Section 2.4 – Multiplying and Dividing Radical Expressions

This booklet belongs to:______________________________Block: ______

- When we multiply radicals, the coefficients multiply together and the radicals multiply together
- The process brings back the general concept of multiplication, **Order Doesn't Matter**

Example: Multiply $2\sqrt{6}\cdot 5\sqrt{3}$

Solution: $2\sqrt{6}\cdot 5\sqrt{3} = 2\cdot 5\cdot\sqrt{6}\cdot\sqrt{3}$

$= 10\sqrt{18}$

$= 10\sqrt{9\cdot 2} \quad\rightarrow\quad 10\cdot 3\cdot\sqrt{2} = \mathbf{30\sqrt{2}}$

Example: Multiply $-3\sqrt{2x}\cdot 4\sqrt{3x},\ x\geq 0$

Solution: $-3\sqrt{2x}\cdot 4\sqrt{3x} = -3\cdot 4\cdot\sqrt{2x}\cdot\sqrt{3x}$

$= -12\sqrt{6x^2} \quad = \quad \mathbf{-12x\sqrt{6}}$

- Next well look at multiplying two term expressions
- There are two methods for these, **Waterbombing** each term (the **DISTRIBUTIVE METHOD**)
- Or
- FOIL (just like Polynomials)

Example: Multiply $(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+\sqrt{2})$

Solution: By *Distributive Method*

$$(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+\sqrt{2}) = (2\sqrt{3}-3\sqrt{2})(2\sqrt{3}) + (2\sqrt{3}-3\sqrt{2})(\sqrt{2})$$

$$(2\sqrt{3})(2\sqrt{3}) - (3\sqrt{2})(2\sqrt{3}) + (2\sqrt{3})(\sqrt{2}) - (3\sqrt{2})(\sqrt{2}) \quad\rightarrow\quad 12 - 6\sqrt{6} + 2\sqrt{6} - 6 = \mathbf{6-4\sqrt{6}}$$

Solution: By *FOIL (First, Outside, Inside, Last)*

$$(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+\sqrt{2}) = \underbrace{(2\sqrt{3})(2\sqrt{3})}_{First} + \underbrace{(2\sqrt{3})(\sqrt{2})}_{Outside} + \underbrace{(-3\sqrt{2})(2\sqrt{3})}_{Inside} + \underbrace{(-3\sqrt{2})(\sqrt{2})}_{Last}$$

$$12 + 2\sqrt{6} + (-6\sqrt{6}) + (-6) \quad\rightarrow\quad 12 - 6 + 2\sqrt{6} - 6\sqrt{6} = \mathbf{6-4\sqrt{6}}$$

Note: You can only use FOIL with two term expressions, anything else use the **DISTRIBUTIVE METHOD**

Multiplying and Dividing with different root indexes (square root vs cube root and so on)

- The most straightforward approach is to convert all radical expressions to exponential notation

Remember that: $\sqrt[b]{n^a} = n^{\frac{a}{b}}$

Simplify $\sqrt[4]{x^2}, x \geq 0$

$$\sqrt[4]{x^2} = x^{\frac{2}{4}}$$
$$= x^{\frac{1}{2}}$$
$$= \sqrt{x}$$

Multiply $\sqrt{x^3} \cdot \sqrt[3]{x}$

$$\sqrt{x^3} \cdot \sqrt[3]{x} = x^{\frac{3}{2}} \cdot x^{\frac{1}{3}}$$
$$= x^{\frac{3}{2}+\frac{1}{3}} = x^{\frac{9}{6}+\frac{2}{6}}$$
$$= x^{\frac{11}{6}} = \sqrt[6]{x^{11}}$$
$$= \sqrt[6]{x^6 \cdot x^5}$$
$$= x\sqrt[6]{x^5}$$

Divide $\frac{\sqrt{x^3}}{\sqrt[3]{x}}$

$$\sqrt{x^3} \div \sqrt[3]{x} = x^{\frac{3}{2}} \div x^{\frac{1}{3}}$$
$$= x^{\frac{3}{2}-\frac{1}{3}} = x^{\frac{9}{6}-\frac{2}{6}}$$
$$= x^{\frac{7}{6}} = \sqrt[6]{x^7}$$
$$= \sqrt[6]{x^6 \cdot x^1}$$
$$= x\sqrt[6]{x}$$

Rationalizing the denominator

- If you have a radical in the denominator it isn't what we consider a rational expression
- This is the case because a radical (that is not a perfect square) is an irrational number
- In order to rationalize the denominator we have to find an equivalent fraction (back to grade 9)
- So we have to multiply the numerator and denominator by the same thing
- Be careful if the denominator is a cube root or higher, it takes a little more thought

Example: Simplify $\sqrt{\frac{2}{7}}$

Solution: $\sqrt{\frac{2}{7}} = \frac{\sqrt{2}}{\sqrt{7}}$

$$\frac{\sqrt{2}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$
$$\frac{\sqrt{14}}{7}$$

- We multiply by 1 in the form of $\frac{\sqrt{7}}{\sqrt{7}}$
- So the appearance changes
- But the value stays the same

- The next example involves denominators with higher roots watch closely

Example: Simplify $\sqrt[3]{\frac{2}{y}}$

Solution:

$$\sqrt[3]{\frac{2}{y}} = \frac{\sqrt[3]{2}}{\sqrt[3]{y}}$$

$$\frac{\sqrt[3]{2}}{\sqrt[3]{y}} \cdot \frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}$$

$$\frac{\sqrt[3]{2y^2}}{y}$$

- We multiply by 1 in the form of $\frac{\sqrt[3]{y^2}}{\sqrt[3]{y^2}}$
- Since we are dealing with a **CUBE ROOT**
- We need the a **perfect cube** in the **denominator**
- So that why its y^2 under the radical
- If it was a $4th\ root$ we would need a power of 4
- And the pattern continues

Using Conjugates to Rationalize

- Conjugates look like this: $(a + b)\ and\ (a - b)$
- When you **multiply** these together you end up with $(a^2 - b^2)$ a **difference of squares**
- And if you think about radicals, since you're **left with squares**, the **radicals** will **multiply out**
- Leaving us with a RATIONAL denominator!
- Again we are using **FOIL!**

Example: Simplify $\frac{3}{2-\sqrt{5}}$

Solution:

$$\frac{3}{2-\sqrt{5}} = \frac{3}{2-\sqrt{5}} \cdot \frac{2+\sqrt{5}}{2+\sqrt{5}} \quad \rightarrow \quad \frac{3(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{3(2+\sqrt{5})}{4-5} \quad \rightarrow \quad \frac{3(2+\sqrt{5})}{-1} = \mathbf{-3(2+\sqrt{5})}$$

Example: Simplify $\frac{\sqrt{x}-2}{\sqrt{x}+1}$

Solution:

$$\frac{\sqrt{x}-2}{\sqrt{x}+1} = \frac{\sqrt{x}-2}{\sqrt{x}+1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}-1} \quad \rightarrow \quad \frac{(\sqrt{x}-2)(\sqrt{x}-1)}{(\sqrt{x}+1)(\sqrt{x}-1)} = \frac{(\sqrt{x}-2)(\sqrt{x}-1)}{x-1} \quad \rightarrow \quad \frac{x-2\sqrt{x}-\sqrt{x}+2}{x-1}$$

$$= \mathbf{\frac{x-3\sqrt{x}+2}{x-1}}$$

- These procedures really relies of the concepts of **Binomial Multiplication** we learned last year

Section 2.4 – Practice Problems

Find the product and simplify the result.

1. $\sqrt{2x}\left(\sqrt{2}-\sqrt{x}\right)$
2. $\sqrt{7y}\left(\sqrt{y}+\sqrt{7}\right)$
3. $\left(2x-\sqrt{3}\right)\left(2x+\sqrt{3}\right)$
4. $\left(2x-\sqrt{3}\right)\left(2x-\sqrt{3}\right)$
5. $\left(\sqrt{x+2}\right)^2$
6. $\left(\sqrt{x}+2\right)^2$
7. $\left(\sqrt{x-3}-4\right)^2$
8. $\left(\sqrt{x-3}-4\right)\left(\sqrt{x-3}+4\right)$

9. $\left(3\sqrt{x}+\sqrt{y}\right)^2$

10. $(\sqrt{x}+3\sqrt{6})(\sqrt{x}-3\sqrt{6})$

Simplify the following

11. $\sqrt{x}\cdot\sqrt[3]{x}$

12. $\frac{\sqrt{x}}{\sqrt[3]{x}}$

13. $\sqrt{x^3}\cdot\sqrt[5]{x^2}$

14. $\frac{\sqrt{x^3}}{\sqrt[5]{x^2}}$

15. $\sqrt[4]{a^3}\cdot\sqrt[3]{a^2}$

16. $\frac{\sqrt{ab^3}}{\sqrt[5]{a^2b^3}}$

17. $\sqrt{16x^3y^3} \cdot \sqrt[3]{8xy^2}$

18. $\dfrac{\sqrt[4]{x^2y^3}}{\sqrt{xy}}$

Simplify, if possible.

19. $\sqrt{2} + \sqrt{5}$

20. $\sqrt{2} \cdot \sqrt{5}$

21. $\sqrt{6} - \sqrt{3}$

22. $\dfrac{\sqrt{6}}{\sqrt{3}}$

23. $\sqrt{3} - 2\sqrt{3}$

24. $\sqrt{3} \cdot 2\sqrt{3}$

25. $\frac{\sqrt{3}}{4\sqrt{3}}$

26. $\frac{\sqrt{3}}{4\sqrt{2}}$

Rationalize the denominator

27. $\frac{1}{\sqrt{2}}$

28. $\frac{1}{\sqrt[3]{2}}$

29. $\frac{3+\sqrt{2}}{\sqrt{2}}$

30. $\frac{5-\sqrt{2}}{\sqrt{3}}$

31. $\frac{1}{3+\sqrt{2}}$

32. $\frac{1}{3-\sqrt{2}}$

33. $\frac{\sqrt{12}}{\sqrt{3}+1}$

34. $\frac{\sqrt{18}}{\sqrt{2}-1}$

35. $\frac{3+\sqrt{2}}{1+\sqrt{2}}$

36. $\frac{\sqrt{5}}{\sqrt{2}-\sqrt{3}}$

37. The volume of a cone is: $V = \frac{1}{3}\pi r^2 h$. If the volume of a cone is $18\pi cm^3$ and the height is $6cm$, what is the radius?

38. The volume of a sphere is: $V = \frac{4}{3}\pi r^3$. If the volume of a sphere is $36cm^3$, what is the radius?

Answer Key – Section 2.4

1.	$2\sqrt{x}-x\sqrt{2}$
2.	$y\sqrt{7}+7\sqrt{y}$
3.	$4x^2-3$
4.	$4x^2-4x\sqrt{3}+3$
5.	$x+2$
6.	$x+4\sqrt{x}+4$
7.	$x-8\sqrt{x-3}+13$
8.	$x-19$
9.	$9x+6\sqrt{xy}+y$
10.	$x-54$
11.	$\sqrt[6]{x^5}$
12.	$\sqrt[6]{x}$
13.	$x\sqrt[10]{x^9}$
14.	$x\sqrt[10]{x}$
15.	$a\sqrt[12]{a^5}$
16.	$\sqrt[10]{ab^9}$
17.	$8xy^2\sqrt[6]{x^5y}$
18.	$\sqrt[4]{y}$
19.	$Not\ Possible$

20.	$\sqrt{10}$
21.	$Not\ Possible$
22.	$\sqrt{2}$
23.	$-\sqrt{3}$
24.	6
25.	$\frac{1}{4}$
26.	$\frac{\sqrt{6}}{8}$
27.	$\frac{\sqrt{2}}{2}$
28.	$\frac{\sqrt[3]{4}}{2}$
29.	$\frac{3\sqrt{2}+2}{2}$
30.	$\frac{5\sqrt{3}-\sqrt{6}}{3}$
31.	$\frac{3-\sqrt{2}}{7}$
32.	$\frac{3+\sqrt{2}}{7}$
33.	$3-\sqrt{3}$
34.	$6+3\sqrt{2}$
35.	$2\sqrt{2}-1$
36.	$-\sqrt{10}-\sqrt{15}$
37.	$r=3cm$
38.	$r=\frac{3\sqrt[3]{\pi^2}}{\pi}cm$

Extra Work Space