}x - 3\right) + 1 \rightarrow \frac{1}{2}x - 6 + 1$ $\quad (f \circ g)(x) = \frac{1}{2}x - 5$

$y = \frac{1}{2}x - 5 \rightarrow x = \frac{1}{2}y - 5$

$x + 5 = \frac{1}{2}y$ $\quad \boxed{y = 2x + 10}$

$f^{-1}(x) \Rightarrow y = 2x + 1 \rightarrow x = 2y + 1 \rightarrow x - 1 = 2y$

$f^{-1}(x) \leftarrow y = \frac{x-1}{2}$

$g^{-1}(x) \Rightarrow y = \frac{1}{4}x - 3 \rightarrow x = \frac{1}{4}y - 3 \rightarrow x + 3 = \frac{1}{4}y$

$g^{-1}(x) \leftarrow y = 4x + 12$

$g^{-1}(f^{-1}(x)) = 4\left(\frac{x-1}{2}\right) + 12$

$= \frac{4x - 4}{2} + 12$

$= 2x - 2 + 12 = \boxed{2x + 10}$

See Website Copy for Detailed Answer Key

Extra Work Space