Section 2.4 – Practice Problems

1. Write an equation for the function that is described by the given characteristics.

a) The shape $f(x) = x^2$, moved $4\ units$ to the left and $5\ units$ downward.

Left Right is in the function
Up down is outside the function

$f(x) = (x+4)^2 - 5$

b) The shape $f(x) = x^2$, moved $2\ units$ to the right, reflected in the $x - axis$, and moved $3\ units$ upward.

$f(x) = -(x-2)^2 + 3$

c) The shape $f(x) = x^3$, moved $2\ units$ to the right and $3\ units$ downward.

$f(x) = (x-2)^3 - 3$

d) The shape $f(x) = x^3$, moved $1\ unit$ downward and reflected in the $y - axis$.

$f(x) = -x^3 - 1$

e) The shape $f(x) = |x|$, moved $6\ units$ upward and $3\ units$ to the left.

$f(x) = |x+3| + 6$

f) The shape $f(x) = |x|$, moved $3\ units$ to the left and reflected in the $x - axis$

$f(x) = -|x+3|$

g) The shape $f(x) = \sqrt{x}$, moved $7\ units$ to the right and reflected in the $x - axis$

$f(x) = -\sqrt{x-7}$

h) The shape $f(x) = \sqrt{x}$, moved $4\ units$ upward and reflected in the $y - axis$

$f(x) = \sqrt{-x} + 4$

2. If $(-3,1)$ or (a,b) is a point on the graph of $y = f(x)$, what must be a point on the graph of the following?

a) $y = f(x+2)$ $(a,b) \to (a-2, b)$

$(-3,1) \to (-5,1)$

b) $y = f(x) + 2$ $(a,b) \to (a, b+2)$

$(-3,1) \to (-3,3)$

c) $y = f(x-2) - 2$

$(-3,1) \to (-1,-1)$

$(a,b) \to (a+2, b-2)$

d) $y = -f(x)$

$(-3,1) \to (-3,-1)$

$(a,b) \to (a,-b)$

e) $y = f(-x)$

$(-3,1) \to (3,1)$

$(a,b) \to (-a,b)$

f) $y = -f(-x)$

$(-3,1) \to (3,-1)$

$(a,b) \to (-a,-b)$

g) $y = f(-x) - 2$

$(-3,1) \to (3,-1)$

$(a,b) \to (-a, b-2)$

h) $y = -f(x+2)$

$(-3,1) \to (-5,-1)$

$(a,b) \to (a-2,-b)$

3. Use the graph of $f(x) = x$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

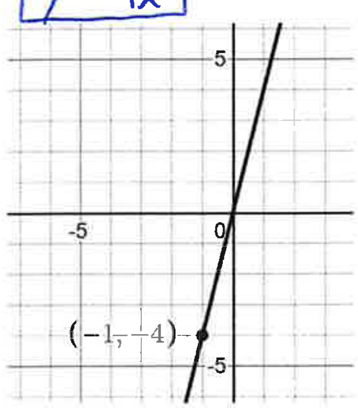
$\boxed{y = 4x}$

a)

Normally for $f(x) = x$

when $x=-1$
$y=-1$

so



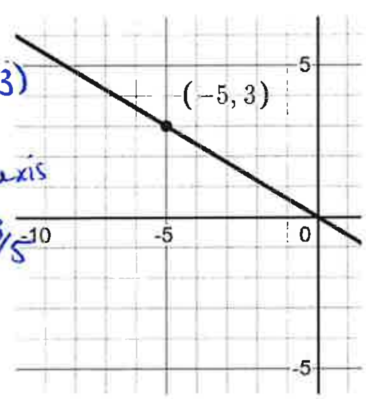
$(-1,-1) \to (-1,-4)$; expansion vertically of 4 (factor

b)

$(-5,-5) \to (-5,3)$

Reflection in x-axis and factor of $3/5$

$\boxed{y = -\frac{3}{5}x}$



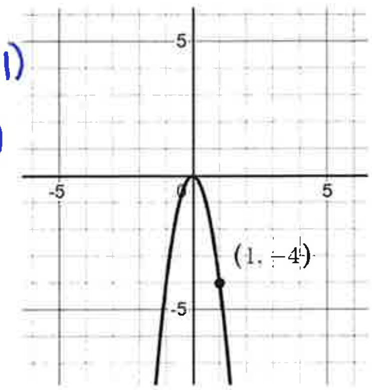
4. Use the graph of $f(x) = x^2$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

a)

Normally: $(1,1)$

$(1,1) \rightarrow (1,-4)$

$\boxed{f(x) = -4x^2}$



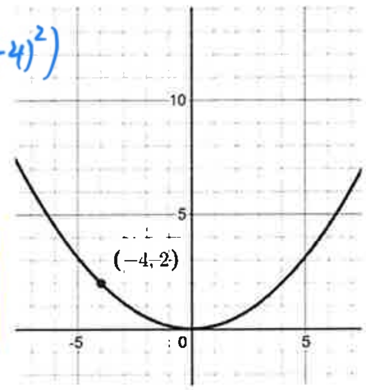
Reflection and vertical expansion by factor of 4

b)

Normally: $(-4,(-4)^2)$

$(-4,16) \rightarrow (-4,2)$

$\boxed{f(x) = \frac{1}{8}x^2}$



Vertical compression by factor of $\frac{1}{8}$

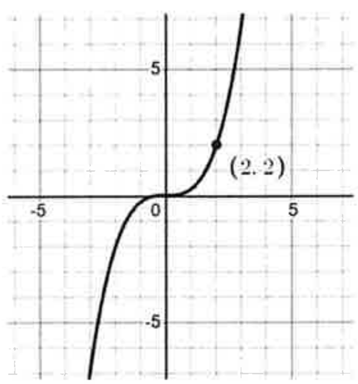
5. Use the graph of $f(x) = x^3$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

a)

Normally

$(2,8)$

$(2,8) \rightarrow (2,2)$



$f(x) = \frac{1}{4}x^3$

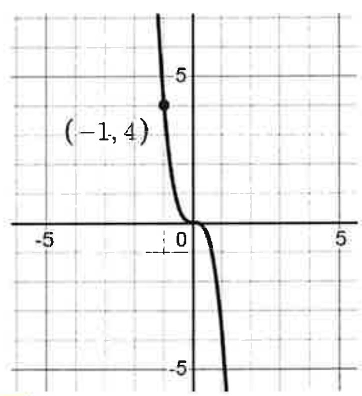
vertical compression by factor of $\frac{1}{4}$

b)

Normally

$(-1,-1)$

$(-1,-1) \rightarrow (-1,4)$

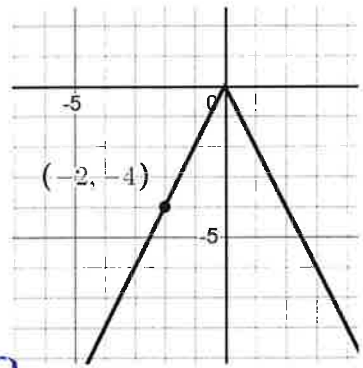


$\boxed{f(x) = -4x^3}$

Reflection and vertical expansion by factor of 4

6. Use the graph of $f(x) = |x|$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

a)



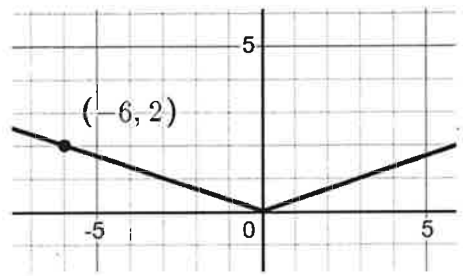
Normally
$(-2,2)$

$(-2,2) \rightarrow (-2,-4)$

$\boxed{f(x) = -2|x|}$

Reflection and vertical expansion by factor of 2

b)



Normally: $(-6,6)$

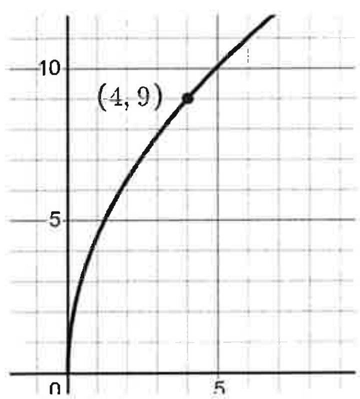
$(-6,6) \rightarrow (-6,2)$

vertical compression by factor of $\frac{1}{3}$

$\boxed{f(x) = \frac{1}{3}|x|}$

7. Use the graph of $f(x) = \sqrt{x}$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

a)



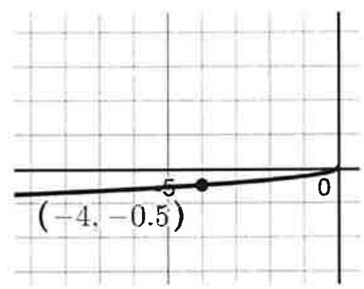
Normally
$(4,2)$

$(4,2) \rightarrow (4,9)$

Expansion by factor of $\frac{9}{2}$

$\boxed{f(x) = \frac{9}{2}\sqrt{x}}$

b)



Normally
$(4,2)$.

$(4,2) \rightarrow (-4,-\frac{1}{2})$

Reflection in y-axis and x-axis vertical compression by factor of $\frac{1}{4}$

$\boxed{f(x) = -\frac{1}{4}\sqrt{-x}}$

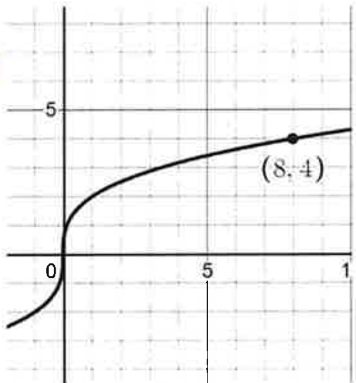
8. Use the graph of $f(x) = x^{\frac{1}{3}}$ to write an equation for each function whose graph is shown. Each transformation includes only reflections or expansions/compressions.

a)

Normal (8,2)

$(8,2) \rightarrow (8,4)$

vertical expansion by factor of 2



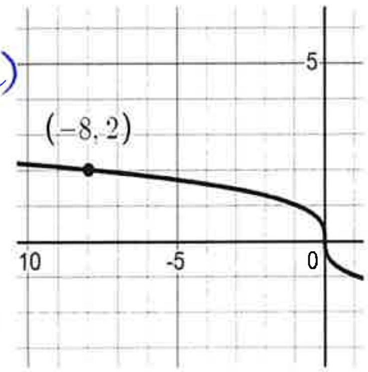
$f(x) = 2x^{\frac{1}{3}}$

b)

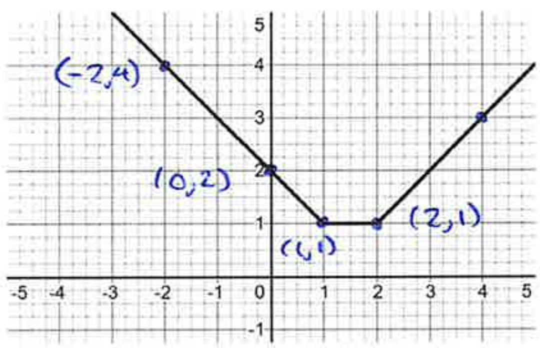
Normal: (8,2)

Reflection in y-axis

$f(x) = (-x)^{\frac{1}{3}}$

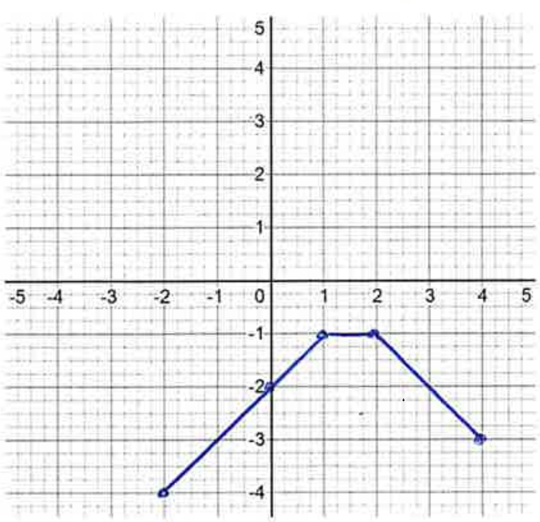


9. Given the graph of $f(x)$ below, sketch the graphs of the following:

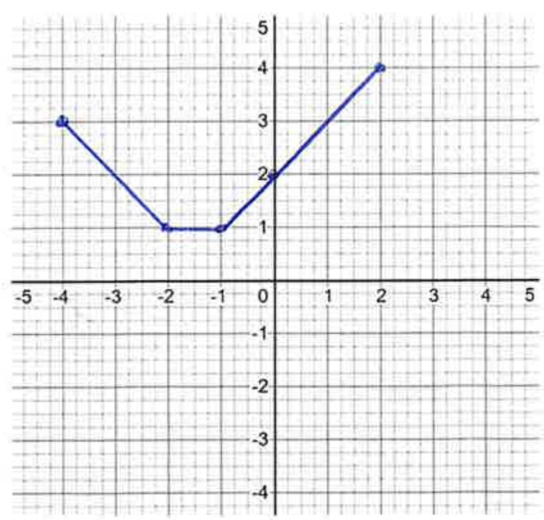


(4,3)

a) $y = -f(x)$ reflects y-values

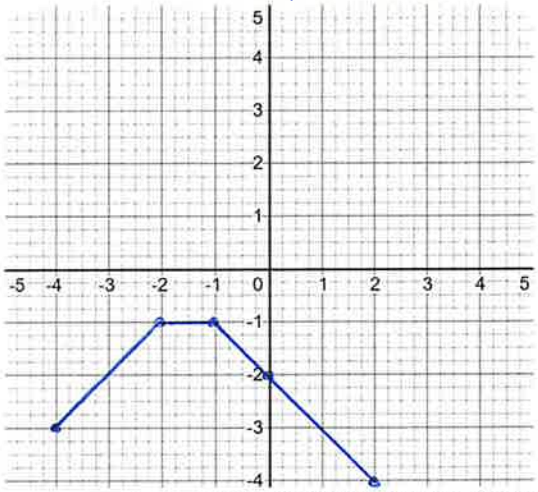


b) $y = f(-x)$ reflects x-values



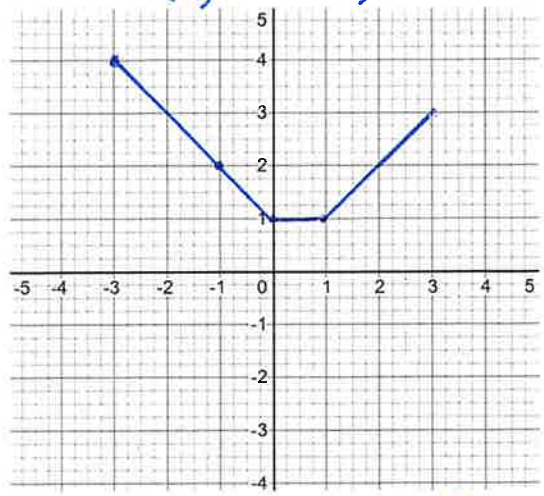
c) $y = -f(-x)$

$(4,3) \to (-4,-3)$
$(-2,4) \to (2,-4)$
$(0,2) \to (0,-2)$
$(1,1) \to (-1,-1)$
$(2,1) \to (-2,-1)$



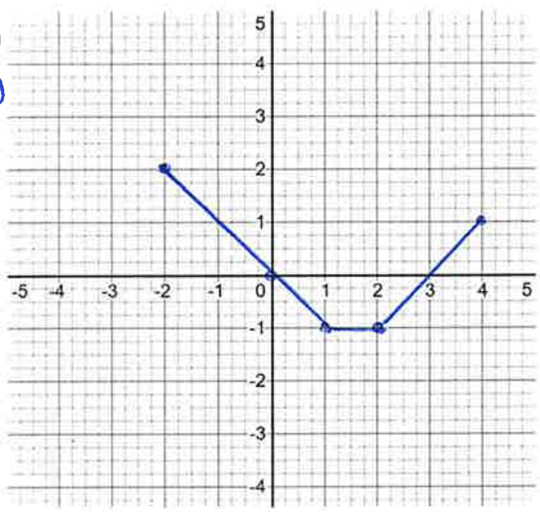
d) $y = f(x+1)$

$(-2,4) \to (-3,4)$
$(0,2) \to (-1,2)$
$(1,1) \to (0,1)$
$(2,1) \to (1,1)$
$(4,3) \to (3,3)$



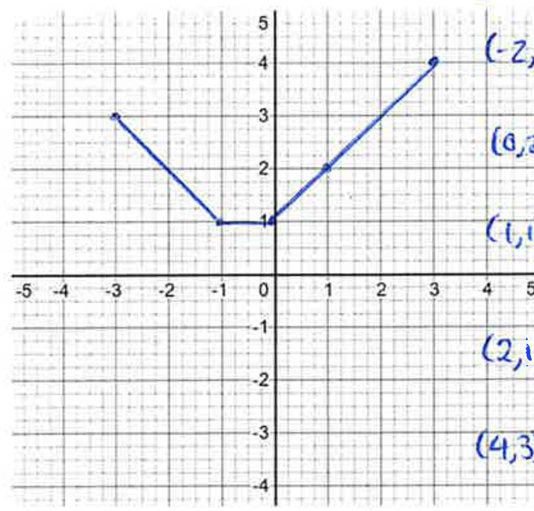
e) $y = f(x) - 2$

$(4,3) \to (4,1)$
$(-2,4) \to (-2,2)$
$(0,2) \to (0,0)$
$(1,1) \to (1,-1)$
$(2,1) \to (2,-1)$



f) $y = f(1-x) \to f(-x+1) \to f[-1(x-1)]$

$(-2,4) \to (2,4) \to \boxed{(3,4)}$
$(0,2) \to (0,2) \to \boxed{(1,2)}$
$(1,1) \to (-1,1) \to \boxed{(0,1)}$
$(2,1) \to (-2,1) \to \boxed{(-1,1)}$
$(4,3) \to (-4,3) \to \boxed{(-3,3)}$



10. If $(-2,4)$ is a point on the graph of $y = f(x-1)$, what must be a point on the following graphs?

a) $y = f(x)$

we have had a transformation one unit right so tracks back to

$(-2,4) \to \boxed{(-3,4)}$

b) $y = -f(x)$

From 10a) we then reflect the y-value

$(-3,4) \to \boxed{(-3,-4)}$

c) $y = f(-x)$

From 10a then reflect x-value

$(-3,4) \rightarrow \boxed{(3,4)}$

d) $y = f(x) + 2$

From 10a then up two units

$(-3,4) \rightarrow \boxed{(-3,6)}$

e) $y = f(x + 2)$

From 10a then left 2 units

$(-3,4) \rightarrow \boxed{(-5,4)}$

f) $y = -f(-x)$

From 10a then reflect both values

$(-3,4) \rightarrow \boxed{(3,-4)}$

11. What is the range of the Absolute Value Function: $f(x) = |4 - x^2|$

Absolute Value cannot have negative y-values so $\boxed{y \geq 0}$

12. If the point $(-1, -2)$ is on the graph $y = f(x)$, what point is on the graph $y = |f(-x)|$?

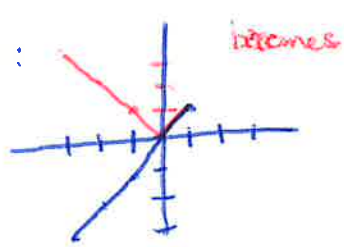
Reflect x-value and absolute value y-value

$(-1,-2) \rightarrow (1,-2) \rightarrow \boxed{(1,2)}$

13. If the range of $y = f(x)$ is $-3 \leq y \leq 1$, what is the range of $y = |f(x)|$?

All negative value become positive

consider:



$\boxed{\text{range: } 0 \leq y \leq 3}$

absolute value

vertical shift 1 unit

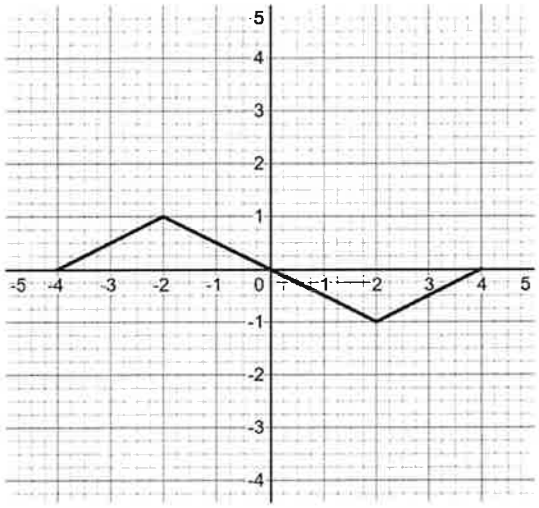
14. If the point $(-3, -6)$ is on the graph $y = f(x)$, what point is on the graph $y = 3|f(x)| + 1$?

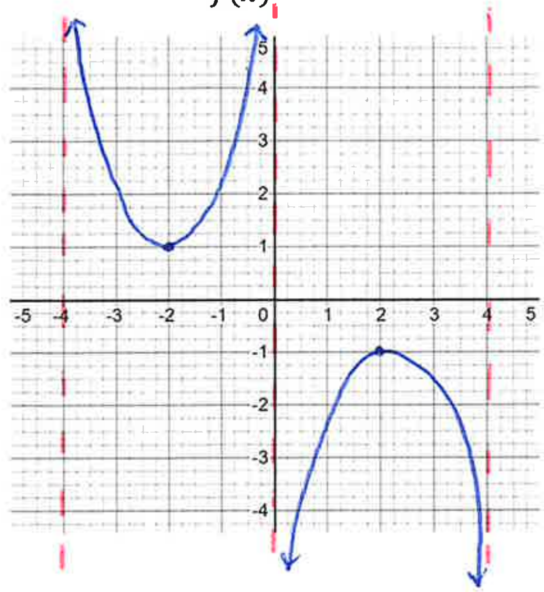
$(-3,-6) \rightarrow (-3,6) \rightarrow (-3,18) \rightarrow \boxed{(-3,19)}$

vertical stretch factor of 3

15. Given the graph of $y = f(x)$, graph the reciprocal function $y = \dfrac{1}{f(x)}$

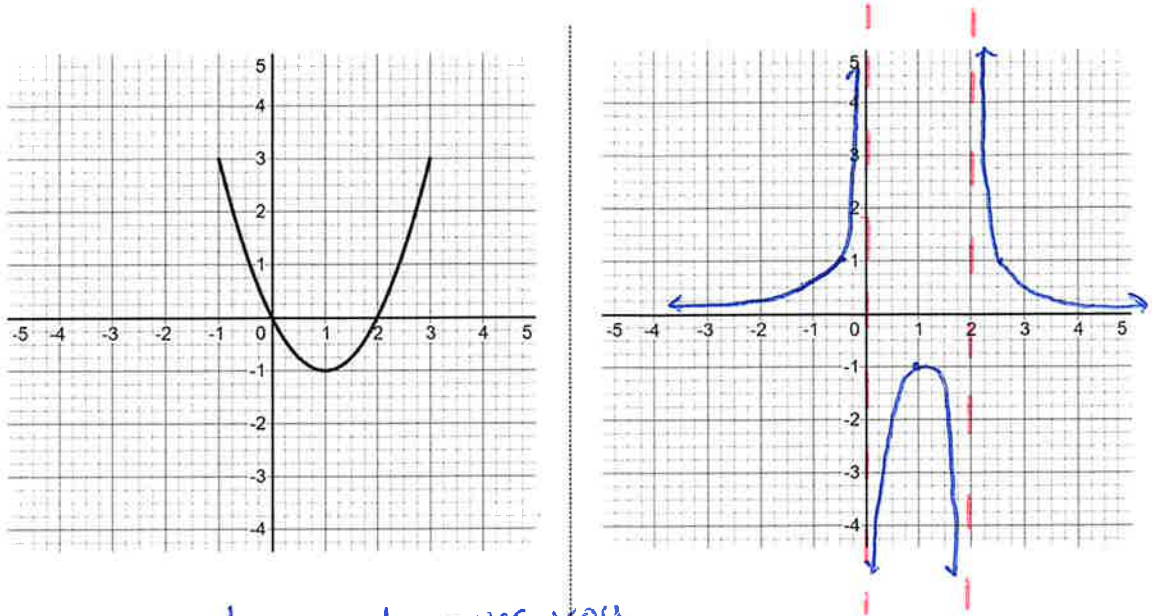
a)





- outputs get written as denominators.
- any y=0 value is a vertical asymptote.

b)



anywhere $y=1$ gives you
an invariant point
(same point on both the original and reciprocal)

16. If $f(x) \geq 1$, what is the reciprocal function $\frac{1}{f(x)}$ value?

If $f(x) \geq 1$ then all outputs are positive numbers
gives all reciprocal values as proper fractions.

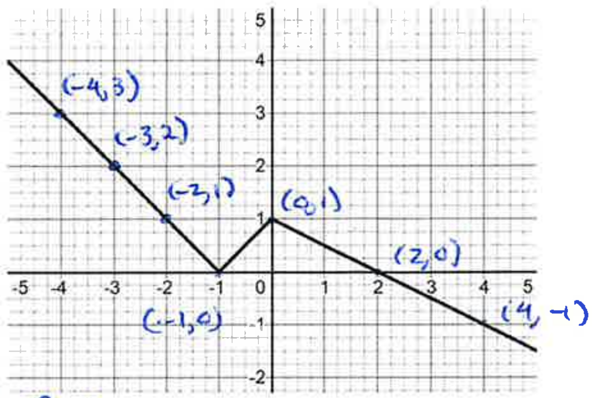
$$\boxed{0 < \frac{1}{f(x)} \leq 1}$$

17. If the graph of $y = f(x)$ has the restriction of $0 < f(x) \leq 1$, what are the restrictions of $y = \frac{1}{f(x)}$?

If $f(x)$ is a proper fraction, then $\frac{1}{f(x)}$ is always
a real number greater than one

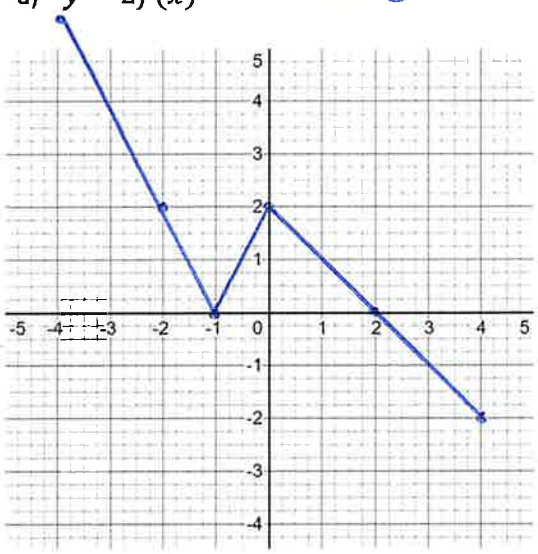
$$\boxed{\frac{1}{f(x)} \geq 1}$$

18. Given the graph of $f(x)$ below, sketch the graphs of the following:



vertical stretch factor of 2

a) $y = 2f(x)$



$(-4,3) \rightarrow (-4,6)$

$(-2,1) \rightarrow (-2,2)$

$(-1,0) \rightarrow (-1,0)$

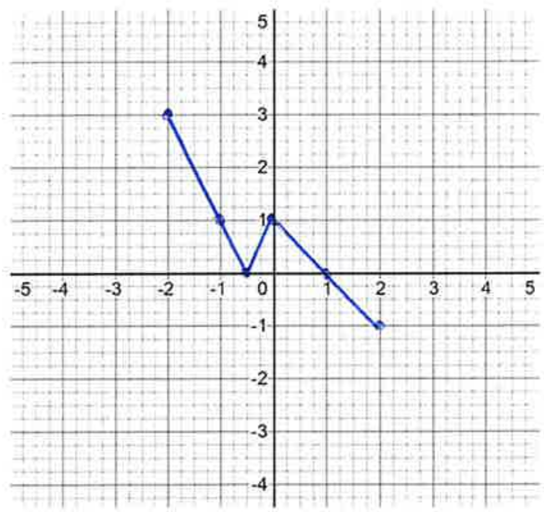
$(0,1) \rightarrow (0,2)$

$(2,0) \rightarrow (2,0)$

$(4,-1) \rightarrow (4,-2)$

horizontal compression by factor of $\frac{1}{2}$

b) $y = f(2x)$



$(-4,3) \rightarrow (-2,3)$

$(-2,1) \rightarrow (-1,1)$

$(-1,0) \rightarrow (-\frac{1}{2},0)$

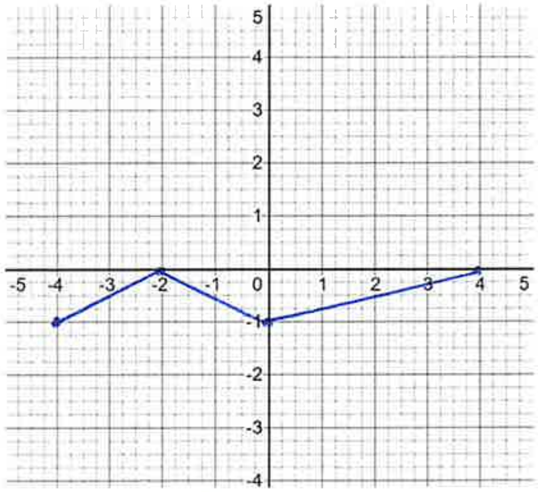
$(0,1) \rightarrow (0,1)$

$(2,0) \rightarrow (1,0)$

$(4,-1) \rightarrow (2,-1)$

c) $y = -f\left(\frac{x}{2}\right)$

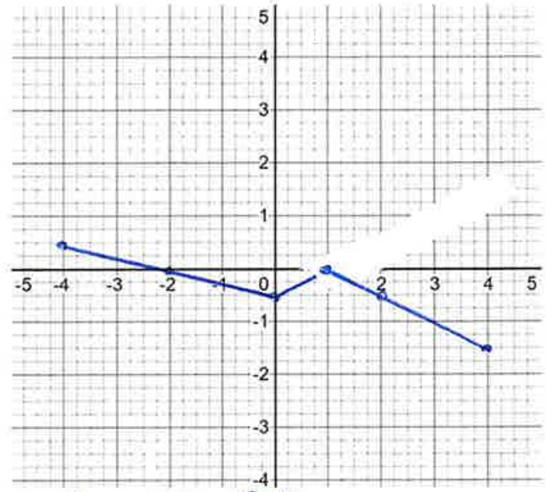
Reflection in y
Horizontal stretch factor 2



(−4,3) → (−8,−3)
(−2,1) → (−4,−1)
(−1,0) → (−2,0)
(0,1) → (0,−1)
(2,0) → (4,0)
(4,−1) → (8,1)

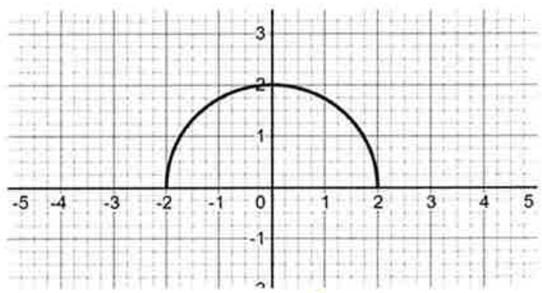
d) $y = -\frac{1}{2}f(-x)$

vertical compression and reflection
x-value reflection



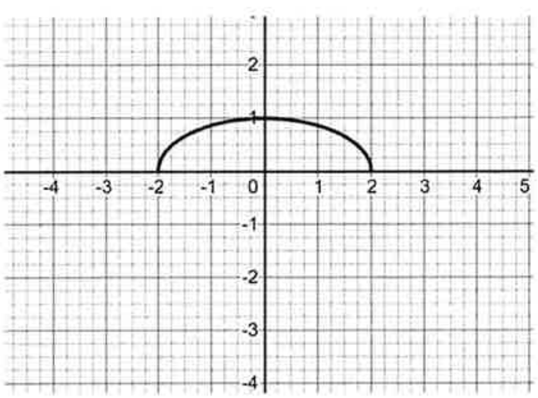
(−4,3) → (4,−3/2)
(−2,1) → (2,−½)
(−1,0) → (1,0)
(0,1) → (0,−½)
(2,0) → (−2,0)
(4,−1) → (−4,½)

19. Given the graph of $f(x)$ below, what equations represent the following graphs



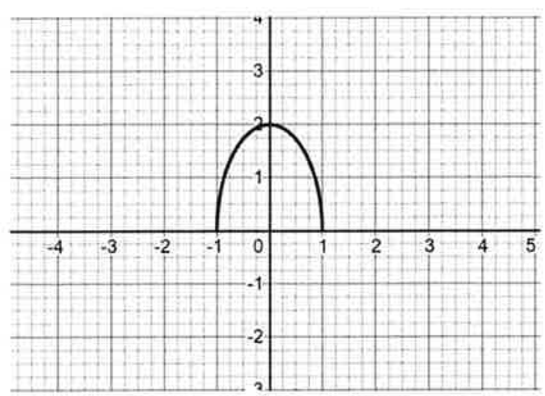
x-values stayed y-value compressed

a) $y =$ ½f(x)



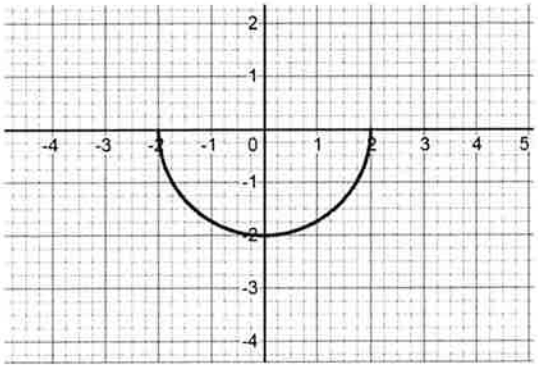
x-values compressed

b) $y =$ f(2x)



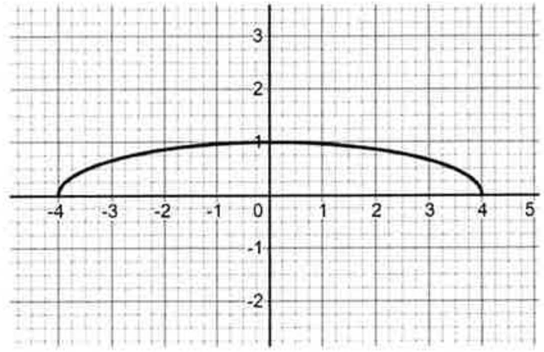
Reflection of y-values

c) $y = -f(x)$

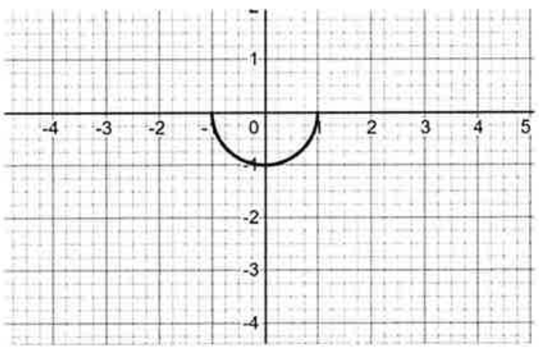


x-values and y-values change

d) $y = \frac{1}{2}f(\frac{1}{2}x)$

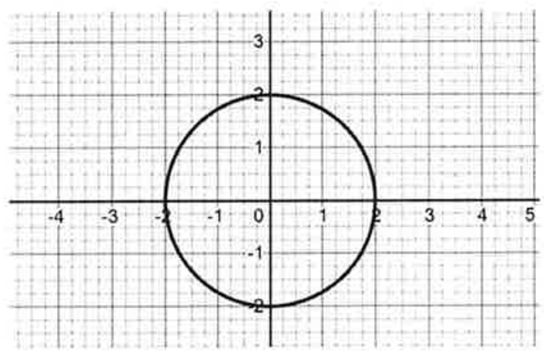


e) $y = -\frac{1}{2}f(2x)$



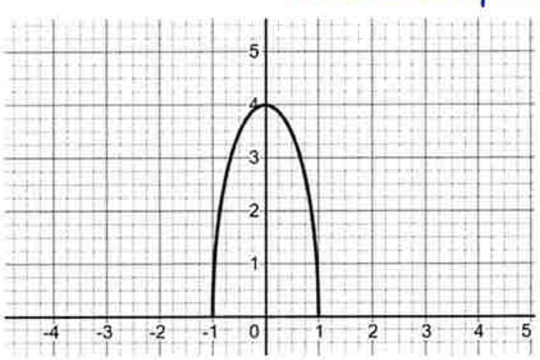
Reflection and double compression

f) $y = \pm f(x)$

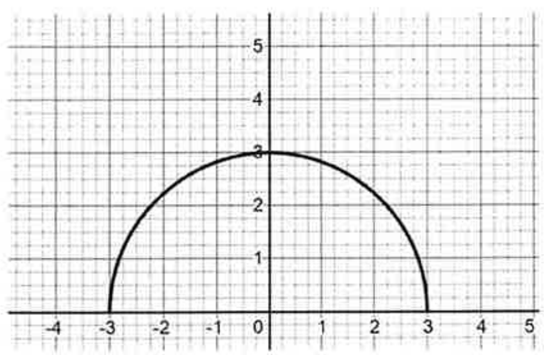


both f(x) and $-f(x)$ present

g) $y = 2f(2x)$ vertical stretch horizontal compression



h) $y = \frac{3}{2}f(\frac{2}{3}x)$



y-value went from $2 \rightarrow 3$

$2y = 3$ $\quad y = \frac{3}{2}$

x-value from $2 \rightarrow 3$ $\quad 2x = 3$ $\quad x = \frac{3}{2}$ but reciprocal

See Website for Detailed Answer Key